RESEARCH ON LEARNING AND TEACHING PROBABILITY WITHIN STATISTICS

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1. INTRODUCTION

Welcome to this special issue of the *Statistics Education Research Journal* (*SERJ*). Those of you familiar with academic time (for lack of a better phrase) will not be surprised to learn that this special issue is a continuation (maybe the culmination) of 'Topic 6: Innovation and Reform in Teaching Probability within Statistics' at the 9th International Conference on Teaching Statistics (more often referred to as ICOTS 9), which took place in mid-July of 2014 in Flagstaff, Arizona, USA. To get a better sense of what took place, over two years ago now, we present the abstract for this Topic:

> Concepts and models of uncertainty and variability are at the heart of statistical thinking and analysis. Hence probabilistic concepts and thinking underpin all of statistics. Chance and Data developments should be intertwined, in harmony and both driven by data, real contexts and problem-solving. As in all of statistics, students bring to formal education in probability concepts and intuitions from everyday real experiences, and the teaching of probability needs to build on these and link formal notions to the everyday, to real contexts and—constantly—to data. In order to enrich conceptual understanding and underpin formal developments with experiential insights, sustainable representations of probability, integrated with data and statistical thinking, will render concepts accessible and more easily exemplified. These invited sessions will address much-needed reforms in teaching probability in harmony with statistics and will explore current innovations and new directions.

Given that three of the four guest editors (Egan, Efi and Peter) of this special issue were also the convenors of Topic 6 at ICOTS 9, it should come as no surprise that we thought our Topic was a success. Perhaps we are overcompensating because we were the only Topic with an explicit focus on probability within a sea of statistical Topics. But we do have the numbers to back up our assertion. Our Topic included six sessions on a diverse range of themes:

- Bayesian inference (probability) goes to school: meanings, tasks and instructional challenges;
- Probability and p-values—probing the problems;
- Interdisciplinarity and innovation;

- Teaching probability to future teachers of mathematics and statistics;
- Modeling distributions to connect chance processes, data production, and data distributions;
- Teachers' awareness of conceptual connections between probability and statistics.

We would like, here, to thank, the organizers of these sessions: Per Nilsson (Sweden), Robyn Reaburn (Australia), Annie Savard (Canada), Marcos Magalhães (Brazil), Hollylynne Stohl Lee (United States) and Dionysia Bakogianni (Greece). They were instrumental in making Topic 6 at ICOTS 9 a success.

Of course, we would be remiss not to thank all of those individuals who presented their research in each of these sessions; 18 presentations (3 per session) from 32 authors and co-authors from across the globe. Further information, including the abstracts, details of the contributors, and the full papers, can be found at the following link: http://icots.info/9/topic.php?t=6. It was the strong interest in and high quality of these presentations that prompted the idea for a special issue that would continue the work highlighted in Topic 6 at ICOTS 9, which brings us to the current special issue.

The 13 articles, from 34 authors and co-authors from across the globe, touch upon and extend specific aspects of the discussion that took place in Flagstaff under the title of "Innovation and Reform in Teaching Probability within Statistics". The title of this special issue is modified to include the terms "research" and "learning". If you read the following brief descriptions of the articles (and the articles themselves) you will see that our modified title "Research on learning and teaching probability within statistics" was a welcome and necessary change.

2. INVITED SUBMISSION AND DISCUSSION

We begin this special issue with an invited paper from Pfannkuch, Budgett, Fewster, Fitch, Pattenwise, Wild and Ziedins, which is entitled "Probability modelling and thinking: what can we learn from practice?". Building on their very successful "Statistical thinking in empirical enquiry" from 1999, Pfannkuch and Wild expand their New Zealand team and tackle the problem of exactly what are the key ideas of probability thinking and modelling. Their aim is to isolate these big ideas as a basis for 21st century pedagogy in probability. Their approach is to interview seven practitioners who are experts in probability modelling and to draw out the key ideas from these interviews.

