YOUNG CHILDREN’S USE OF SUBJECTIVE THINKING IN RESPONSE TO PROBABILISTIC TASKS

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ABSTRACT

This paper reports on a study that investigated young children’s responses to a range of probabilistic tasks. A central aspect of the study was our examination of the children’s use of subjective thinking. Most research that has been conducted in relation to young children’s probabilistic thinking has focused on the extent to which young children can identify the most and least likely outcome of experiments. There is, however, limited research into the types of judgements children use when making these identifications. For example, while a small number of studies have reported on children’s use of subjective thinking, there is an absence of research focusing on the role of subjectivity and the range of beliefs on which these judgements are based. In this research, the subjective thinking of children aged 5–6 years in Ireland was examined to address this gap in current knowledge. The data were collected through task-based group interviews and analysed using thematic analysis. Results suggest that a range of personal beliefs and experiences influence young children’s probabilistic thinking including the physical position of objects, personal affinity for one possible outcome, a desire to win, and the influence of previous experiments.

Keywords: *Statistics education research; Probabilistic thinking; Subjective thinking; Task-based group interviews; Early childhood; Kindergarten*

1. INTRODUCTION

The probability of an event relates to the likelihood of an event occurring and an understanding of this aspect of probability enables individuals to assess the likelihood of various events (Nikiforidou, 2018). The study described in this paper aimed to examine the types of probabilistic thinking used by young children when asked to judge the probability of an event. The purpose of the research was to investigate the extent to which the children who took part in the study used subjective thinking when determining the probability of an event, and to examine the range of beliefs and ideas on which these subjective judgements were based. Limited attention has been given to the influence of subjective thinking on probabilistic judgements and we aim to address this gap in current knowledge. Throughout this paper, the term *probabilistic thinking* is used to describe young children’s thinking in response to any probabilistic task. Although children may not always use probabilistic thinking when responding to probabilistic situations, all thinking about probability can be interpreted as evidence of development towards probabilistic thinking (Jones et al., 1999). The term *young children* refers to children aged 3 to 8 years.

2. LITERATURE REVIEW

Probability is not a new mathematical concept. As such, this literature review includes recent research studies along with literature that gives a historical perspective. We draw largely on the work of international researchers due to the absence of research that has been undertaken into probabilistic thinking in Ireland, particularly relating to young children.

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2.1. PROBABILITY AND YOUNG CHILDREN

Piaget and Inhelder (1975, first published in 1951) are widely recognised as the first researchers to study the development of probabilistic thinking in children, and their research paved the way for further research into this area (Nikiforidou, 2018). They concluded that children's ability to engage in probabilistic thinking is linked to their cognitive development and that the systematic understanding of probability commences between the ages of 9 and 12 years (Piaget & Inhelder, 1951/1975). The results of a number of subsequent studies have echoed Piaget and Inhelder's finding that young children have a limited understanding of probability as it is developmental in nature (e.g., Davies 1965; HodnikČadež & Škrbec, 2011). Several researchers, however, have since presented research findings to support the contention that children possess basic notions of probability from a young age, thereby refuting the position of Piaget and Inhelder (e.g., Nikiforidou et al., 2013). Groth et al. (2019) argued that while recent research strongly points to the potential of young children to engage in probabilistic thinking, a stronger theoretical base is required to understand *how* young children learn and develop understandings about this complex topic.

2.2. TYPES OF PROBABILISTIC THINKING

Jones et al. (1997) conducted a two-year study of children's probabilistic thinking and postulated that four types of probabilistic thinking exist: subjective, transitional, informal quantitative, and numerical. Nikiforidou (2018) described this study as pioneering research, and we draw on her summary of each of these forms of thinking. Subjective thinking involves making intuitive judgements based on imagination and personal preferences. Transitional thinking refers to inflexible attempts to begin to quantify probabilities. Informal quantitative thinking describes the use of more generative strategies in quantifying probabilities. Finally, numerical thinking relates to the use of valid numerical measures to describe probabilities. Jones et al. (1997) presented these types of probabilistic thinking as levels through which children progress as they develop more complex understandings of probability. In the absence of a more recent framework for describing the development of young children's probabilistic thinking (Supply et al., 2020), empirical studies in the field of probability have continued to draw on this framework since it was first established over twenty years ago (e.g., Groth et al., 2019; Nikiforidou, 2018; Nikiforidou & Pange, 2010). The current study used the work of Jones et al. (1997) to examine the types of thinking used by young children when making probabilistic judgements. The central focus of this paper is to examine the children's use of subjective thinking. We next examine this type of thinking and its influence when making probabilistic judgements.

