# OPPORTUNITIES TO LEARN MEAN, MEDIAN, AND MODE AFFORDED BY TEXTBOOK TASKS

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## ABSTRACT

*This research paper examines tasks related to mean, median, and mode in seven Swedish textbook series for students aged between 10–13 years. The tasks were analysed based on context, mathematical properties, input and output objects, and transformations. These categories allowed for a thorough analysis of the opportunities afforded to students to understand these measures. The analysis revealed that most tasks focus on the mean and on procedural transformations with quantitative values. The findings suggested that the textbooks do not afford enough explicit context for students to develop a deep understanding of the mathematical properties of different measures of central tendency. By analysing various textbooks, a broader understanding of the learning opportunities afforded to students was gained. The discussion includes the implications of these results for task design.*

***Keywords:*** *Statistics education research; Tasks about mean, median, and mode; Textbook analysis; Mathematical properties; Opportunities to learn; Input objects; Transformations; Output objects*

## 1. INTRODUCTION

Textbooks play a crucial role in mathematics education. Teachers in many countries rely on textbooks as their primary resource to guide decisions about what content to teach and what students should learn about the content (Fan et al., 2013; Glasnovic Gracin, 2018; Jones & Pepin, 2016; Pepin & Haggarty, 2001; Remillard, 2005). Textbook tasks are vital for students' learning because teachers and students working on textbook tasks is the main activity in many math classes (Neuman et al., 2014). Textbooks also represent the current math content that should be taught (Rezat & Sträßer, 2015). Consequently, the content and the way it is presented in textbooks can significantly impact how students understand math concepts (Bryant et al., 2008).

Research suggests a connection between the opportunities students are given to learn (OTL) content and their achievement in learning the content (Cogan et al., 2001; Tarr et al., 2006). Textbooks are an important source of information to consider when assessing a student's OTL because the textbooks offer various learning opportunities (Jones & Pepin, 2016; Sayers et al., 2019; Törnroos, 2005). Researchers have developed and used different approaches to analyse the OTL afforded by textbooks (Rezat & Sträßer, 2015; Sayers et al., 2019; Tarr et al., 2006). They found that every task has the potential to create a learning situation, but that potential is dependent on the OTL generated by the task's treatment of mathematics content (Hadar, 2017; Kieran et al., 2015; Tarr et al., 2006; Törnroos, 2005; Watson & Thompson, 2015).

This study examines the OTL provided by tasks related to mean, median, and mode (measures of central tendency) in Swedish textbooks for students aged 10–13 years. The focus is on the mathematical properties of the input object, transformation, and output object. These concepts are explained in the following section.

## 2. BACKGROUND

First, we present an overview of textbook analysis and the theoretical concept of OTL. This is followed by the distinction between contextual and non-contextual tasks and a focus on different

*Statistics Education Research Journal, 22(3), Article 6. https://doi.org/10.52041/serj.v22i3.655*

mathematical properties of the measures of central tendency. In subsequent sections, the themes and categories in the deductive content analysis are described and theoretically connected to the research subject and the aims of the analysis.

## 2.1. TEXTBOOK ANALYSIS AND OPPORTUNITIES TO LEARN

By reviewing a variety of textbook analyses, Charalambous and colleagues (2010) classified three different analytical approaches, namely horizontal, contextual, and vertical. The horizontal approach examines the textbook as a whole for its general characteristics such as number of pages, page size, and content sequencing and development. This approach provides less information about the treatment of mathematical content than the other two approaches. The contextual approach focuses on how textbooks are used by students or teachers in instructional activities. The vertical approach studies how mathematical concepts are treated through a deep analysis of the concepts. This study employs a vertical analysis to address what mathematical properties of measures of central tendency are present in textbook tasks. The ambition is not to compare the different textbook series but to give a general view of what OTL these Swedish textbooks present for mean, median, and mode.

The concept of OTL originated from the report on the First International Mathematical Study (FIMS), in which its meaning was that a student had an opportunity to study certain content (Husén, 1967). OTL was identified as one factor that might influence students' scores. Since then, OTL has continuously been used in comparative studies such as the Program for International Student Achievement (PISA) and the Trends in International Mathematics and Science Study (TIMMS) as a validity check to compare content taught in different countries (Floden, 2002). OTL has been considered helpful in educational research to analyse the content or cognitive skills promoted by textbooks or the curriculum (Floden, 2002). This latter research entailed using OTL within textbook analysis as one way to identify the implemented curriculum—how the curriculum was applied in mathematics teaching using textbook tasks (e.g., Charalambous et al., 2010; Törnroos, 2005).

Task design is important for tasks to provide the intended OTL. Carefully designed tasks can provide students with opportunities to practice skills at different levels, such as performing a calculation or practicing critical thinking (Callingham & Watson, 2017). For example, an important consideration for teaching inference is providing contexts that enhance students' opportunities to develop their understanding that they are drawing inferences about populations using samples selected from the populations (Leavy, 2010). Hence, it is crucial that an inferential task directs student's attention to looking beyond the data. Asking for comparisons between different populations is one way to guide the learner to reason beyond the data (Leavy, 2010). The textbook analysis in this research investigates students' OTL about mean, median, and mode from solving textbook tasks (e.g., Charalambous et al., 2010). This kind of analysis has the potential to find what limitations textbooks might have (e.g., Hadar, 2017).