Their article is followed by three invited discussions. Chance and Roy (tertiary statistics educators) talk about the focus of their introductory course on simulation and its role in helping students to understand modelling. They point out the importance of the combination of probabilistic and statistical thinking—the deductive and inductive reasoning that should be integrated into a successful pedagogy. They also highlight the various aspects of intuition of young students that needs to be acknowledged and maybe challenged in any successful teaching of probability and statistics. Shaw (a practicing statistician) points out the reality that much statistical work consists of applying known models and methodologies to particular problems in order to obtain timely results. A familiarity with a wide range of models and methodologies, and an understanding of their assumptions, is a key to success as a practicing statistician, and some aspect of this is a necessary component of a university education in statistics and probability. Cordani (a statistics educator at tertiary and pre-tertiary levels) makes the important point that the authors' study was carried out in New Zealand in a context where university students have a strong practical and theoretical exposure to ideas of probability and statistics from their early school days. In a country such as Brazil the reality is that most university students have only a limited background in probability and statistics from their schooling, as

disparate topics presented in the abstract and focusing on calculation and formality. There is much work to be done in curriculum change before school curricula reach the breadth of the one in New Zealand.

Moving beyond this invited paper and responses, the remaining 11 articles in our special issue represent extensions of the research that was presented in Topic 6 at ICOTS 9. As you will read, the articles have been organized according to research regarding: (1) Elementary, Middle and High School Students, (2) University Students, and (3) Teachers (Prospective and In-Service).

3. ELEMENTARY, MIDDLE AND HIGH SCHOOL STUDENTS

Our special issue contains five articles that deal with research on learning and teaching probability within statistics at the school level. In particular, three articles focus on middle grade levels and two articles focus on senior grades.

The papers of Henriques and Oliveira, and Kazak, Fujita and Wegerif emphasize the importance of informal statistical inference in bridging statistics and probability at school level. The two studies differ on both the students' grade level and their focus. In particularly, Henriques and Oliveira focus on middle grade students' statistical investigations of data produced by students, while Kazak, Fujita and Wegerif focus on primary grade students' interpretations regarding probability distributions. They both highlight the potentialities of the tasks utilized in the classroom as well as the use of appropriate software tools as important factors in assisting students in developing statistical reasoning and its connection to ideas of probability. Particularly for primary students, Kazak, Fujita and Wegerif also emphasize the importance of teacher scaffolding and peer-to-peer interactions in the development of inferential reasoning. Furthermore, the dialogic discourse analysis utilized by Kazak, Fujita and Wegerif in the context of primary school students seems to be a useful tool in identifying and understanding shifts in students' reasoning and conceptual perspectives. It thus constitutes a viewpoint that can assist researchers in creating the conditions to support such shifts.

The articles by Saldanha and Prodromou also deal with probability and statistics at the school level, but focus on high school students. Saldanha reports on a classroom teaching experiment that engaged a group of high school students in designing sampling simulations within a computer microworld. The paper investigates the conceptual operations and challenges entailed in construing contextual situations as stochastic experiments, and in viewing the outcome of random sampling as a statistical quantity. It is based on a classroom teaching experiment which explored the thinking of a group of students as they engaged with tasks involving the design and use of computer simulations for carrying out statistical inference. The simulation-design activities aimed to foster students' abilities to conceive of contextual situations as stochastic experiments, and to engage them with the logic of hypothesis testing. Prodromou's article focuses on the reasoning about conditional probability and sampling procedures shown by a small group of students. The paper examines how students reason about conditional probabilities in problem situations, creating two-way tables to illustrate reduced sample spaces and the number of favorable outcomes before any numerical calculation of conditional probabilities. It also gives some examples of the difficulties students face when making conditional probability judgments.

The last article in this section of our special issue also deals with probability at the school level, but does so in a more abstract nature when compared to the previous four. In this article, Sharma contends that while research on misconceptions is abundant, more attention could be paid to the development of students' probabilistic thinking in the classroom. In line with this reasoning, the paper describes a lesson sequence for developing

students' probabilistic understanding. In line with the theme of this special issue, she demonstrates how the context can be used to explore the relationship between experimental and theoretical probabilities in a classroom setting.

4. UNIVERSITY STUDENTS

Following from the research presented in Topic 6 and in ICOTS 9 generally, this special issue includes three articles dealing with research on learning and teaching probability in a post-secondary or tertiary environment.

The paper of Lesser, Wagler and Salazar reminds us of the important role played by language in the learning and teaching of probability. The topic is replete with terms that have a specific technical meaning but are also used in general language in a sometimes different way. Classic examples are the terms "independence" and "fair" (in addition to terms such as "significant" that are important in the related area of statistics). Lesser's team investigates Spanish-language background learners in an introductory statistics class at a large university. The students are using an applet in probability, presented in a bilingual mode. Appropriate linguistic and other support can benefit such learners, as the authors show; indeed, this can benefit learners generally, even if they are native English speakers. A focus on careful usage of language, together with appropriate pedagogical approaches, such as small-group work and use of (maybe bi-lingual) glossaries, can be a positive step towards learning in probability.