2.3. SUBJECTIVE THINKING

Probability can be viewed from classical, frequentist, and subjective perspectives (Estrada et al., 2018; Kazak & Leavy, 2018). The subjective approach represents a perspective of probability in which probability is interpreted as "'a degree of belief', where biases, heuristics and intuitions interplay" (Nikiforidou, 2018, p. 22). Limited research has been conducted into young children's conception of probability from the subjective approach (Kazak, 2015). This narrow research base, however, has produced conflicting opinions pertaining to the benefits of this approach to probability. Threlfall (2004) described the subjective approach as an inappropriate form of judgement that leads to decisions that are based on feelings rather than on analysis. In contrast, more recent research has indicated young children's capability to use their personal beliefs when evaluating the probability of simple events and highlighted that this approach should not be viewed as an obstacle to learning as subjective judgements can be developed into more powerful ideas through engaging with appropriate probabilistic situations (Huber & Huber, 1987; Kazak & Leavy, 2018).

A distinction must be made between the *subjective approach* and *subjective thinking*. While the subjective approach represents one of three perspectives of probability, subjective thinking can be used to inform and justify probabilistic judgements in situations involving theoretical probability, often prior to the development of quantitative thinking (Jones et al., 1997). Building on the work of Jones et al., Groth et al. (2019) postulated that this early notion of probabilistic thinking may vary from one individual to another and is often dominated by egocentrism (Groth et al., 2019). Further research is required to investigate how children may be supported in developing more robust understandings, and thereby move away from subjectivity and towards quantitative judgements (Kafoussi, 2004). Kafoussi indicated that supporting children in understanding the notion of randomness may enhance children's probabilistic thinking and support them in this progression from subjective judgements to quantification. Kafoussi's findings resonate with those of a study by Nikiforidou and Pange (2010), who indicated that a link may exist between the use of subjective judgements and a lack of understanding of the process of randomisation. Providing opportunities for children to reflect on their personal beliefs and to compare these subjective judgements to formal probabilistic concepts has also been identified as an important stage in the development of young children's probabilistic thinking (Helmerich, 2015). Through this process of reflection, children may begin to recognise when and how to mobilise subjective thinking.

The research reported in this paper aims to explore children's use of subjective thinking, as defined by Jones et al. (1997), when identifying the most and least likely outcome of an experiment. The research also examines the impact of subjectivity on their understanding of likelihood. Previous research has been conducted in which particular factors influencing children's subjective thinking in response to probabilistic situations have been presented. For example, Konold (1989) presented the outcome approach and analysed children's responses to four problems, examining children's thinking for outcome-oriented responses. No study to date, however, has synthesised these subjective influences and presented a complete analysis of the factors upon which the children based their subjective judgements.

3. METHODS

3.1. PARTICIPANTS

This study was conducted in a school in a middle-class urban area where one of the authors was teaching. A sample of 16 children, comprising an equal number of boys and girls, were randomly selected from a Senior Infant class. Children at this class level are typically in their second year of primary education and aged 5 to 6 years. The participating children had received no formal probabilistic instruction prior to their engagement in this study as children in Ireland are not introduced to the topic of probability until Third class (children at this class level are typically 8 to 9 years old). The children were interviewed in groups of four in a room in the school with which they were familiar, away from the busy classroom environment. Informed consent was sought and received from the principal of the school in which the study took place and from the parents of the children involved. This was achieved through providing detailed information pertaining to the study to all relevant parties and a meeting was facilitated during which opportunities were provided for questions to be asked. Subsequently, details relating to the study were discussed with the children in an age-appropriate way and a child-friendly visual form consisting of dichotomous questions, phrased using simple language, and accompanied by images was used to seek their assent. We were aware that the interview presented a new situation for the children and steps were taken to ensure they felt safe revealing their thinking in this new setting. The interviews were conducted by the lead author who was the children's teacher. We were cognisant that teacher-led studies can heighten the power imbalance between researcher and participant, which can result in children agreeing to partake in activities that they might not feel comfortable with or viewing the interview as a test situation (Alderson, 2004; Department of Children and Youth Affairs, 2012). Throughout this study, the children were reminded there would be no negative consequences should they wish to withdraw at any point, and they were praised for their efforts throughout the interview.