## 2.2. CONTEXTUAL AND NON-CONTEXTUAL TASKS

This study analyses tasks in printed student textbooks (e.g., Remillard, 2005). The unit of analysis in this study is a task, which is defined as every written marked division of a proposed student activity in a textbook (e.g., Jones & Jacobbe, 2014, Reinke, 2019; Watson & Thompson, 2015).

Contextual tasks have been categorised in many different ways in research, mostly according to their purpose (e.g., Greatorex, 2014; Kieran et al., 2015). The purpose of interest in this study is how educational features are included in a task (e.g., Charalambous et al., 2010; Watson & Mason, 2006). One feature is when a contextual task explicitly focuses on the mathematical properties of a concept (e.g., Chick, 2007; Kieran et al., 2015; Liljedahl et al., 2007; Mason, 2011). Another feature is when the task description implicitly focuses on properties by, for example, including a solution process that requires drawing on those properties (e.g., Kieran et al., 2015; Reinke, 2019; Verschaffel et al., 2000; Watson & Thompson, 2015). These examples of features exemplify how contextual tasks can enable learning by directing students' attention to the properties of the concept and thereby afford the possibility for students to develop conceptual knowledge (e.g., Chick, 2007; Kieran et al., 2015; Leavy, 2010; Liljedahl et al., 2007; Reinke, 2019; Stein et al., 2007; Verschaffel et al., 2000; Watson & Mason, 2006; Watson & Thompson, 2015). In this study, tasks that afford OTL about mathematical properties are defined as contextual tasks (e.g., Kieran et al., 2015; Mason, 2011; Wijaya et al., 2015).

In contrast, a non-contextual task does not treat, refer to, or contain any extra-mathematical elements (e.g., Reinke, 2019). These kinds of tasks are often without a context and are named bare tasks; however, there are also tasks that have a context but provide little cognitive demand to students (Wijaya et al., 2015). These tasks are like dressed-up bare tasks where the arithmetical operations are apparent to students (Verschaffel et al., 2000; Wijaya et al., 2015).

In the analysis of tasks for this study, tasks are categorised as contextual or non-contextual depending on whether they afford OTL about mathematical properties or not. The theoretical foundation for how mathematical properties will be identified in this study is described in Section 2.7.

## 2.3. PROCEDURAL AND CONCEPTUAL TASKS

A considerable number of textbook tasks in general are procedural (e.g., Glasnovic Gracin, 2018; Hadar, 2017; Wijaya et al., 2015). Such tasks are often solved by routine and without necessarily considering any mathematical properties of the concepts that underlie the procedures (Brousseau, 1997). Many tasks about mean, median, and mode focus on developing procedural aspects rather than developing conceptual knowledge for aspects of these measures (Cai, 1998; Konold & Pollastek, 2004; Lampen, 2015; Landtblom, 2018). Conceptual knowledge is domain-specific knowledge that involves abstract and general knowledge of concepts and is dependent on awareness of mathematical properties (e.g., Canobi, 2009; Rittle-Johnson & Alibali, 1999; Strauss & Bichler, 1988). To afford students' development of conceptual knowledge, textbook tasks need to develop concepts consistently, a process dependent on the presence of mathematical properties (Watson & Thompson, 2015). Task design can trigger students' focus on conceptual aspects through the formulation of a question or use of representations (Watson & Thompson, 2015). Shahbari and Tabach (2020) observed a shift from an arithmetic, procedural view to a conceptual view of mean after students were given tasks with an explicit focus on developing conceptual knowledge. Consequently, being able to identify underlying mathematical properties is essential for conceptual knowledge (Usiskin, 2012).

## 2.4. MATHEMATICAL CONCEPTS

The theoretical foundation in this study proceeds from a framework in which a concept is based on central mathematical ideas that in turn are "built on a set of objects, transformations, and their properties" (Lithner, 2008, p. 261). The object—the entity transformed—can be a number, a variable, a diagram, etcetera. The input object transforms into the output object through transformations in an often-iterative process (Lampen, 2015; Lithner, 2008). One example given by Lithner (2008) was the transformation that takes place when the input object of 2 (a number) is entered into a function $f(x) = x^3$ (a transformation) to yield an output object of 8 (a number). Depending on their relevance, mathematical properties are classified as surface or intrinsic. For example, when deciding which of the two rational numbers, 9/15 and 2/3, is larger, the size of the numbers 9, 15, 2, and 3 is a surface property because it is not sufficient to answer the question using this property (Lithner, 2008). The intrinsic property is captured by the quotient.

Knowledge about concepts such as mean, median, and mode is dependent on awareness of properties, both mathematical and statistical (e.g., Burrill & Biehler, 2011; Strauss & Bichler, 1988). The common mathematical idea for mean, median, and mode is that each provides a measure of central tendency for a dataset, but each relies on different properties (Byström & Byström, 2011). For instance, to calculate the median, the intrinsic property is that the values can be ordered by rank. This requires that the variable in the dataset (the input object) must be ordinal or quantitative. For example, in a data set representing the number of siblings for each individual in a group of people (e.g., 3, 5, 7, 4, 0, 1, 1, 0, 2, 1), the numbers can be ordered, and consecutive numbers are equidistant. A surface property of these values is the size of the numbers. However, if the dataset contains outliers, the size of the values would be an intrinsic property. When the median is calculated, the transformation yields a median of 1.5. This number, 1.5, is the output object and has the mathematical properties that the measure does not exist in the dataset or in the physical world.

