A MULTI-INSTITUTIONAL STUDY OF THE RELATIONSHIP BETWEEN HIGH SCHOOL MATHEMATICS ACHIEVEMENT AND PERFORMANCE IN INTRODUCTORY COLLEGE STATISTICS

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ABSTRACT

In this study we examined the effects of prior mathematics achievement and completion of a commercially developed, National Science Foundation-funded, or University of Chicago School Mathematics Project high school mathematics curriculum on achievement in students' first college statistics course. Specifically, we examined the relationship between students' high school mathematics achievement and high school mathematics curriculum on the difficulty level of students' first college statistics course, and on the grade earned in that course. In general, students with greater prior mathematics achievement took more difficult statistics courses and earned higher grades in those courses. The high school mathematics curriculum a student completed was unrelated to statistics grades and course-taking.

Keywords: *Statistics education research; Post-secondary education; Course-taking*

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1. INTRODUCTION

Statistics has received increasing recognition as an important part of both secondary and post-secondary education (we define post-secondary as institutions offering a Bachelor's degree). At the post-secondary level, an increasing number of students are taking courses in statistics, many of which satisfy the now common quantitative literacy graduation requirement for undergraduates. In fall 2005, an estimated 260,000 U.S. undergraduate students enrolled in a statistics course, an increase of over 40,000 students from 1995 (Lutzer, Rodi, Kirkman, & Maxwell, 2007). This number is likely an underestimate as it is limited to enrollment in courses offered by mathematics and statistics departments, and thus, does not count students who take statistics courses in other departments. These courses are often taught in psychology, biology, or economics departments and may differ from those typically offered by mathematics and statistics departments in the emphasis placed on understanding statistical theory and the importance of possessing a good working knowledge of calculus.

In this study, we explored the relationship between students' prior mathematics achievement in high school and achievement in their first college statistics course. Mulhern and Wylie (2005) identified six aspects of mathematical thinking relevant to success in statistics: calculation, algebraic reasoning, graphical interpretation, proportionality and ratio, probability and sampling, and estimation. Intuitively, student achievement in high school mathematics would impact these aspects of mathematical thinking and students' subsequent achievement in college statistics courses. Yet there has been little empirical study of the impact of students' prior mathematics achievement in high school on their achievement in college statistics courses. Moreover, the fact that the mathematics preparation of many college bound students is inadequate, as evidenced by numerous statistics; for example, 25% of all U.S. college freshman enroll in a developmental mathematics course (i.e., a non-credit bearing course that should have been completed in high school) (National Center for Education Statistics, 2002). This raises important questions about the adequacy of students' preparation for college statistics coursework.

To date, much of the research examining achievement in college statistics coursework has focused on factors such as student attitudes and anxiety toward statistics (see Baloglu & Zelhart, 2003, for a review), sex (e.g., Schram, 1996), and statistical reasoning (Garfield & Ben-Zvi, 2009; Zieffler, Garfield, delMas, & Reading, 2008). The role of classroom characteristics on statistics achievement has also been studied, and has included the influence of working in small-groups (e.g., Delucchi, 2007; Perkins & Saris, 2001) and of on-line learning environments (e.g., Everson, 2005). The absence of research examining the role of students' prior mathematics achievement on college statistics performance is, in part, attributable to arguments that mathematical reasoning and statistical reasoning are distinct cognitive processes, resulting from differences between mathematics and statistics as disciplines (Garfield & Gal, 1999). Gal and Garfield (1997) distinguish the discipline of statistics from that of mathematics in four ways: the role of context in interpreting data, the indeterminacy of data, the reduced need for accurate computation and execution of mathematical procedures in light of technological advances, and the need to make judgments and inferences and evaluate the quality of those judgments and inferences to solve statistical problems.

The Gal and Garfield (1997) framework provides an important jumping-off point for beginning to understand the nature of the relationship between mathematical and statistical reasoning. However, the absence of research examining the role of students' prior mathematics achievement on college statistics performance is surprising given the wide-spread recognition of the importance of mathematical preparation in other quantitatively-oriented fields like economics and finance (Johnson & Kuennen, 2006). In short, our view is that the practice of statistics is more than a simple application of mathematics and requires reasoning skills not required in mathematics, but it seems unlikely that the ability to reason statistically (and be successful in college statistics courses) is unrelated to a student's ability to reason mathematically (reflected in prior mathematics achievement). However, as noted above, little work has been done to examine this relationship.