3.2. DATA COLLECTION

Ginsburg (2016) posited that young children possess a system of mathematical thinking that needs to be recognised and understood. Assessing young children's thinking presents inherent challenges however (Lee, 2013). The most commonly used measures in the area of early mathematics development are often lengthy and unsystematic or short and inadequate (Weiland et al., 2012). In particular, pencil-and-paper tests are widely considered an inappropriate assessment tool for use with young children (e.g., Ellerton & Clements, 1995; Maher & Sigley, 2020). The reading and writing involved in completing a written test may result in children underperforming and may limit the insight provided into their thinking (Cheeseman & McDonough, 2013). Additionally, children may encounter difficulties interpreting questions, symbols, and diagrams (Lowrie et al., 2012; Smith et al., 2011). They may also provide a platform from which children could perform well by applying a mechanical approach that they do not understand (Ginsburg, 2009).

Our choice of data collection method, namely task-based group interviews, aimed to address some of the challenges of exploring young children's thinking. Interviewing children about their mathematical thinking enables researchers to look beneath the surface and can reveal strengths and weaknesses in a child's learning that otherwise may go undetected (Ginsburg, 1997). Task-based interviewing involves the interviewer and participants interacting in relation to one or more tasks which are introduced in a pre-planned manner (Goldin, 2000). All tasks used in this study related to the same probability concept, most and least likely, to allow for a fine-grained analysis of the children's thinking. The children were presented with 12 tasks where they were required to identify the most and least likely outcome (see Appendix). The tasks varied in complexity and a range of random generators, including dice, spinners, and bags containing sets of objects with different attributes, were presented.

In this study, task-based interviews were conducted in small groups. Group interviews have been shown to generate richer responses than individual interviews and may provide opportunities for children to share ideas with each other, hear opposing views, and challenge each other's thinking (Littleton & Mercer, 2013). Throughout the study, we also remained cognisant of the potential limitations of group interviews. For example, in a group interview it can be difficult to ascertain if children are sharing their own thoughts or if they are agreeing with the views of others, repeating these ideas with little understanding. We did not analyse children's comments in isolation, tracking their thoughts throughout each task and considering the potential impact of the group on their thinking.

To facilitate the replicability and generalisability of this study, interview scripts were created that explicitly prescribed the tasks to be discussed, listed the questions to be asked, and detailed the sequence to be followed (Goldin, 2000). The majority of the questions were open-ended to provide opportunities for the children to share their thinking and engage in argumentation (Ginsburg, 1997; Makar et al., 2015). The interview scripts also identified additional questions to pose in response to a range of comments that could be made by the children. These contingency questions were particularly useful as they enabled the interviewer to clarify and probe the children's thinking, producing rich data which otherwise may not have been retrieved (Ginsburg, 1997).

Data from the task-based group interviews were drawn upon to generate the most accurate interpretation possible of the children's thinking, acknowledging the lens of the researchers when seeking to interpret children's internal thinking from external representation (Gandhi, 2018). Data sources included children's drawings, photographs of their use of concrete materials, and video recordings of the children's task-based group interviews which facilitated analysis of children's spoken comments along with gestures and facial expressions. In addition, brief field notes were taken during the interviews with more detailed journal entries written immediately after each interview and during initial viewings of the video recordings.

3.3. DATA ANALYSIS

A combined technique of inductive and deductive thematic analysis, termed a hybrid approach by Fereday and Muir-Cochrane (2006), was adopted. Firstly, deductive codes of *subjective*, *transitional, informal quantitative*, and *numerical* were generated from the findings of Jones et al. (1997), as previously discussed. A rubric was utilised to increase the consistency and reliability of this coding process. It was comprised of a list of codes accompanied by code definitions and examples of children's thinking that represented each code. Even though the rubric was modelled primarily on the study by Jones et al. (1997), research linked to this study and conducted by other established researchers in the field also informed its design (e.g., Jones et al., 1999; Polaki et al., 2005; Tarr & Jones, 1997). Second, inductive codes were derived from the data through the process of inductive thematic analysis. While these codes were based on the interpretations of the researchers, regular meetings were held in which emerging codes were discussed. This provided multiple perspectives and decreased researcher bias that can occur if one researcher takes sole responsibility for exploring patterns in the data. The frequency of codes was also examined and connections among these codes were considered. This led to the identification of sub-themes relating to a number of key themes. This paper focuses on the key theme of children's subjective thinking and discusses the sub-themes that emerged from an analysis of the codes that related to subjective thinking. These sub-themes were the physical position of objects, personal affinity for one possible outcome, a desire to win, and the influence of previous experiments.