## 2.5. INPUT OBJECTS

In this article, the input object for the mean, median, and mode is the variable in the dataset. Mathematically, the variable's level, whether nominal, ordinal, interval, or ratio, is determined by the nature of the variable and the mathematical operations that can be performed on the data values (e.g., Byström & Byström, 2011; Leavy, 2010). Values of all levels can be grouped or categorised. Additionally, ordinal, interval, and ratio values follow a natural order that makes ranking values possible (Byström & Byström, 2011; Leavy et al., 2009). Finally, arithmetic calculations are possible for values on an interval or ratio level; what distinguishes these two levels is that the ratio level contains a true zero (Byström & Byström, 2011). In this study, interval and ratio levels are categorised as quantitative because the appearance of a true zero is not of interest to the research. There are no tasks at the school level of interest that deal with this property.

One distinction between ordinal values and quantitative values is that ordinal level values are not equidistant (e.g., Mayén & Diaz, 2010). Having unequal distances between the values, therefore, makes mathematical calculations meaningless (e.g., Kitto et al., 2019). Thus, arithmetic calculation of mean or standard deviation is inappropriate for ordinal variables because they lead to incorrect conclusions, for example, about statistical significance (e.g., Byström & Byström, 2011; Kitto et al., 2019). Boundaries for data analysis, however, are not always strict, and some researchers find it appropriate to calculate the mean for some ordinal values such as grades, but not for other variables such as data given on a Likert-type scale (e.g., Allen & Seaman, 2007; Göb et al., 2007). Thus, it is important to afford students OTL about the differences between ordinal and quantitative levels for measures of central tendency. Research suggests that students otherwise demonstrate uncertainty and confusion regarding these two levels (Mayén & Diaz, 2010). This study considers it inappropriate to calculate the mean of ordinal values, although determining the median is possible if the median is a value in the dataset. As described, the properties of the variable depend on the data level, which in turn determines how input data can be transformed. Because calculations cannot be conducted on qualitative data, there are fewer categories of transformations and output objectives to investigate (Leavy, 2010). In the analysis in this study, input objects are categorised as nominal, ordinal, or quantitative.

## 2.6. TRANSFORMATIONS

According to Lithner's framework, a "*transformation* is what is being done to an object" (2008, p. 261). In this study, a transformation procedure is used to turn an input object into a measure of central tendency. A specific formula is applied to calculate the mean; finding the median involves ranking values in a specific order; and finding the mode involves grouping values by frequency. This study categorises each measure as shown in Table 1. The transformation for mean is calculation; the transformation for median is rank ordering (and calculation for data sets with even numbers of values when calculation is needed); and the transformation for mode is grouping.

However, there are different approaches to tasks focused on measures of central tendency that require different transformations. Such tasks can help learners to understand the measures better (e.g., Watson & Thompson, 2015). For instance, a missing value in a dataset with a given mean can be found through a reverse calculation, which requires considering both input and output objects and their interplay in the transformation process (Groth, 2007; Groth & Bergner, 2006; Konold & Pollastek 2004; Watson, 2006). A different type of task, a debugging task, involves interpreting a mathematical activity or formula such as identifying errors in a calculation (Glasnovic Gracin, 2018). Tasks of this kind are categorised as "debugging" in this study.

*Table 1. Themes and categories for the analysis of tasks along with anchor examples*

| Theme | Category | Anchor examples |
|---|---|---|
| Context | Non-contextual[(1)] | 3, 5, 7, 6, 4. What is the median? |
| | Contextual[(2)] | Anne has 2 cookies and Bill has 5. How many cookies do they have on average? |
| Input objects | Nominal | Green, blue, red, red. What is the mode? |
| | Ordinal | S, S, S, M, M. What is the median? |
| | Quantitative | 3, 4, 5, 8. Calculate the mean. |
| Transformations | Grouping | 2, 3, 3, 4, 3, 1. Calculate the mode. |
| | Rank-ordering | 2, 3, 3, 4, 3, 1. Calculate the median. |
| | Calculation | 2, 3, 3, 4, 3, 1. Calculate the mean. |
| | Debug[(3)] | Nora calculates the mean for the dataset 11, 12, 9, 8, and 0. Her answer is 10. What has she done wrong? |
| | Equal distribution[(4)] | Three people paid 9, 10, and 5 euros, respectively, shopping for food. Redistribute their costs for equal payment. |
| | Reverse calculation[(5)] | The numbers 5, 6, 7, and *x* have a median of 5.5 and a mode of *x*. Which number is *x*? |
| | Validate values[(6)] | Look at the values in the frequency table (the values are colors). Why can you not find a median within this dataset? |
| Output objects | Mathematical property[(7)] | • In the dataset 1, 6, 5, 4, and 6, the mean can take a value greater than or equal to 1 and less than or equal to 6.<br>• For the data set 1, 2, and 3, the mean is 2; adding a value of 4 gives a mean of 2.5.<br>• The average number of siblings for the students in a class equals 2.4.<br>• The dataset 0, 0, 1, and 3 has a mean of 1. |
| | Measures affected by distribution[(8)] | A group of people has the following age distribution: 5, 9, 4, 83, 18, 2, 27, 15, 84, and 4. Is the mean or the median most representative of the dataset? |
| | Levels of measure affect possible transformations of values[(9)] | Why is it impossible to calculate the median for the dataset: green, blue, red, and red? |
| | The number of modes varies[(10)] | Decide the mode for the data set 3, 4, 3, 1, 4, 2, 8, and 5. |
| | Measure corresponds to more than one dataset[(11)] | The dataset 1, 2, 3, 4, and 5 has a mean of 3.<br>The dataset 3, 3, 3, and 3 has a mean of 3. |