Studies that have examined mathematics preparation have largely been limited to examining student achievement in business statistics courses. For example, Johnson and Kuennen (2006) examined the impact of students' mathematical skills (basic and more advanced) and course-taking in college mathematics on the grades earned in an introductory course in business statistics. The most interesting finding was that students' scores on a basic skills mathematics test were a significant predictor of statistics grades, whereas scores on the mathematics portion of the ACT college entrance exam (www.act.org) and whether or not a student had taken calculus were not significant predictors. This suggests that a strong background in more advanced mathematical topics, like calculus, is not necessary to be successful in applied statistics but that the ability to perform more fundamental mathematical skills is important. In contrast, Green, Stone, Zegeye and Charles (2009) found that students who had taken a rigorous sequence of mathematical courses in college (including calculus) were twice as likely to receive an A in a required course in business statistics as students who had taken the minimum prerequisite mathematics courses (which did not include calculus). Collectively, this evidence suggests that some mathematical proficiency is needed to do well in college statistics but the role of more advanced mathematical skills is less clear.

An important, and to the best of our knowledge unstudied, aspect of student preparation for college work in statistics is the mathematical preparation students receive in high school. With the exception of students' scores on the mathematics portion of the ACT or SAT, variables reflecting students' high school mathematics preparation, such as the curriculum a student completed, have not been studied. This is surprising given the widely accepted view that high school mathematics should prepare college bound students for college mathematics (Mathematical Sciences Education Board, 2004; National Research Council, 2004), and the growing body of research examining this relationship (e.g., Harwell et al., 2009; Post et al., 2010; Schoen & Hirsch, 2003). This study contributes to this literature by examining the role of students' high school mathematics achievement, and the impact of completing a particular high school mathematics curriculum, on college statistics performance. The results have important implications for high schools preparing students for post-secondary study and for post-secondary institutions advising students on appropriate statistics coursework.

In some high schools, statistics is offered as a separate elective course; for example, the College Board's Advanced Placement (AP) statistics course, which is a non-calculus based introduction to college level statistics. In 2010, nearly 130,000 U.S. high school students took the AP statistics exam, with 34% of these students earning a score of four or higher (College Board, 2010a), qualifying them to receive college credits at many post-secondary institutions. More typically, statistics material is integrated into the high school mathematics curriculum. The National Council for Teachers of Mathematics (NCTM) outlined learning goals for statistics and probability (at all grade levels) in their document The Principles and Standards for School Mathematics (NCTM, 1989, 2000). It should also be noted that the mathematics portion of the SAT college entrance exam includes items assessing knowledge of data analysis, statistics, and probability (College Board, 2010b).

We identified three general categories of high school mathematics curricula: National Science Foundation (NSF)-funded curricula, the University of Chicago School Mathematics Project (UCSMP) curriculum, and a broad category of commercially developed (CD) curricula. The NSF-funded curricula were developed based on the Principles and Standards for School Mathematics (NCTM, 1989, 2000), and include topics in data analysis and probability as well as algebra, geometry, and topics in discrete mathematics at all grades levels. The data analysis and probability standard states that students should be able to

1) Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
2) Select and use appropriate statistical methods to analyze data;
3) Develop and evaluate inferences and predictions that are based on data, and;
4) Understand and apply basic concepts of probability. (NCTM, 2000, p. 4)

As an example, statistical topics covered in one NSF-funded curriculum, Contemporary Mathematics in Context (Core-Plus) (Coxford et al., 1998), include descriptive statistics, simulation, correlation and regression, sampling distributions, and probability (see Appendix A for a full list of statistical topics covered in the Core-Plus curriculum). Other NSF-funded curricula included in the present study are the Interactive Mathematics Program (IMP) (Fendel, Resek, Alper, & Fraser, 1998) and Mathematics Modeling Our World (MMOW) (Garfunkel, Godbold, & Pollack, 1998), both of which include similar statistical topics at all grade levels.

The UCSMP mathematics curriculum emphasizes problem solving, everyday applications, and the use of technology in the mathematics classroom at all grade levels (Usiskin, 1986). First developed in the early 1980s, UCSMP is currently in its third edition and has received both public (NSF) and private (Amoco) funding. In addition to courses in Algebra, Geometry, Advanced Algebra, and Precalculus & Discrete Mathematics, the UCSMP secondary curriculum includes a course in Functions, Statistics, & Trigonometry (FST). This course is typically taken after a student has completed the Advanced Algebra course and before they enroll in the Precalculus & Discrete Mathematics course. Statistical topics covered in the FST course include data collection, descriptive statistics, graphical representations of data, probability, simulation, and sampling distributions. Of the thirteen chapters in the current FST textbook, three are dedicated to these topics.