4. RESULTS

The children's probabilistic thinking was analysed in terms of subjective, transitional, informal quantitative, and numerical thinking using the analytical approach described above. Although a detailed analysis of the children's thinking across each of these types of probabilistic thinking is beyond the scope of this paper, we focus on the children's use of subjective thinking during these interviews. We found that 10 of the 16 children used subjective thinking on at least one occasion during the interviews. We were aware these comments could not be analysed in isolation because many of the children may have been influenced by their peers, as we discuss later. Furthermore, many of the children may have been operating across more than one form of probabilistic thinking. This is true in all areas of mathematics as children are continually learning and may be operating across levels or transitioning between levels (Clements & Sarama, 2014). The most consistent factors upon which the children based their subjective judgements were the physical position of objects, personal affinity for one possible outcome, a desire to win, and the influence of previous experiments. Each of these types of subjective judgements will be unpacked in the sections that follow.

4.1. PHYSICAL POSITION OF OBJECTS

The use of concrete manipulatives is recommended when engaging children in mathematical tasks because they facilitate rich thinking and reasoning by providing opportunities for exploration, manipulation, interaction, discussion, observation, and conceptualisation (Nikiforidou, 2018, 2019; Swan & Marshall, 2010). A range of random generators including dice, spinners, and bags of discrete objects were utilised throughout the tasks (see Appendix). The most common form of subjective reasoning used by the children arose from their engagement with these random generators as they justified their choice of the most/least likely outcome with reference to the physical position of particular items on a number of occasions:

Sarah: Yellow because, probably, all the blues will like be, all the yellows will probably be in the corners and all the blues will be in the inside *(bag of bears).*

Ben: Not really because the six is on the bottom, so it, it's definitely easy to win *(dice)*.

Mark: Because the orange is at the bottom and the spinner, emm, doesn't really go at the bottom *(spinner)*.

This form of subjective reasoning was observed during each of the interviews and was used by seven of the children. Reliance on this form of subjective thinking has been attributed to a difficulty experienced by young children in understanding that a randomising process distributes the position of items (Nikiforidou & Pange, 2010). As such, the child does not recognise the equiprobability of an object taking up various positions, believing that positions are in some way predetermined and that items in certain positions are therefore more likely to occur. For example, during Task 3.3, which involved a bag that contained two red counters and one yellow counter, one group of children discussed the relevance of the position of the counters in the bag. The children were asked to identify the least likely outcome when drawing a counter from the bag and the following conversation ensued:

Teacher: What colour are you least likely to get?
Amy: Yellows.
Emma: Yellow.
Tom: Yellow.
Amy: Yellows.
Teacher: Why?
Emma: Because there's only one yellow and the yellow could be at the bottom or the side.
Amy: Eh, 'cause there's two reds and one yellow. The yellow could be at the bottom and one of the reds could be at the bottom but the other red could be at the top.

Teacher: OK. What do you think Shane?
Tom: It doesn't matter where they are because we always shake the bag.
Shane: I think the yellow because there's only one yellow and there's two reds.

In the extract, Emma and Amy seem to be using both subjective and informal quantitative reasoning to explain why yellow is the least likely outcome. As previously discussed, children often operate across levels and may be in a period of transition as they develop new understandings. However, in building upon Emma's statement, Amy's contribution to the idea is quantitative. Her referral to the position may indicate her subjective thinking but may also be an echo of Emma's idea while her own thinking was guided by the relative quantities of the counters.

Tom disagreed with the reference that Emma and Amy made to the position of the counters in the bag. He seemed to recognise that the position of the counters would not impact their chance of occurrence. In a study by Kafoussi (2004), children made similar references to the process of executing the experiment and these justifications were interpreted as representing an acceptance of the notion of randomness and its impact on the experiment. As previously mentioned, Jones et al. (1997) established a framework for describing children's probabilistic thinking across four levels: subjective, transitional, informal quantitative, and numerical. Within this framework, however, there is no acknowledgement that a child may begin to recognise and understand the notion of randomness prior to quantifying likelihood. Polaki et al. (2005) extended the framework, stating that children exhibiting transitional thinking may acknowledge uncertainty without quantifying it. We contend that this level of probabilistic thinking within the framework of Jones et al. (1997) should be further extended to include an acknowledgement of the impact of randomness on experiments.