[(1)] (e.g., Reinke, 2019); [(2)] (e.g., Mason, 2011); [(3)] (e.g., Glasnovic Gracin, 2018); [(4)] (e.g., Konold & Pollastek, 2004); [(5)] (e.g., Groth, 2007); [(6)](e.g., Konold & Pollastek, 2004); [(7)] (Strauss and Bichler, 1988)*;* [(8)](e.g., Wild, 2006); [(9)](e.g., Byström & Byström, 2011); [(10)]( e.g., Groth & Bergner, 2006); [(11)] (e.g., Kitto et al., 2019)

Moreover, task design can highlight the inherent mathematical properties of a concept through the expected transformation (Watson & Thompson, 2015). For example, the metaphor of equal distribution or fair share is commonly used to explain the mean and leads to a specific transformation that builds on the mathematical property that the sum of deviations from the mean equals zero (Konold & Pollastek, 2004; Strauss & Bichler, 1988). Such tasks are categorised as equal distribution, are often used for discontinuous variables, and may require higher levels of abstraction depending on the mathematical properties of the output value (Konold & Pollastek, 2004). Fair share tasks can be used to promote understanding that a mean value does not always exist in physical reality, such as a mean of 2.3 siblings in a family (Watson & Moritz, 2000). The latter focuses on an answer, the output object, and affords the opportunity to explore the relation between the calculated measure and the context of the given data set (e.g., Watson & Moritz, 2000). These kinds of tasks are categorised as “validate values” in this study.

## 2.7. OUTPUT OBJECTS

In the theoretical framework for this study, the output object is the result of transforming the input object (Lithner, 2008). As such, the output object from a task of interest to this study is a measure of central tendency with corresponding mathematical properties that students can observe. Strauss and Bichler (1988) created a list of seven properties for averages. These properties involve mathematical, abstract, or representative characteristics of a group of values. Strauss and Bichler did not distinguish among mean, median, and mode but used the term, average, to describe all three measures because average is often associated with each of the measures (Watson & Moritz, 2000). Because arithmetic operations can be used to perform calculations with quantitative data, measures calculated using quantitative data have more mathematical properties than measures calculated using qualitative data.

Mathematical properties of average identified by Strauss and Bichler (1988) include: (1) the average is located between the extreme values; (2) the sum of the deviations from the average is zero; (3) the average is influenced by values other than the average, meaning that "the average of 0, 5, and 10 is 5, and the addition of 10 to the numbers being averaged changes the average to 6.25" (Strauss & Bichler, 1988, p. 66). Properties of an abstract character are: (4) the average may not be one of the values in the data set; (5) the average can be a rational number with no physical counterpart; and (6) a zero value in the input object must be considered in the calculation. Finally, the representative property of average is: (7) the average value represents the values that are averaged. Table 1 in this text provides examples of output objects for four of these seven properties.

The fourth and fifth properties listed above apply to the mean and median. Meanwhile, properties one and three apply to all three measures of central tendency. Property seven means that the mean is affected by all individual observations, even outliers. This means that the median and/or mode better represent such data sets because these measures are not affected by outliers (e.g., Batanero et al., 1994; Byström & Byström, 2011; Groth & Bergner, 2006). Therefore, it is important to identify possible outliers when examining a distribution of values (according to property seven), a key aspect highlighted in previous research (Biehler et al., 2018; Wild, 2006) that is connected to how variability among values affects the skewness of the distribution (Bakker, 2004; Konold & Pollastek, 2004; Wild, 2006). The categorisation of the property that median and mode are not sensitive to outliers is found in Table 1 as "measures affected by distribution."

Aside from the above properties, the concept of mode has two unique mathematical properties. First, a dataset can have zero modes or more than one mode. Second, the mode is the only measure that can be used with values at a nominal level (Groth & Bergner, 2006). These properties are categorised as "the number of modes varies," and "levels of measure affect possible transformations of values." The latter category encompasses nominal, ordinal, and quantitative values. The last category in Table 1 pertains to a measure corresponding to more than one dataset (e.g., Kitto et al., 2019).

## 2.8. RESEARCH QUESTIONS

The focus of this study is on analysing the mathematical properties inherent in specific tasks. By examining what opportunities to learn (OTL) measures of central tendency are presented in Swedish textbook tasks for students aged 10–13, we can gain insight into their potential for learning. Focusing on representative textbooks from a country can be considered a unique signature of the textbooks for this particular country (Charalambous et al., 2010). The research aims to answer two questions: (1) What is the distribution among non-contextual and contextual tasks? (2) What opportunities to learn (OTL) about a) input objects, b) transformations, and c) output objects do textbook tasks afford, and what does the distribution among input objects, transformations, and output objects look like?

# 3. METHOD

## 3.1. SAMPLE OF TEXTBOOKS

This study analysed 17 textbooks from seven different Swedish textbook series, all of which are relevant to the investigation of tasks related to mean, median, and mode. By choosing several textbook series in this national analysis, the intention is to present what OTL about mean, median, and mode the textbook tasks afford. The sample was purposefully chosen to ensure that there was enough data to provide a broad and representative cross-section of the learning potential of textbook tasks (Denscombe, 2017; Silver, 2017; Son & Diletti, 2017).