Although most CD curricula include statistical topics, the depth and breath of this coverage is limited. Generally, topics in data analysis and probability are included as a single chapter in Algebra I and II textbooks; for example, *Merrill Algebra 2 with Trigonometry* (Foster, Gordon, Gell, Rath, & Winters, 1995), often as the last chapter. In some cases statistical topics are included as a single unit in a chapter containing other units; for example *Algebra: Structure and Method Book One* (Brown et al., 1997). The most frequently addressed topics include measures of central tendency, measures of dispersion, and normal distributions as well as probability.

We hypothesize that because the NSF-funded curricula explicitly include statistical topics at all grade levels, students who completed one of these NSF-funded curricula in high school will be better prepared for college statistics than students who completed the UCSMP curriculum or a CD curriculum. To better understand the role of high school mathematics preparation on students' college statistics performance, we posed two research questions:

1) Is there a relationship between students' high school mathematics achievement and the (a) difficulty level of their first college statistics course, and (b) grade earned in their first college statistics course, controlling for background variables (sex, ethnicity, college major)?

2) Is there a relationship between the type of high school mathematics curriculum completed (NSF-funded, UCSMP, CD) and the (a) difficulty level of students' first college statistics course, and (b) grade earned in students' first college statistics course, controlling for background variables (sex, ethnicity, college major)?

Student high school mathematics achievement variables included ACT mathematics test score, years of high school mathematics completed, and high school mathematics GPA. We focus on the first college statistics course because the majority (90%) of students in our sample took a single statistics course at the college level, whereas 7% of students took two statistics courses, and the remaining 3% took three or more courses (up to eight courses).

2. METHOD

2.1 RESEARCH DESIGN

A retrospective, cohort, cluster design was used in which the difficulty level and grade of students' first college statistics course were examined. Based on their high school mathematics curriculum, students were categorized into one of three curriculum cohorts (NSF-funded, UCSMP, CD). To accurately categorize students, we contacted each of the approximately 300 high schools represented in the student sample to obtain descriptions of their mathematics programs, the mathematics courses offered, and the textbooks used. This information was then coordinated with each student's high school transcript. Although this effort required significant resources, it was necessary to ensure that categorizing a student as having completed a particular curriculum was accurate. All other student data, including their high school mathematics transcripts, college statistics course-taking and performance, and demographic information, was obtained from each student's post-secondary institution.

2.2 POPULATION AND SAMPLE

The target population consisted of college students in the United States who completed at least three years of high school mathematics in a CD, NSF-funded, or UCSMP curriculum and took at least one statistics course at the post-secondary level. The population was restricted to students who completed at least three years of high school mathematics because post-secondary students will generally be expected to have completed at least three years (Boyer, 1983) of high school mathematics, as reflected in the mathematics portions of the ACT and SAT exams (ACT, 2009; College Board, 2010c).

The sampled population consisted of n = 5,308 students from 23 purposively sampled post-secondary institutions (clusters). However, three institutions were excluded because their within-institution sample sizes were too small to provide adequate data for parameter estimation, reducing the number of students to 5,296. Sampled students enrolled in college during the fall 2002 and fall 2003 terms and had graduated from a Minnesota high school. These terms were selected because they represented the first groups of students who had the opportunity to complete four full years of the published (i.e., not pilot materials) NSF-funded curricula.

The remaining 20 four-year institutions were located in the upper midwest of the United States. Seven of the 20 post-secondary institutions were publicly-funded universities and are part of one of four states' college systems, whereas the remaining 13 institutions were privately-funded liberal arts colleges. All institutions are not-for-profit.

Four institutions are located in a large metropolitan area with the remainder located in suburban and rural settings. Eight institutions were classified as "most selective" according to the Carnegie Foundation's Undergraduate Profile Classification (Carnegie Foundation, 2009b), indicating that test scores for first year students place these institutions in the top-fifth of post-secondary institutions offering Bachelor's degrees. Eleven institutions were classified as "selective," which means that test scores for first year students place these institutions in the middle two-fifths of post-secondary institutions. The remaining institution was classified as "inclusive," which indicates that the institution either did not report test score data or does not make admissions decisions based on test score data.

Due to missing data, the sample was further reduced to n = 3,599 students. The variable with the most missing data was college major with 13% of students (n = 707) not providing values, most of whom were in "Other" majors, which included technologies, technicians, trades, military, and culinary arts. A missing values analysis of all variables revealed no significant differences in means between the data when all available values were included and the data set containing the n = 3,599 students who provided complete data. In addition, refitting our statistical models to subsets of students after filtering on missing values (e.g., including "Other" college majors) produced similar patterns of findings (regarding statistical significance and the direction of effects) to those based on all available data. Together, these analyses suggest that missing data did not seriously distort our findings.