In contrast to the other children, Shane did not refer to the position of the counters and based his identification of the least likely outcome on the relative quantities. His views were not influenced by the comments shared by his peers, suggesting that he was confident in his belief that the quantity of each colour impacted their likelihood of occurrence rather than their physical position in the bag. Applying the framework by Jones et al. (1997), his thinking can be classified as representing informal quantitative thinking.

4.2. PERSONAL AFFINITY FOR ONE POSSIBLE OUTCOME

During the interviews, evidence was gathered of children basing their choice of the most and least likely outcome on their personal preferences. These judgements largely related to the children's favourite colour. For example, in Task 1.1, which involved a bag containing five green bears and one red bear, the children were asked to identify the most and least likely outcome. Amy identified red as the most likely outcome, stating that "it's my favourite and it's the same as my uniform." Amy referred to her personal preferences on one other occasion during the interview. In Task 4.2, the children were presented with a spinner that was shaded one-half orange, one-third blue, and one-sixth green. Amy again stated that she didn't know which outcome was most likely as "there's loads more orange, but I don't really like orange." In contrast to the first occasion when Amy allowed her personal preference to dominate her thinking, it appeared that Amy held conflicting views. She seemed to understand that the orange segment was the largest and that this increased its likelihood of occurrence but her preference for the other colours resulted in a reluctance to identify orange as the most likely outcome. A small number of other research studies have also found that children's preferences for certain colours may influence their probabilistic judgements (e.g., Skoumpourdi et al., 2009). Antonopoulos and Zacharos (2013) described the influence of such preferences as a cognitive obstacle that can make it difficult to understand fundamental probabilistic concepts. The findings of our study support this notion that personal preferences can inhibit probabilistic judgements because only three instances when the children's thinking was influenced by their personal preferences were associated with the correct identification of the most or least likely outcome. In contrast, on nine occasions such a preference was mobilised in a context, which led to the misidentification of the most or least likely case.

4.3. DESIRE TO WIN

The children's desire to win appeared to impact their identification of the most and least likely outcome. In three of the 12 tasks, the children were split into two teams and each team was assigned one of two possible outcomes, resulting in one team having a greater chance of winning. Across all interview groups, all children who incorrectly identified the most and least likely outcomes of these tasks were children who were assigned to the team that had the lesser chance of winning. An analysis of each of these incorrect identifications shows that in all cases, following the completion of the task, the children who had initially incorrectly identified their own team as being more likely to win proceeded to correctly identify the most and least likely outcome. These children provided clear justifications for their change in thinking, with reference to quantification, when they were no longer personally associated with the outcome. Although all of these errors cannot be attributed directly to the children's desire to win, the potential relationship is worthy of further research.

One example of a child whose desire to win appeared to influence his identification of the most and least likely outcomes was Mark. For example, Task 2.1 involved a spinner that was split into quarters, with one quarter representing a dog and three-quarters representing cats. Each team was given a pet and when the spinner landed on their pet, they earned a treat. The first team to earn five treats for their pet was the winner. Mark was assigned to the team that had a pet dog and he identified the dog as the most likely outcome of the spinner. He appeared to be preoccupied with the spinner, however, and expressed a desire to change it, informing the interviewer that she should have included two cats and two dogs on the spinner. This highlighted an inconsistency in his thinking because his belief that the dog was the most likely outcome should imply that he was satisfied with the spinner in its existing format. Furthermore, following the game, he stated that the cat was the most likely outcome because the spinner contained three cats and one dog. Mark appeared to draw on both subjective and informal quantitative ideas in justifying his thinking. Antonopoulos and Zacharos (2013) analysed the results of a number of research studies and concluded that as young children's probabilistic thinking develops, prior to abandoning their subjective judgements, they begin to accompany these judgements with reference to quantification.

While our findings suggest that this type of subjective thinking is associated with errors in children's judgements pertaining to the likelihood of an event occurring, further research in the form of case studies would enable a more in-depth analysis to be undertaken. Such case studies would allow the influence of subjective thinking on children's identifications of the most and/or least likely outcome to be understood and described, thereby providing an insight into the extent to which subjective thinking leads to "incoherent" ideas, as described by Hawkins and Kapadia over 35 years ago (1984, p. 374).