All textbooks were for school years 4–6 (ages 10–13) and followed the current Swedish guidelines for measures of central tendency. The textbook series included tasks on measures of central tendency for all three school years in the series or one or two school years in the series, which aligns with when these measures are introduced in the Swedish guidelines. The unit of analysis for this study was on a task level, with a total of 1,392 tasks available for analysis. Details about the textbooks used in analyses can be found in the Appendix.

## 3.2. CONTENT ANALYSIS

In this deductive content analysis, we focused on the context, input objects, transformations, and output objects exemplified in Table 1. We used only the categories determined beforehand to analyse tasks and excluded information about whether data was presented through diagrams or tables because the format of data presentation did not relate to our research questions. With this category system as a central instrument of analysis, we aimed to draw inferences about the tasks and the OTL they afford (Mayring, 2015). We focused on the central and intrinsic mathematical properties of the concept(s) addressed by tasks (Lithner, 2008). Therefore, we analysed tasks based on the themes of context, input objects, transformations, and output objects, as well as relevant categories within each theme.

Tasks can be viewed as contextual or non-contextual, which can be somewhat dependent on whether one considers only the task description or includes consideration of different solutions (Hong & Choi, 2018). We chose to include solutions in our analysis because mathematical properties can be ascertained through the transformation of input values. Tasks can have implicit mathematical properties that are not immediately evident, as noted by Shahbari and Tabach (2020). Non-contextual tasks do not explicitly provide any OTL properties, and the theme of context as contextual or non-contextual allows us to measure the distribution of mathematical properties in different measures.

Table 1 was used as a coding guide, and each task was scored dichotomously for each category as (1) if the category was present and (0) if the category was not present. Some tasks were assigned values of (1) for multiple categories, resulting in sums greater than the total number of tasks for some categories reported in the Results.

## 3.3. STATISTICAL ANALYSIS

To ensure validity and reliability, proper analysis methods and coding systems were established (e.g., Son & Diletti, 2017). In this analysis, a category system was implemented to aid in comparing findings and in strengthening reliability (e.g., Mayring, 2015). Two analyses were conducted to compare categories for mean, median, and mode tasks. The first analysis determined frequencies and percentages for the presence of categories among the tasks for the three measures. The second analysis involved a chi-square test for homogeneity to determine the significance of differences between the distribution of proportions for measures of central tendency among categories of contextual/non-contextual tasks (refer to Tables 2 in the Results). Even though the selection of tasks was not random, the tasks focused on measures of central tendency from the Swedish textbooks selected for this study can be considered to be representative of tasks in Swedish textbooks for students aged 10–13 more generally. The null hypothesis is that there is no difference between the distributions of proportions for measures of central tendency among the categories. We note that the chi-squared test cannot be applied to the results of question 2a (Section 4.2), question 2b (Section 4.3), and question 2c (Section 4.4) regarding OTL about input objects, transformations, and output objects due to the lack of data for some categories. It is also worth considering that the level of measure of the values affects which transformations are possible, resulting in different mathematical properties that may arise.

To ensure a more stable analysis, the process was broken down into several steps and discussed with other mathematics education researchers throughout. The author conducted the initial coding, followed by 14 independent raters who coded the mathematical properties of five randomly selected task samples each. In 77% of these tasks, two coders independently worked on the same task. The level of agreement among coders was 82%.

## 4. RESULTS

To gain a general understanding of what OTL are afforded on a general level, we compiled results from all seven textbook series. First, we will discuss the distribution of contextual and non-contextual tasks. We will then address the different mathematical properties associated with the tasks, including input objects, transformations, and output objects.

### 4.1. DISTRIBUTION OF NON-CONTEXTUAL AND CONTEXTUAL TASKS

The distribution of non-contextual and contextual tasks related to mean, median, and mode is presented in Table 2

*Table 2. Frequency of observed (and expected) non-contextual and contextual tasks, chi-square statistic for each cell, percentage of observed task type by measure, and chi-square test results*

| Tasks | Mean | Median | Mode | Total | $\chi^2$ (2, $N$ = 1392) |
|---|---|---|---|---|---|
| Non-contextual | 247 (342.11)<br>[26.44]<br>28.42% | 124 (106.29)<br>[2.95]<br>45.92% | 177 (99.60)<br>[60.15]<br>69.96% | 548<br>39.37% | 147.67* |
| Contextual | 622 (526.89)<br>[17.17]<br>71.57% | 146 (163.71)<br>[1.92]<br>54.07% | 76 (153.40)<br>[39.05]<br>30.04% | 844<br>60.63% | |
| Total no. of OTL | 869 | 270 | 253 | 1392 | |

* $p < .001$

According to the findings presented in Table 2, many of the tasks (60.63%) are contextual. Moreover, there is an imbalance in the distribution of the measures of central tendency in tasks, with nearly two-thirds of the tasks focusing on the mean. The percentage of contextualized and non-contextualized tasks varies depending on the measure of central tendency used. When working with the mean, contextualized tasks are more prevalent (71.57% contextualized vs. 28.42% not contextualized) compared to working with the median (54.07% contextualized vs. 45.92% not contextualized) or the mode, where the number of contextualized tasks drops significantly (30.04% contextualized vs. 69.96% not contextualized). These findings are supported by the statistical tests that provide evidence of differences in the distributions of the two categories of contexts among the three measures, with the mode components contributing disproportionately more to the chi-square statistic value of 147.67 (60.15 for non-contextual tasks and 39.05 for contextual tasks) and the median components contributing the least (2.95 for non-contextual tasks and 1.92 for contextual tasks).