Our sample is not random in the traditional sense (Cochran, 1977). However, we believe that following the purposive sampling arguments of Shadish, Cook, and Campbell (2002), and the similarity of the 20 sampled institutions to those of U.S. institutions represented in the undergraduate profile classification of the Carnegie Foundation (on institutional selectivity and nature of the student body), our results are generalizable to students attending post-secondary institutions beyond our sample.

2.3 VARIABLES

Archival data reflecting variables at the high school and college level were collected from each of the post-secondary institutions. Student-level predictor variables were the mathematics curriculum a student completed (NSF-funded 1 = yes, 0 = no; UCSMP 1 = yes, 0 = no so CD served as the reference group), ethnicity (African American 1 = yes, 0 = no; Hispanic 1 = yes, 0 = no; Asian 1 = yes, 0 = no; so Caucasian students served as the reference group), sex (1 = male, 0 = female), and college major (1 = STEM, 0 = non-STEM). Students' majors were classified as STEM (Science, Technology, Engineering or Mathematics) or non-STEM using the Carnegie Foundation's Undergraduate Instructional Program Classification (Carnegie Foundation, 2009a). Other predictors were students' ACT mathematics score, years of high school mathematics completed (3, 4, 5), and their GPA in high school mathematics courses. Years of high school mathematics completed was coded such that a value of three reflects completion of an Algebra II course, a value of four reflects completion of a Pre-calculus course, and a value of five reflects completion of a Calculus I course in high school. Students' high school mathematics GPAs were captured using a twelve-point scale ranging from A (scale value of 4.0), A- (scale value of 3.67), down to F (scale value of 0.0).

One dependent variable reflected the difficulty level of the first statistics course a student completed in college, allowing for the possibility that students with differing prior mathematics achievement, who completed different high school mathematics curricula, would enroll in courses of varying difficulty. This variable was constructed after a careful review of information about college statistics courses that included a course description, prerequisites, a syllabus, and the department in which the course was taught (e.g., Statistics, Nursing). Combining this information produced the following difficulty scale for college statistics courses:

Level 1: Least difficult (prerequisite is usually 1-2 yrs of high school algebra, content limited to simple methods and procedures like *t*-tests and ANOVA, little or no focus on statistical theory);

Level 2: Moderately difficult (at least one college prerequisite, moderately-difficult content such as continuous/discrete probability distributions and mathematical expectation, moderate attention paid to statistical theory, procedures covered typically include multiple regression);

Level 3: Most difficult (typically two college prerequisites, advanced content like moment generating functions and likelihood ratio test principle, considerable emphasis on statistical theory, linear models, and multivariate and Bayesian methods).

The students in our sample took 6,090 statistics courses across eight semesters of post-secondary coursework, 88.2% of which were taken at Level 1, 7.6% at Level 2, and 4.6% at Level 3. A review of the difficulty level of students' first course showed that 93.4% of students began at Level 1, 5.7% began at Level 2, and the remaining 0.9% began at Level 3. This led us to aggregate Level 2 and Level 3, producing the following dichotomous scale representing the difficulty level of students' first course in post-secondary statistics:

Level 1: Applied introduction to statistics, prerequisite is usually 1-2 yrs of high school algebra and content is limited to simple methods (e.g., *t*-tests, ANOVA, correlation/regression, sampling distribution of the mean) with little or no focus on statistical theory, course is typically taught in a service department (e.g., psychology, public health, business);

Level 2: Theoretical introduction to statistics, prerequisite is usually one semester of calculus and content includes advanced topics (e.g., moment generating functions, likelihood ratio test principle, multivariate and Bayesian methods) with greater emphasis on statistical theory, course is typically taught in a mathematics or statistics department but may be taught in a service department.

The difficulty variable was coded such that Level 2 = 1 and Level 1 = 0.

A second dependent variable reflected student achievement levels captured through the grade students earned in their first college statistics course. Grades were again captured using a twelve-point scale ranging from A (scale value of 4.0), A- (scale value of 3.67), down to F (scale value of 0.0), which was treated as showing an interval scale of measurement.

2.4 DATA ANALYSIS

We used descriptive analyses to describe patterns in the data and inferential analyses to address the research questions. To model the difficulty data, a hierarchical generalized linear model (Raudenbush & Bryk, 2002) was fitted, where students were treated as nested within colleges. Preliminary analyses revealed that only the intercepts varied across institutions, and, because of the small number of institutions (j = 20), no college-level covariates were included in the model. Slopes from the student level covariates were treated as fixed across colleges and the fitted model for the difficulty level outcome had the following form:

$$\log(y_{ij}) = \gamma_{00} + \sum_{p}^{P} \gamma_{0p} X_{pij} + u_{0j} \tag{1}$$

where, $\log(Y_{ij})$ is a vector of outcomes reflecting the log-odds of the difficulty level of the first course for the i^{th} (i = 1, 2, … , n_j) student in the j^{th} (j = 1, 2, … , J) post-secondary institution, $\boldsymbol{\gamma_{00}}$ is the precision-weighted average log-odds of beginning with difficulty Level 2, $\boldsymbol{\gamma_{0p}}$ is a precision-weighted average slope capturing the effect of the p^{th} covariate X_p, and u_{oj} is the random effect for intercepts.