4.4. THE INFLUENCE OF PREVIOUS EXPERIMENTS

Many individuals hold the belief that the likelihood of an event occurring is related to the outcomes of previous events (Chiesi & Primi, 2009). In this study three of the 16 children indicated that they held a belief that previous outcomes could have an impact on future events:

Ben: I think I'm, well, going to lose because I landed on a six last time and that means I might land on a six again.

Conor: Because it won last time.

Mark: Emm, lose because I already won.

From the extract, it appears that Ben and Conor exhibited a *positive recency effect* whereby they seemed to hold the belief that a previous outcome is more likely to occur again (Bryant et al., 2018; Chiesi & Primi, 2009). Contrastingly, it appears that Mark exhibited a *negative recency effect* as he believed that because he won previously, he was less likely to win again (Bryant & Nunes, 2012; Chiesi & Primi, 2009). Although the results of previous studies (e.g., Chiesi & Primi, 2009; Shaughnessy, 1992) indicated that the probabilistic reasoning of most children is influenced by previous experiences, the results of this study suggest that young children are less inclined to display recency effects than previously understood because 13 of the 16 participants did not exhibit either of these effects. It must also be noted that the three children who referred to previous outcomes of experiments when explaining their thinking participated in three separate interviews. None of the other children appeared to be influenced by hearing their justifications, suggesting that they may have recognised the independence of events. While these recency effects were not explicitly mentioned in the framework by Jones et al. (1997), therefore, we classified such justifications as reflecting subjective thinking. This decision emerged from the well-established ideas of Kahneman and Tversky (1972) who described these recency effects as subjective because they result in the disregard of consideration of the sample size, regardless of whether they lead to reasonable ideas or estimations. It must be acknowledged that the frequentist probabilistic approach is based directly on this logic of carrying out experiments and considering the results to calculate the probability. Despite the aforementioned probabilistic conceptions such as recency effects, the use of experimentation to determine the likelihood of events can nevertheless be useful, particularly when the experiment can be repeated a large number of times or when the theoretical probability is unknown (Helmerich, 2015).

4.5. MITIGATING FACTORS: INTERACTIONS WITHIN THE GROUP

In many cases, children moved beyond their initial affective responses to express accurate probabilistic reasoning as they progressed through the interview tasks. One aspect of the research setting that facilitated the children in refining their thinking was the interaction among the children. This is not surprising because talk between children has been shown to be valuable for the construction of knowledge because it offers opportunities for children to engage critically and constructively with each other's ideas, leading children to justify their thinking, counter-challenge ideas, and offer alternative hypotheses (Littleton & Mercer, 2013). Our decision to conduct the interviews in small groups emanated from our understanding of the impact that interaction and collective sensemaking can have on children's thinking. It has been reported in studies pertaining to other mathematical constructs that providing children with opportunities to work together can lead to collective sense-making, which can make learning more meaningful than when engaging in tasks in isolation (Rivera & Becker, 2011).

The children in this study actively engaged in discussion during the interviews and interactions among the children appeared to have positive implications on their probabilistic thinking. For example, the following extract from Task 1.3 (drawing a bear from a bag containing four red bears, three green bears, and one purple bear) seemed to show John refining his subjective probability as he interacted with his peers:

Teacher: Which one are you most likely to get?
Daniel: Red, there's four of them.
John: Purple, it's my favourite.
Daniel: There's four reds, three green.
Teacher: Hannah, which colour do you think we're most likely to get?
Hannah: The red.
Teacher: Why?
Hannah: Because there's four of them and there's three greens and one purple.
Daniel: And one purple.
Teacher: OK. Kate, which colour are we most likely to get?
Kate: Red because there's four reds and three greens, well, you have a chance of getting green but you only have a little chance of getting purple, but you have a bit of a lot of chance of getting red because red have most of the bears.
Teacher: John, what do you think?
John: It could be any. Red most likely but it could be any.
Hannah: Yeah mostly red.
John: Yeah, most likely red but if we closed our eyes and just mixed them up it could be any. But red probably because it has four and green has three and purple has one.

The discussion that unfolded among the children appeared to lead to a development in John's probabilistic thinking. John initially stated that he felt that purple was the most likely outcome because purple was his favourite colour. Subsequently, the other children shared the view that red was the most likely outcome due to the number of red bears in the bag. Kate also referred to the uncertainty of probability, stating that all of the outcomes were possible, but that red had the greatest chance of occurrence. John listened to the contributions of the other children and appeared to combine their ideas and develop further their justifications, leading to the refinement of his thoughts. This interaction highlights that enabling children to share and discuss their ideas with others can lead to a development in their thinking (Lampert & Cobb, 2003).