### 4.2. INPUT OBJECTS

In Table 3, afforded OTL about the measures of mean, median and mode for input objects of value type are presented using frequencies and percentages.

There is evidence of differences in distribution among the measures of central tendency. Upon analysing the total distribution, we found that OTL about measures of central tendency for different types of data input objects focus predominantly on a quantitative level (91.54% in total). Specifically, tasks involving the mean (97.81%) and median (92.65%) provide quantitative data. For mode, the percentage of quantitative values is 69.92%. The table shows that nominal and ordinal levels each make up approximately 4% of the total. Table 3 also reveals that tasks involving the mean and median are categorised as having nominal and ordinal values. In tasks with nominal values, the focus was on whether the measure was appropriate to the given data—no calculations were asked for in these tasks. In the tasks with ordinal values, qualitative values were treated as quantitative.

*Table 3. Frequency and percentage of total of input objects*

| Input objects | Mean | Median | Mode | Total |
|---|---|---|---|---|
| Nominal values | 1 | 3 | 57 | 61 |
| | 0.12% | 1.10% | 21.42% | 4.34% |
| Ordinal values | 18 | 17 | 23 | 58 |
| | 2.07% | 6.25% | 8.65% | 4.12% |
| Quantitative values | 850 | 252 | 186 | 1288 |
| | 97.81% | 92.65% | 69.92% | 91.54% |
| Total no. of OTL | 869 | 272 | 266 | 1407 |

Finally, among the tasks dealing with qualitative values and the mode, 21.42% were categorized as nominal and 8.65% as ordinal. When looking at the mode on a nominal level across the total, we see that the OTL about this property is 4.05% (57 out of 1407).

## 4.3. TRANSFORMATIONS

In Table 4, we present the provided OTL about measures of central tendency based on the transformations needed to solve the tasks.

*Table 4. Frequency and percentage of total of transformations*

| Transformation | Mean | Median | Mode | Total |
|---|---|---|---|---|
| Grouping | n/a | n/a | 204<br>80.63% | 204<br>14.48% |
| Rank-ordering | n/a | 209<br>77.41% | n/a | 209<br>14.83% |
| Calculation | 594<br>67.04% | n/a | n/a | 594<br>42.16% |
| Equal distribution | 57<br>6.43% | n/a | n/a | 57<br>4.05% |
| Debug | 11<br>1.24% | 1<br>0.37% | 1<br>0.40% | 13<br>0.92% |
| Reverse calculation | 192<br>21.67% | 35<br>12.96% | 27<br>10.67% | 254<br>18.03% |
| Validate values | 32<br>3.61% | 25<br>9.26% | 21<br>8.30% | 78<br>5.54% |
| Total no. of OTL: | 886 | 270 | 253 | 1409 |

n/a = not applicable

Based on the analysis of OTL, it was found that 1007, or 71.47%, of all transformations were linked to the definitions of mean (calculation), median (rank-ordering), or mode (grouping). Out of the three measures, the mode had the highest percentage of transformations linked to the definition at 80.63%. Alternative transformations not linked to the definition for mean (32.96%) were more prevalent than those for median (22.59%) and mode (19.37%). The most common transformation was the reverse calculation (18.03%), particularly for the mean (21.67%).

## 4.4. OUTPUT OBJECTS

In Table 5, afforded OTL about measures of central tendency were presented according to the intrinsic mathematical properties of output objects.

*Table 5. Frequency and percentage of total of output objects*

| Output objects | Mean | Median | Mode | Total |
|---|---|---|---|---|
| Measure located between extreme values | 8<br>1.13% | 0<br>0% | ~~0~~ n/a<br>~~0%~~ | 8<br>0.82% |
| The sum of the deviations from the measure is zero | 57<br>8.09% | n/a | n/a | 57<br>5.87% |
| Measure influenced by values other than the measure | 6<br>0.85% | 4<br>2.23% | 0<br>0% | 10<br>1.03% |
| The measure does not equal values from the dataset | 378<br>53.62% | 85<br>47.49% | n/a | 463<br>47.68% |
| Measure not from physical reality | 46<br>6.52% | 12<br>6.70% | n/a | 58<br>5.97% |
| Consider zero as a value | 120<br>17.02% | 36<br>20.11% | 34<br>39.08% | 190<br>19.56% |
| Measure representative of all values measured | 6<br>0.85% | n/a | n/a | 6<br>0.62% |
| Average affected by the distribution | 20<br>2.84% | 20<br>11.17% | 15<br>17.24% | 55<br>5.66% |
| Levels of measure affect possible transformations of values | 3<br>0.43% | 4<br>2.23% | 4<br>4.60% | 11<br>1.13% |
| The number of measures vary | n/a | n/a | 16<br>18.39% | 16<br>1.65% |
| A measure corresponds to more than one dataset | 61<br>8.65% | 18<br>10.06% | 18<br>20.69% | 97<br>9.99% |
| Total no. of OTL: | 705 | 179 | 87 | 971 |

n/a = not applicable

From Table 5, it is evident that most of the mathematical properties related to the mean have a higher percentage (705/971 or 72.61%) than those related to the median (179/971 or 18.43%) and mode (87/971 or 8.96%). Upon analysing the total number of output objects, it was found that almost half of them (47.68%) were about the mathematical property that the measure did not equal a value in the dataset, followed by considering zero as a value (19.56%), and that a measure can correspond to more than one dataset (9.99%). This latter percentage is higher for the mode. Some output objects have a low percentage, such as a measure located between the extreme values (0.82%), which occur only in tasks for the mean. The property that a measure is influenced by values other than the measure has a low percentage (1.03%). The unique property of the mode, that the number of modes can vary, has a total percentage of 1.65%, which corresponds to18.39% tasks about the mode.