To model the grade data, a hierarchical linear model (Raudenbush & Bryk, 2002) was fitted, where students were treated as nested within colleges. Again, because of the small number of institutions no college-level covariates were modeled. Likewise, preliminary analyses revealed that only the intercepts varied across colleges as such slopes for student-level covariates were treated as fixed across institutions. The fitted model for the grades outcome had the following form:

$$y_{ij} = \gamma_{00} + \sum_{p=1}^{P} \gamma_{0p} X_{pij} + u_{0j} + r_{ij} \tag{2}$$

where, Y_{ij} is a vector of outcomes reflecting the grade in the first course for the i^{th} (i = 1, 2, … , n_j) student in the j^{th} (j = 1, 2, … , J) post-secondary institution, $\boldsymbol{\gamma_{00}}$ is the precision-weighted average grade, $\boldsymbol{\gamma_{0p}}$ is a precision-weighted average slope capturing the effect of the p^{th} covariate X_p, u_{oj} is the random effect for intercepts, and r_{ij} is the student level residual term. The student-level covariates included in the models fit to the grade and difficulty outcomes were identical except that the difficulty variable was used as a covariate in the grade model. The grade data effect sizes were calculated following

$$\delta_p = \frac{\delta_{0p}}{\sqrt{\tau_{00} + \sigma^2}} \tag{3}$$

where, δ_p is the standardized effect size for the p^{th} covariate, γ_{0p} is the precision-weighted average slope capturing the effect of the p^{th} covariate, τ_{00} is the variance component associated with the unconditional random intercepts, and σ^2 is the variance of the unconditional student-level error term (Raudenbush, Spybrook, Congdon, Liu, & Martinez, 2009). An a priori power analysis using equation (3) and the Optimal Design Software (Raudenbush et al.) revealed a statistical power of approximately .75 to detect a fixed-effect for high school mathematics curriculum of moderate size (.40) (Spybrook, Raudenbush, Congdon, & Martinez, 2009).

To control for compounding of Type I error rates we used an adjusted Type I error rate attributed to Sidak (1967). The adjusted error rate was computed as $\alpha' = 1 - (1 - \alpha)^{1/k}$, where α is the unadjusted Type I error rate, k = number of statistical tests, and α' is the adjusted Type I error rate. For the two-level models that follow the adjusted error rate was $\alpha' = 1 - (1 - \alpha) = .008$, where $\alpha = .10$, and k = 11 for the grade outcome and 10 for the difficulty level outcome. All of the multilevel analyses were performed using the lme4 package in R 2.10.1 (The R Project for Statistical Computing).

3. RESULTS

2.5 DESCRIPTIVE ANALYSES

We began by performing descriptive analyses to explore patterns in the data. As noted previously, 90% of the students in our sample took one statistics course at the college level, whereas 7% of students took two courses and the remaining 3% took three or more courses. Regarding the difficulty of the first statistics course taken, 93% of students began with a course equal to difficulty Level 1 (applied introduction) and 7% began with a course equal to difficulty Level 2 (theoretical introduction).

Of the students in our sample, 67% completed a CD curriculum in high school, 19% completed the UCSMP curriculum, and 14% completed a NSF-funded curriculum. Descriptive statistics for variables in our statistical models can be found in Table 1. Table 2 presents descriptive statistics for enrollment, 25th and 75th percentiles of ACT mathematics scores for first year students, percentage of African American students, and percentage of STEM majors across institutions. Within institution samples sizes (n_j) ranged from 14 to 1,213 (M = 265, SD = 284). Table 3 contains bivariate correlations between variables, most of which are relatively small.

Table 1. Descriptive statistics for student variables

	Mean[a]	Std Dev	Minimum	Maximum
Difficulty level of first course	0.07	0.249	0.00	1.00
Grade of first course	3.05	0.834	0.67	4.00
ACT mathematics score	24.23	4.476	11.00	36.00
High school mathematics GPA	3.25	0.643	0.66	4.00
Years of high school mathematics	4.07	0.740	3.00	5.00
UCSMP	0.19	0.400	.00	1.00
NSF-funded	0.14	0.349	.00	1.00
African American	0.03	0.156	.00	1.00
Asian	0.07	0.255	.00	1.00
Hispanic	0.02	0.129	.00	1.00
Male	0.41	0.493	.00	1.00
STEM major	0.30	0.459	.00	1.00

Note. Difficulty level of the first course 1 = Level 2, 0 = Level 1; Curriculum variables (UCSMP, NSF-funded) were coded so that CD students served as the reference group; Ethnicity variables were coded so that Caucasian students served as the reference group; Male 1 = yes, 0 = no; STEM major 1 = yes, 0 = no.