5. DISCUSSION

This research examined young children's use of subjective judgements when engaging in probabilistic tasks. While the findings offered here are consistent with the results of other studies that have reported on the role of subjectivity on young children's probabilistic judgements, this study offers new insights into the range of values and beliefs on which young children base their subjective judgements. In this study, these judgements related to the physical position of objects, personal affinity for one possible outcome, a desire to win, and the influence of previous experiments. The extracts analysed in this paper were chosen to exemplify the influence that each of these factors had on the children's probabilistic thinking, focusing in particular on how they guided their identifications of the most and least likely outcomes of events. Subjective beliefs play an important role in children's initial exploration of probabilistic concepts and it is upon these beliefs that children can build and construct new forms of thinking (Skoumpourdi et al., 2009). Responses to probability tasks that are based solely on personal thoughts and ideas, however, tend to run counter to normative probabilistic thinking (Groth et al., 2019). Therefore, it follows that such judgements often lead to errors, as evidenced in the analysis presented in this paper. Our findings suggest that children need to be supported in reflecting on their subjective judgements and in evaluating their appropriateness in relation to probability tasks.

A number of the participating children referred to both subjective and quantitative judgements simultaneously, highlighting that they were in a period of transition towards making more robust probabilistic judgements. Further research is required to identify ways in which children can be supported in building upon these subjective judgements and in transitioning towards more formal probabilistic judgements based on quantification rather than personal beliefs. Antonopoulos and Zacharos (2013) argued that providing opportunities for young children to engage in probabilistic tasks may have positive implications on their probabilistic understandings and may result in children replacing some of their subjective judgements with increased reference to quantification. The results of our study suggest that children would benefit from exploring probabilistic tasks in small groups as the sharing of ideas among children can provide opportunities for children to learn from each other and to build upon each other's ideas. Such opportunities may also include explicit discussion of subjective thinking to support children in evaluating the factors on which they base their judgements.

6. CONCLUSION

6.1. IMPLICATIONS OF THE STUDY

The results of our study suggest that children need to be supported to reflect on the appropriateness of their subjective judgements in relation to particular tasks. Antonopoulos and Zacharos (2013) argued that providing opportunities for young children to engage in probabilistic tasks may have positive implications on their probabilistic understandings and may result in children replacing some of their subjective judgements with increased reference to quantification. Such opportunities may also include explicit discussion of subjective thinking to support children in evaluating the factors on which they are basing their judgements. This study also illuminated the potential of task-based group interviews as an assessment tool for exploring the probabilistic thinking of young children. We recommend the use of task-based group interviews to gain deep understandings of young children's probabilistic thinking in the classroom and to reveal insights that may not be revealed through the use of other methods of assessment.

6.2. LIMITATIONS OF THE STUDY

Marshall and Rossman (2011) attested that all research projects have limitations. This piece of research was a small-scale study involving 16 participants and its scope was limited by the sample size. The findings of this study are applicable only to the setting in which it took place and similar results can only be expected in another context that is sufficiently congruent to the current setting (Lincoln & Guba, 1985). Consequently, the findings of this study are not representative of the entire population of Senior Infant children in Ireland because the study was conducted in a middle-class school in an urban area. A larger-scale study with a more substantial sample size across a range of school settings would bolster findings and may shed light on other elements relating to the subjective thinking of young children. It must also be acknowledged, however, that the small sample size used in this study permitted a detailed analysis of the participating children's engagement in probabilistic tasks and offered an authentic snapshot of their probabilistic thinking.

6.3. FURTHER STUDY

The study reported in this paper was conducted in relation to the *probability of an event* construct. Extending the study to examine the children's probabilistic thinking in relation to the other probabilistic constructs, such as *sample space* and *probability comparison*, could further our understanding of young children's potential to engage with probabilistic scenarios. A further possibility would be to carry out a teaching intervention to assess the impact of formal probabilistic instruction on the development of young children's probabilistic thinking. It may also be worthwhile to involve children from Junior Infants to Second class in these studies because these are the class levels at which children in Ireland do not receive formal probabilistic teaching. It would also allow for comparisons to be made concerning the potential of young children from 4 to 8 years old to engage with probabilistic tasks, providing useful information regarding the development of probabilistic thinking. Such studies may provide further evidence to support a restructuring of the probabilistic thinking content of the Irish primary mathematics curriculum, in particular regarding the age at which children are introduced to the topic of probability. Another opportunity for research pertaining to young children's probabilistic thinking is longitudinal research. This could be of benefit to the field as it would provide insights into the potential impact of children's early experiences of probability on their overall probabilistic learning and may contribute further to a review of the policy in Ireland that delays the introduction to the topic of probability until children are approximately 9 years old.