## 5. DISCUSSION

The goal of this research was to examine the OTL about mean, median, and mode afforded by Swedish textbooks and to determine how this information was presented in a country-specific way (Charalambous et al., 2010). The study revealed that, regardless of the focus of the information provided, there were fewer OTL about median and mode due to the skewed distribution of the three measures of central tendency among the tasks analysed.

The tasks were analysed based on context, including both the task's educational features and its solution. The results indicated that approximately 40% of the tasks were non-contextual, whereas 60% were contextual. We should note that some of the bare non-contextual tasks that focused only on calculation and not on the mathematical properties nonetheless generated mathematical properties connected to the measure. Therefore, these tasks were considered to be contextual and were referred to as implicit contextual tasks. In contrast, tasks that had a context that explicitly focused on the mathematical properties of a concept were referred to as explicit contextual tasks. Thus, implicit contextual tasks can afford more than procedural knowledge, and teachers should make such afforded information explicit to develop students' conceptions (Brousseau, 1997; Leavy, 2010; Shabbari & Tabach, 2020; Usiskin, 2012).

When comparing the measures of central tendency, there were fewer occurrences of OTL about median and mode compared to mean. Notably, there were relatively few contextual tasks related to the mode. This was surprising given that the tasks were taken from seventeen textbooks in seven textbook series, emphasizing the low number of tasks related to the mode. This study's findings align with earlier research, such as Groth and Bergner's 2006 study, which also found the mode to have a less prominent role when studying measures of central tendency.

To determine the contextual nature of a task, we found it useful to analyse the input object, transformation, and output object (e.g., Lampen, 2015; Lithner, 2008). By examining how these concepts were handled in various instances, it was possible to identify the central mathematical ideas emphasized. One notable observation regarding input objects was the prevalence of quantitative variables and the scarcity of nominal values in tasks related to the mode. This suggests that mode is commonly perceived as an easy concept, but in reality, it can be challenging to understand (e.g., Groth & Bergner, 2006; Landtblom & Sumpter, 2021). The distribution of tasks in textbooks also supports the idea that mode applies only to numerical data. Unfortunately, these tasks may not effectively promote student understanding of mode or their analysis of qualitative data (e.g., Cogan et al., 2001; Stein et al., 2007; Tarr et al., 2006).

Previous research has indicated that textbook tasks tend to emphasize quantitative measures and the mean (e.g., Groth & Bergner, 2006; Landtblom, 2018), which is consistent with our findings. Some tasks focused on mean and median using data on nominal or ordinal levels. The tasks, however, on a nominal level focused on the level of measure and whether the choice of variable was suitable/possible or not. Hence, these tasks highlighted central mathematical ideas about input objects. The tasks on an ordinal level treated ordinal values as quantitative without further reflection (e.g., Kitto et al., 2019).

More research is needed to better understand the motivations of textbook authors when producing their material. It is interesting to look at their intentions in relation to relevant mathematical concepts and how these intentions affect the decisions made, such as the distribution of the number of tasks between mean, median, and mode, respectively. Their insights regarding data levels are also of interest, for instance, the reason there are a low number of tasks with nominal values. Moreover, it would be interesting to investigate their thoughts regarding mathematical properties. The results show that when properties occur to varying degrees among the tasks, the student may not pay attention to the property if they are not explicitly asked about it. An example of how to direct attention is to ask why one can only determine mode if color is the input object or why you cannot determine median for a set of data.

In terms of transformations, most tasks only involved procedures. This aligns with previous research indicating that statistics instruction primarily focused on calculation for measures of central tendency (e.g., Cai, 1998; Konold & Pollastek, 2004; Lampen, 2015; Landtblom, 2018). Some tasks, however, did allow for engagement with the mathematical properties of the input object, transformation, or output object. These tasks afforded opportunities for students to connect mathematical concepts and transformations, but these tasks were not common in the examined teaching materials. Limiting exposure to these tasks can hinder students' learning of key concepts (e.g., Hadar, 2017; Shahbari & Tabach, 2020; Watson & Thompson, 2015). When alternative transformations are included in tasks, they offer a chance for students to develop a deeper understanding by exploring connections between levels of measures and different transformations (e.g., Chick, 2007; Leavy et al., 2009). These types of tasks were rare and require active selection by teachers or awareness by textbook authors.

When it comes to output objects, one noticeable property was that the values of the measures of central tendency often do not match a data value in the dataset. This property, however, was not always explicitly highlighted in tasks, sometimes it remained implicit. The same held true for other properties, such as measures not representing a physical reality or considering zero as a data value, which also appeared quite frequently. In some tasks, these data were transformed without any warning to the student and only became clear in the solution. This means that students might think they have the correct answer without really understanding the underlying property of the measure (e.g., Shahbari & Tabach, 2020; Usiskin, 2012). To address this situation, tasks could be designed to explicitly focus on these properties, such as providing an example where the mean of flower petals is 8.16 and the student must explain what that value means. Other properties could also be highlighted through task design or transformations. Highlighting these properties through task design or transformations is crucial in effectively addressing implicit properties in textbook tasks.