[a]Mean values that are less than 1 can be interpreted as percentages, such as 19% of the sample completed a UCSMP high school mathematics curriculum.

Table 2. Descriptive statistics for institution variables (j = 20)

	Mean	Std Dev	Minimum	Maximum
Enrollment	6384.15	8170.60	440.00	37383.00
ACT math 25th percentile	21.85	2.54	19.00	28.00
ACT math 75th percentile	26.70	2.39	23.00	32.00
Percentage of African American students	2.15	2.46	.00	7.69
Percentage of STEM majors	7.22	4.90	.00	19.55

Table 3. Correlations between student variables

	1	2	3	4	5	6	7	8	9	10	11	12
Difficulty level	1.0											
Grade	-.00	1.0										
ACT math	.14**	.34**	1.0									
HS math GPA	.10**	.41**	.49**	1.0								
Years HS math	-.05**	.21**	.44**	.28**	1.0							
UCSMP	.03	.01	.07**	.01	.02	1.0						
NSF-funded	-.05**	-.04**	-.15**	-.03	-.04**	-.20**	1.0					
African Amer	-.03*	-.08**	-.17**	-.08**	-.07**	-.02	.05**	1.0				
Asian	-.00	-.07**	-.11**	-.01	.02	-.02	.05**	-.04**	1.0			
Hispanic	-.02	-.02	-.05**	-.05**	-.04**	-.00	.02	-.02	-.04**	1.0		
Male	-.02	-.09**	.15**	-.09**	.09**	.02	-.00	.02	-.01	-.01	1.0	
STEM major	.28**	.11**	.29**	.24**	.13**	.05**	-.07**	-.02	.02	.01	.07**	1.0

Note. HS = high school; Difficulty level 1 = Level 2, 0 = Level 1; Curriculum variables (UCSMP, NSF-funded) were coded so that CD students served as the reference group; Ethnicity variables were coded so that Caucasian students served as the reference group; Male 1 = yes, 0 = no; STEM major 1 = yes, 0 = no.
*$p < .05$. **$p < .01$.

2.6 INFERENTIAL ANALYSES

Difficulty level of the first college statistics course First, an unconditional model was fitted to the difficulty outcome. The average (across institutions) log-odds of a college student beginning with a course of difficulty Level 2 (theoretical introduction) as opposed to Level 1 (applied introduction) was -4.33. This means that for a "typical" institution (i.e. with a random effect of 0) the expected log-odds of taking a first statistics course of difficulty Level 2 is -4.33, which corresponds to an odds ratio of 0.01.

The results for the conditional model (see Table 4) revealed that the type of high school mathematics curriculum a student completed and variables reflecting prior mathematics achievement (with one exception) were unrelated to the log-odds of students beginning their statistics coursework at difficulty Level 2, as were the ethnicity and sex variables. ACT mathematics score and whether a student was a STEM major were significant predictors of the log-odds that a student would begin their statistics coursework with a course of difficulty Level 2. Being a STEM major was associated with an expected increase in the log-odds of a student beginning their statistics coursework at difficulty Level 2 of 1.11, with the other predictors held constant. Converting to an odds ratio produces 3.03. Thus, the odds that a student begins their coursework at difficulty Level 2 is three times higher for STEM majors than non-STEM majors. The effect of ACT mathematics score on the odds a student begins with a statistics course at Level 2 is considerably smaller; for example, at no point on the ACT mathematics scale does the predicted probability of a student beginning with a course of difficulty Level 2 exceed 0.50.

Grade earned in the first college statistics course In order to determine whether a multilevel model was necessary to analyze the grade outcome, an unconditional model was fitted first. This initial analysis showed that there was enough variation in average grades between the post-secondary institutions to continue modeling ($\hat{\rho}_{ICC} = .04$). Next we constructed a predictive model to account for the variation in grades. The results of this analysis are presented in Table 5. Effect sizes for all statistically significant findings are presented.

