RESEARCH ON REASONING ABOUT VARIABILITY: A FORWARD

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We are very pleased to introduce this special issue of the Statistics Education Research Journal (SERJ), which presents cutting-edge research in an area of increasing importance: *Reasoning about variability*. The notion of variability and the importance of its role in statistics have been documented by David Moore, the well-known statistician and former president of IASE and ASA. Moore (1990) describes statistical thinking as recognizing the omnipresence of variability and considering appropriate ways to quantify and model the variability of data. Wild and Pfannkuch (1999) further describe the centrality of variability in statistical thinking, as revealed by their studies of expert statisticians solving statistical problems: "*Variation is the reason why people have had to develop sophisticated statistical methods to filter out any messages in data from the surrounding noise*" (p. 236).

We note that the terms *variability* and *variation* are often used in the teaching and research literature interchangeably, and this may add to a confusion regarding this complex area. Reading and Shaughnessy (2004) address this problem and offer the following definitions for the two terms: "*Variation* is a noun used to describe the act of varying or changing condition, and *variability* is one noun form of the adjective *variable,* meaning that something is apt or liable to vary or change" (p. 201). However, they note that educators and researchers often refer to *variability* as the characteristic of the entity that is observable, and the term *variation* as the measuring of that characteristic. So a distinction is made between what is observed (what is varying) and what is measured. Moore (1997) points out that both variability and the measuring and modeling of that variability are important, however he does not distinguish between terms used to describe these phenomena. The papers in this special issue do not necessarily follow these definitions; however they mostly refer to "variability" to represent how data vary.

Despite the attention paid by statisticians and statistics instructors to this important topic, to date little has been published about how people, particularly novices and statistics students, actually reason about variability, or how this type of reasoning develops in the course of statistics instruction. Examples for challenging questions that call for careful attention by researchers and educators are: What are the simplest forms of variability that children can understand? What are instructional tasks and technological tools that promote the understanding of variability? What are the common misconceptions regarding variability? What are the difficulties that people encounter when dealing with variability in data? What does correct reasoning about variability look like? What are ways to assess understanding of variability? How does an understanding of variability connect and effect understanding of other statistical concepts and types of reasoning? What are useful methodologies for studying the understanding of variability? What type of understanding of variability is sufficient for a statistically literate person?

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In response to these challenges, "*Reasoning about Variability*" became the theme of the third and most recent international research forum on Statistical Reasoning, Thinking and Literacy (SRTL-3), held in July 2003 at the University of Nebraska, USA. The SRTL-3 forum built on the two previous SRTL forums, held in 1999 in Israel and in 2001 in Australia. The five papers appearing in this special issue include one invited paper (Gould) and four peer-refereed papers (Hammerman & Rubin, Ben-Zvi, Bakker, Reading) that are based in part on presentations and discussions at SRTL-3.

A unique aspect of the SRTL forums is the opportunity to bring together a small number of researchers whose work is focused in a particular area. They have an opportunity to present their research in extended sessions that permit lengthy discussions among the participants. In addition, many researchers present video clips of students discussing and explaining their actions and reasoning; this allows for intensive review and discussion of research methods and results by all participants.

At SRTL-3, a variety of researchers from diverse backgrounds and countries presented studies that examined different aspects of reasoning about variability. For example, some looked at the inherent variability of data, variability as represented in a univariate or bivariate distributions, the role of variability in comparing groups, students' understanding of particular measures of variability (e.g., the standard deviation), and variability in different sampling contexts (Lee, 2003). The studies presented involved students from elementary school through college, and some studies also examined teachers' reasoning about variability. After five days of presentations and discussions, the participants were reinforced in their belief that variability is a complex topic to understand, learn and teach, and that its understanding is a fundamental component of statistical reasoning and thinking. We also began to design a hypothetical learning trajectory that may be useful to help guide teachers as they aim to develop students' understanding of this topic. We see the role of innovative technological tools, appropriate teacher guidance and curricular tasks, as well as using good data sets, as crucial in helping students develop this idea.

The first paper in this Special Issue, by *Gould*, is based on his opening address at SRTL-3 and designed to provide a statistician's view of the importance of noticing, understanding and analyzing variability and using measures of variation when making sense of data. This paper frames issues and provides examples that help to appreciate the complexity of the conceptual issues that researchers and educators have to grapple with. The paper by *Hammerman & Rubin* focuses on secondary level teachers who are learning statistics and attempting to cope with variation in data using the innovative Tinkerplots software (Konold & Miller, 2004). The paper by *Ben-Zvi* provides a detailed qualitative analysis of the ways by which two seventh grade students started to develop views (and tools to support them) of variability in comparing groups task using various statistical representations. The paper by *Bakker* describes activities and use of technological tools by advanced 8th-grade students and how their reasoning about variability and variation is developing. The paper by *Reading* suggests hierarchies to assess high school students' understanding of variation, one for more qualitative descriptions and the other for more quantitative descriptions of variation.

The research studies presented in this special issue have several common features. Their topics reflect the shift in emphasis in statistics instruction, from statistical techniques, formulas, and procedures to developing statistical reasoning and thinking. These studies employ various types of qualitative methodologies, which appear to have uncovered many interesting points about how students and teachers reason about variability. Most of them use extended teaching experiments, or represent cases where researchers collaborated with teachers in field settings or designed specialized learning episodes or environments, to be able to elicit detailed and deep data about students' actions and reasoning.

Most of the studies in this special issue emphasize the role of technology (statistical software or specially designed tools) in developing students' statistical reasoning about the variability of data. This is not surprising, given how the discipline of statistics has depended on technology and how technology has been driving changes in statistical practice. Although there are many technological tools available, including graphing calculators, computers, and the World Wide Web, there is still a lack of research on how to best use these tools and how they affect student learning. Regardless of the type of technology used or the level of the students studied, all the studies attempt to understand the

development of conceptual models that students (or teachers) use to reason about data and its variability. Together, these studies help us understand the complexity of the idea of variability, and its interconnectedness to core statistical ideas of data, sampling, distribution, and center.

The forthcoming issue of SERJ (May 2005) will also include a special section of papers related to reasoning about variability which will enable us to continue the attention to research on this important area. Among them will be two invited papers which will reflect on the collection of papers in this special issue, by *Cliff Konold* (University of Massachusetts, Amherst, USA) and *Maxine Pfannkuch* (University of Auckland, New Zealand). We hope that their responses will lead to productive discussions about the importance of the notion of variability in statistics education as well as about ways to further study and improve its development in students at different educational levels and contexts.

We appreciate having the opportunity to put together these papers and responses for publication in SERJ, and especially value all the contributions of the co-editors, in particular, *Iddo Gal* (University of Haifa, Israel), who oversaw this special issue and offered many suggestions to improve the quality of the papers. We also thank all of the participants at SRTL-3 who contributed to discussions of earlier versions of these papers and to those who served among the reviewers of these papers. We invite readers with comments and suggestions to contact us. Finally, we invite researchers to review information (see "Forthcoming Conferences" in this issue) on the forthcoming SRTL-4, to be held in 2005 in New Zealand, which will be devoted to *Reasoning about Distributions*.

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