In this study, a limitation identified was the use of non-probability stratified sampling. The sample was found to be satisfactorily representative, and no subjective judgment was made. Altering the sample, however, could lead to slightly different results. Despite this limitation, interesting patterns were still observed. It is worth noting that although the data analysed in this study came from Swedish textbooks, the methodology and analytical approach used could be applied to textbooks or other tasks from other countries. For instance, further studies could analyse how TIMSS and PISA cross-national surveys of educational achievement incorporate measures of central tendency.

## 6. CONCLUSIONS

The results of this study suggest several conclusions regarding the use of Swedish textbooks for tasks related to mean, median, and mode. Firstly, the mean should be the focus of a higher percentage of tasks compared to the median and mode due to its connection with more mathematical properties. Many of these properties, however, are implicit and may be mistaken as serving solely to develop procedural knowledge without proper identification (e.g., Brousseau, 1997). This possibility is highlighted by Star (2005), who divides procedural knowledge into categories of surface and deep. The distinction between surface and deep procedural knowledge is important because procedures often are thought of as playing a secondary role in students' learning (Star, 2005). The result exemplifies how an implicitly afforded mathematical property can promote learning that could result in deep procedural knowledge. Secondly, tasks related to the mode often involve quantitative values, although the mode is primarily used for qualitative and nominal values. Unfortunately, textbooks do not address adequately this primary use of mode. Additionally, the textbooks did not address the uniqueness of the mode in terms of the number of modes that may exist for a set of data. Thirdly, the number of tasks that afford mathematical properties was low, indicating a need for more attention to OTL basic knowledge such as determining the reasonableness of a measure or its relationship to distribution. Finally, although ordinal level values appear in tasks, textbooks do not provide sufficient guidance on specific properties of ordinal data. Providing tasks that highlight the differences between ordinal and quantitative values can help students and teachers address common challenges in understanding these concepts (e.g., Groth & Bergner, 2013; Kitto et al., 2019; Leavy et al., 2009; Mayén & Diaz, 2010).

Overall, the Swedish textbook tasks focus heavily on procedural knowledge, both superficial and deep, particularly for calculating the mean. This emphasis on operations may limit opportunities for students to develop conceptual understanding of mean, median, and mode (e.g., Bryant et al., 2008; Cogan et al., 2001; Fan et al., 2013; Leavy, 2010; Rezat & Sträßer, 2015; Stein et al., 2007; Tarr et al., 2006). It is important to consider the impact of textbook tasks on students' learning, and future studies could explore different types of tasks and analyse students' mathematical reasoning to determine how they use different mathematical properties in their solutions.

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# APPENDIX

## ANALYZED TEXTBOOK SERIES

### TEXTBOOK SERIES 1

Undvall, L., Melin, C., & Ollén, J. (2016). *Matematikboken alfa* [Math book alpha]. Liber.
Undvall, L. & Melin, C., (2016). *Matematikboken beta* [Math book beta]. Liber.
Undvall, L. (2016). *Matematikboken gamma* [Math book gamma]. Liber.

### TEXTBOOK SERIES 2

Asikainen, K., Nyrhinen, K., Rokka, P., & Vehmas, P. (2016). *Mera favorit matematik 5B* [More favorite mathematics 5B]. Studentlitteratur.
Asikainen, K., Nyrhinen, K., Rokka, P., & Vehmas, P. (2017). *Mera favorit matematik 6B* [More favorite mathematics 6B]. Studentlitteratur.

### TEXTBOOK SERIES 3

Olsson, I., & Forsbäck, M. (2013). *Eldorado: Matte. [5B]. Grundbok*. [Eldorado: Math. [5B]. Basic book.]. Natur & kultur.
Olsson, I., & Forsbäck, M. (2013). *Eldorado: Matte, [6B]. Grundbok*. [Eldorado: Math. [5B]. Basic boo*k*]. Natur & Kultur.

### TEXTBOOK SERIES 4

Picetti, M., Falck, P., & Sundin, K. (2012). *Matte direkt borgen 4A*. [Math direct the fort 4A] (2nd ed.). Sanoma Utbildning.
Picetti, M., & Falck, P (2013). *Matte direkt borgen 5A* [Math direct the fort 5A] (2nd ed.). Sanoma Utbildning.
Picetti, M., Falck, P., Carlsson, S., & Liljegren, G. (2012). *Matte direkt borgen 6A* [Math direct the fort 6A] (2nd ed.). Sanoma Utbildning.

### TEXTBOOK SERIES 5

Alseth, B., Nordberg, G., & Røsseland, M. (2012). *Pixel: 4A grundbok* [Pixel: 4A basic book]. Natur och kultur.
Alseth, B., Nordberg, G., & Røsseland, M. (2012). *Pixel: 5A grundbok* [Pixel: 5A basic book]. Natur och kultur.
Alseth, B., Nordberg, G., & Røsseland, M. (2009). *Pixel: 6A grundbok* [Pixel: 6A basic book]. Natur och kultur.

### TEXTBOOK SERIES 6

Sjöström, B., & Sjöström, J. (2012). *Prima formula matematik 5* [Prima formula mathematics 5]. Gleerup.

### TEXTBOOK SERIES 7

Hernvald, A., Kryger, G., Persson, H., & Zetterqvist, L. (2011). *Mattespanarna 4B* [The math scouts 4B]. Liber.
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