Table 4. Multilevel results of the difficulty level of the first college statistics course

Between-Student Model				
Effect	*b*	SE(*b*)	*z*-value	*p*-value
Intercept	-8.908*	1.067	-8.35	<0.001
ACT math score	0.176*	0.025	7.03	<0.001
Years of HS math	0.019	0.134	0.14	0.888
HS math GPA	-0.122	0.173	-0.70	0.482
UCSMP	-0.048	0.205	-0.24	0.814
NSF-funded	-0.111	0.313	-0.35	0.723
African American	-0.034	0.823	-0.04	0.967
Asian	0.337	0.311	1.08	0.279
Hispanic	-0.057	0.698	-0.08	0.935
Male	-0.072	1.176	-0.41	0.680
STEM	1.109*	0.197	5.64	<0.001

Note. Difficulty level of the first course 1 = Level 2, 0 = Level 1; Curriculum variables (UCSMP, NSF-funded) were coded so that CD students served as the reference group; Ethnicity variables were coded so that Caucasian students served as the reference group; Male 1 = yes, 0 = no; STEM major 1 = yes, 0 = no.
*$p < .008$.

Table 5. Multilevel results for the grade earned in the first college statistics course

Between-Student Model				
Effect	*b*	SE(*b*)	*t*-value	Effect Size
Intercept	0.978*	0.102	9.59	----
Difficulty of first course	-0.165*	0.025	-6.68	-0.197
ACT math score	0.038*	0.004	10.28	0.045
Years of HS math	0.053	0.019	2.74	
HS math GPA	0.406*	0.023	17.72	0.486
UCSMP	-0.012	0.031	-0.40	
NSF-funded	-0.008	0.036	-0.21	
African American	-0.092	0.081	-1.14	
Asian	-0.197*	0.048	-4.08	-0.236
Hispanic	-0.028	0.092	-0.30	
Male	-0.127*	0.026	-4.89	-0.152
STEM	-0.001	0.031	-0.04	

Note. Difficulty level of the first course 1 = Level 2, 0 = Level 1; Curriculum variables (UCSMP, NSF-funded) were coded so that CD students served as the reference group; Ethnicity variables were coded so that Caucasian students served as the reference group; Male 1 = yes, 0 = no; STEM major 1 = yes, 0 = no.
*$p < .008$.

These results indicate that the type of high school mathematics curriculum a student completed was unrelated to the grade earned in the first course. Students' prior mathematics achievement was the strongest predictor of the grade earned in the first college statistics course, with both ACT mathematics score (b = 0.05) and high school mathematics GPA (b = 0.41) significant predictors. Despite ACT mathematics score being a statistically significant predictor, its practical significance seems to be relatively small. Students with ACT mathematics scores greater than 21 (the U.S. average) are predicted to receive a B (3.0) or higher in their first college statistics course. Further, at no point on the ACT mathematics scale are students predicted to fail their first college statistics course by earning less than a C-. Another significant predictor was the difficulty level of the first course (b = -0.17), with students tending to earn lower grades in more difficult courses. Asian students (b = -0.20) also earned lower grades in the first course than Caucasian students, and male students (b = -0.13) earned lower grades than female students.

4. DISCUSSION

In the present study we sought to examine the relationship between students' high school mathematics preparation (achievement and curriculum) and performance in their first college statistics course. In response to our research questions, our findings can be summarized in two ways. First, students with stronger prior mathematics achievement generally took more difficult statistics courses and earned higher grades in those courses than students with weaker prior mathematics achievement. This finding suggests that a stronger mathematical background benefits students in college statistics. Whereas ACT mathematics score was a significant predictor of the difficulty outcome, the size of the effect was small, with students who earned a perfect score on the ACT mathematics exam (i.e., 36) only having a predicted probability of taking a theoretical introductory course of 0.25. None of the other prior mathematics achievement variables were significant predictors of the difficulty outcome.

In fact, the best predictor of a student beginning their statistics coursework at difficulty Level 2 was college major. As previously noted, the odds of beginning with a theoretical introductory course were three times higher for STEM majors than their non-STEM major peers. This result suggests that a student's major (STEM vs. non-STEM) plays a key role in the type of introductory statistics course with which they begin. Research on the impact of college major on mathematics course-taking shows that over 90% of students take the minimum mathematical requirement(s) of their major (Harwell, et al., 2009). This finding may explain the current finding in that STEM majors take more difficult college statistics courses, because their major requires it, compared to non-STEM majors.

Our results also show that high school mathematics GPA and ACT mathematics scores were significant predictors of students' grades in their first college statistics course. Students' high school mathematics GPA appeared to be the strongest predictor with a medium size effect of d = .49, supporting the inference that prior mathematics achievement (reflected in success in high school mathematics courses) is related to college statistics achievement. Although scores on the ACT mathematics test had a statistically significant relationship with students' grades, the practical significance of that relationship was small. This result is consistent with work by Johnson and Kuennen (2006) showing that scores on the ACT mathematics test did not predict grades in an introductory business statistics course and suggests that the mathematical skills measured by the ACT test only modestly overlap with those needed in college statistics. The ACT mathematics test is designed to measure mathematical reasoning and contains 60 items divided into three subtests covering topics in pre-algebra, elementary and intermediate algebra, coordinate and plane geometry, and trigonometry. As such only half of the items on the ACT mathematics test measure aspects of mathematical reasoning that would seem to be related to achievement in a statistics course.