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APPENDIX

Task 1 – A Bag Full of Bears

Task 1.1
Show the children a bag containing 5 green bears and 1 red bear.
Questions: If you pick a bear from the bag, what colour are you most/least likely to get? Why?
Activity: Pick a bear 10 times to see what colour you get. Replace the bear and shake the bag each time.
Questions: How many greens and how many reds came out? Why do you think that happened?

Task 1.2
Show the children a bag containing 3 blue bears and 3 yellow bears.
Questions: If you pick a bear from the bag, what colour are you most/least likely to get? Why?

Task 1.3
Show the children a bag containing 1 purple bear, 3 green bears and 4 red bears.
Questions: If you pick a bear from the bag, what colour are you most/least likely to get? Why?

Task 2 – Terrific Treats

Task 2.1
Split the children into two teams. Tell one team that they have a pet dog and tell the other team that they have a pet cat.
Game Instructions: We are going to play a game. Every time the spinner lands on your pet, you earn a treat. The winner is the first team to earn five treats for their pet.
Show the children a spinner than has 4 equal segments. 1 segment has a picture of a dog and 3 segments have a picture of a cat.
Questions: Who is most/least likely to win? Why?
Play the game.
Questions: How many treats did you earn? Are you surprised that happened? Why do you think that happened?

Task 2.2
Game Instructions: Let's play the game again but this time let's use a dice.
Show the children a dice that has dogs on four sides and cats on two sides. Allow them time to examine and discuss the dice.
Questions: Who is most/least likely to win? Why?
Play the game.
Questions: How many treats did you earn? Are you surprised that happened? Why do you think that happened?

Task 2.3
Questions: Was the dice fair? Why/Why not?
Get the children to work together to design a fairer dice.
Questions: Why is your dice fair? If you roll your dice which pet are you most/least likely to land on? Why?

Task 3 – The Pirate Race

Task 3.1
Game Instructions: I will give you a dice and if you can roll a three you will win some treasure.
Questions: Do you think you will win? Why/Why not?
Give each child a turn to roll the dice and repeat above question each time.

Task 3.2
Game Instructions: Let's change the game. This time I will give you a piece of treasure as long as you don't roll a six.
Questions: Do you think you will win? Why/Why not?
Give each child a turn to roll the dice and repeat above question each time.
Questions: When we roll a dice, what number are we most/least likely to get?

Task 3.3
Split the children into two teams. Ask each team to choose a pirate and to place their pirate at their choice of starting position – green or orange.
Game Instructions: We are going to play a game called 'The Pirate Race'. The pirates are racing to the island. Each time your colour comes out of the bag, you can move one place closer to the island. The first pirate to get there wins the treasure chest full of gold. Here is the bag of counters. There are 2 red counters and 1 yellow counter in the bag.
Questions: Which pirate is most/least likely to make it to the island first? Why?
Play the game.
Questions: Which pirate made it to the island first? Are you surprised that happened? Why do you think that happened?

Task 4 – Tower Spin

Task 4.1
Show the children a spinner that has 4 equal segments.
Questions: If I spin the spinner which colour is it most/least likely to land on? Why?

Task 4.2
Split the children into two teams. Show them a spinner that is one-half orange, one-third blue and one-sixth green.
Game Instructions: We are going to use this spinner to play the tower game. Each team must pick a colour and then take turns spinning. If the pointer lands on your colour, you earn a cube. The winning team is the team that builds the tallest tower.
Questions: Which colour would you like to choose? Why?
Ask the teams to try to agree on a colour for each team. If an agreement can't be reached discuss the reasons for this difficulty.
Questions: Which team do you think will win? Why?
Play the game. Discuss what happened and if the children are surprised by the outcome.

Task 4.3
Questions: Was the spinner fair? Why/Why not?
Get the children to design a fairer spinner to show the 3 colours.
Questions: Why is your spinner fair? If you spin your spinner, which colour are you most/least likely to land on? Can you design a spinner that we could use with just two colours, one for each team? Discuss.