Primi, Donati and Chiesi detail a mediation model that was estimated to derive the total, direct, and indirect effects of mathematical competence on statistics achievement taking into account probabilistic reasoning ability. The participants in their research were enrolled in an introductory statistics course (for psychology students). As the authors describe in their article, their research suggests, in line with the theme of this special issue, that "interventions designed to promote the mathematical prerequisites necessary to probabilistic reasoning can have a positive effect on achievement in statistics."

Lastly in this group of papers, Kuzmak presents an argument that understanding of probability and statistics is dependent upon building a "mature" understanding of common random phenomena such as the rolling of dice or the blind drawing of balls from an urn. Kuzmak's research, which draws on the analysis of undergraduate students at an American university, explores a cognitive schema representing a mature understanding that is contrasted to a diversity of observed immature understandings. Kuzmak also touches upon teaching to explicitly build the mature cognitive schema proposed, which nicely acts as a bridge to our last set of articles.

5. TEACHERS (PROSPECTIVE AND IN-SERVICE)

The last three articles found in our special issue focus on teachers, both prospective teachers currently in training and teachers who are already active in the teaching profession, maybe undertaking in-service courses. Certainly, those in the first group are also university students, but these students have a specific emphasis on their future as educators, and this translates into a particular focus in these studies on the teaching and learning of probability.

The article by Gómez-Torres, Batanero, Díaz and Contreras describes the development of a questionnaire designed to assess the probability content knowledge of prospective primary school teachers. Focusing on three different meanings of probability (classical, frequentist and subjective), the questionnaire content is based on curricular guidelines and primary school textbooks in Spain. Responses from 157 prospective primary school

teachers provide information about various aspects of participants' probability content knowledge.

Lee, Doerr, Tran and Lovett investigate the role of probability in developing learners' abilities to understand and use simulation approaches in the repeated sampling approach to statistical inference. Their research considers how learners and teachers build a (conceptual) model for a repeated sampling approach to inference that highlights the role of probability models, uses both the power of physical models and capabilities of computer simulations, and can be applied to a range of inference contexts. The setting of the study is two institutions, a team of four instructors and 27 graduate students. Data were collected while the students worked on designed model applications, and analyzed based on various representations. The authors identify the key conceptualizations of a repeated sampling approach to inference and show how they can be used by learners to develop a model for inference via the repeated sampling approach.

The final article, by Savard and Manuel, investigates in-service secondary mathematics teachers; it presents empirical classroom data on teachers' instructional goals as supported by their practices. The authors view the understanding of chance or probability as a crucial aspect of students' decision making processes in the development of statistical reasoning. Their results show, however, that teachers seem to focus on the mathematical context of a given problem, emphasizing procedural aspects rather than the understanding of probability. Moreover, this article reveals the potential of the blend of classroom observation with discussion groups with colleagues in investigating teachers' practices and conceptualizations, and consequently in helping researchers to establish links between research and practice.

6. CONCLUDING REMARKS

The papers in this special issue illustrate the complexity of teaching probability within statistics and consequently in developing conceptual connections between the two areas. Among the papers we can see various teaching approaches and teaching material that aim to support students to overcome this complexity and to build on correct and powerful probability intuitions and effective thinking processes. However, despite the reported positive results from these approaches and materials, their implementation and use was carried out by the researchers themselves. What is needed in addition is empirical data from practicing teachers who use innovative approaches and materials to support their students in integrating probability and statistics when dealing with problem solving and decision making situations. Such empirical data could give us further insight into the actual teaching practices and the associated challenges related to these approaches for "teaching probability within statistics," predominantly by mathematics teachers. Furthermore, this could also help researchers to deal more effectively with teachers' preparation and professional development and consequently to have a stronger impact on statistics classrooms.

We hope that you enjoy this special issue as much as much as we, the guest editors, have enjoyed putting it together, and that you find it useful in your continued thinking about learning and teaching probability within statistics. We look forward to continuing the discussion at the forthcoming ICOTS 10 in Kyoto! Lastly, the other members of the editorial team would like to congratulate Dionysia, who welcomed a healthy and lovely baby boy into the world while working on this issue!