Similarly, the number of years of high school mathematics a student completed was unrelated to both the difficulty level of the course taken and the grade subsequently earned, which is consistent with previous research (Johnson & Kuennen, 2006) and suggests that exposure to more advanced mathematics courses (e.g., calculus) is not necessary to do well in statistics. For the difficulty variable this finding is particularly interesting because it suggests that students who take calculus in high school do not necessarily enroll in calculus-based statistics courses in college. In addition, the non-significant findings regarding ethnicity (with one exception for the grade data) suggest that students of different ethnic backgrounds are equally prepared for college statistics. The finding that female students earned higher grades than their male peers in college statistics is consistent with previous research regarding sex effects in statistics achievement (Schram, 1996). This finding is contrary to the sex effect traditionally observed in mathematics, where males have generally been found to outperform their female peers. This may be related to the composition of statistics courses, with less relative emphasis on formal abstraction and a greater reliance on contextually based applications.

Second, the type of high school mathematics curriculum a student completed was unrelated to the difficulty level of a student's first college statistics course as well as the grade earned in that course. Thus, greater exposure to statistical content in high school does not lead to students taking more difficult statistics courses in college or earning better grades in those courses. The non-significant findings for curriculum may in part be due to factors we were not able to observe such as the fidelity of implementation of the curricula or variations in teaching quality and assessment practices within- and across-institutions. Likewise, we do not have data assessing the extent to which teachers (high school, college) actually covered the statistical content present in the curricula. However, we have anecdotal information suggesting that many high school mathematics teachers do not feel comfortable teaching statistics and frequently skip this material. Another possibility is that the integrated nature of the NSF-funded curricula may make it difficult for students to learn distinct statistical concepts, leaving them as prepared for college work in statistics as their peers who likely received little exposure to statistical concepts in high school. Of course, it is also possible that the statistical material embedded in the NSF-funded curricula do not promote more understanding than the statistical material appearing in the CD and UCSMP curricula.

In sum, these findings do not resolve the debate between those who believe that mathematical and statistical reasoning are distinct types of reasoning and those who do not, because the grades students' earn in a statistics course are a measure of many things, only some of which can be attributed to a student's ability to reason statistically. But these results do provide preliminary evidence that there is a relationship between students' mathematics achievement and statistics achievement, although the exact nature of this relationship remains unclear.

That being said, the present findings regarding the effect of curriculum are consistent with previous results regarding the impact of high school mathematics curricula on performance in students' first college mathematics course (e.g., Harwell et al., 2009; Post et al., 2010). In general, studies to date examining the impact of high school mathematics curricula and the difficulty of a student's first college mathematics class (defined as a course whose difficulty level equals or exceeds College Algebra/Pre-Calculus) or the grade earned in that class have not found any relationship. Other work (Harwell, LeBeau, Post, Dupuis, & Medhanie, 2011) has found that students completing an NSF-funded curriculum are more likely to begin college with a developmental mathematics course compared to students completing a CD or UCSMP curriculum. The current results extend these previous findings to include performance in college statistics coursework.

Future research should further examine the amount and nature of mathematical preparation necessary to be successful in statistics, particularly the importance of higher-level mathematical skills. For example, the literature remains unclear about the advantages gained in statistics from exposure to calculus. In the present study, students who had taken calculus in high school did not outperform their peers in university statistics work (even in calculus-based statistics courses), however other studies (e.g., Green et al., 2009) found that students who had a more rigorous sequence of mathematical courses (including calculus) did outperform their peers who had experienced a less rigorous sequence of courses that did not include calculus. It is possible that the mathematical skills that students gain in a calculus course are not directly applicable to an introductory statistics course (particularly in an applied course), but that taking calculus is related to a student having a positive attitude about and less anxiety toward mathematics, which translates into that student having less anxiety toward statistics than students with less advanced coursework in mathematics. Similarly, future research would benefit from an examination of the aspects of mathematical proficiency that contribute to a better understanding of specific statistical concepts. For example, it seems plausible that a good working knowledge of calculus would not be related to a student's understanding of measures of central tendency, but that it may be related to a student's understanding of sampling distributions. Studies that measure specific aspects of both mathematical and statistical thinking would significantly contribute to our understanding of the relationship between mathematical and statistical reasoning.

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