CHALLENGES IN SEEING DATA AS USEFUL EVIDENCE IN MAKING PREDICTIONS ON THE PROBABILITY OF A REAL-WORLD PHENOMENON

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ABSTRACT

This study investigates the relationship between deterministic and probabilistic reasoning when students experiment on a real-world situation involving uncertainty. Twelve students, aged eight to nine years, participated in an outdoor teaching activity that called for reflection on the growth of sunflowers within the frame of a sunflower lottery, where students were involved in the process of creating their own empirical data of the growth. However, the study shows not only that the students do not make use of data for predicting the outcome of an uncertain event, but also how this can be explained by students' attention to deterministic features of the situation, brought to the fore within an ecology context and connected to a conceptual principle of 'sharing'.

Keywords: *Statistics education research; Deterministic reasoning; Probabilistic reasoning; Ecology context; Sharing; Experimentation; Outdoor mathematics; Driving question*

1. INTRODUCTION

In recent decades, probability has emerged as a mainstream area in school mathematics curricula around the world (e.g., National Council of Teachers of Mathematics, 2000; Swedish National Agency for Education, 2012). This is especially opportune because as members of a society we increasingly meet chance variation and random phenomena not only in mathematics but also in the media, in meteorological and financial forecasting, and in social activities such as sports and gambling (Jones et al., 2005). Referring to Gal (2005), we can talk about an increasing need to educate for probability literacy.

Much of the previous research on probability teaching and learning has investigated students' understanding in relation to the classical interpretation of probability. The classical interpretation is based on the assumption that each outcome of a random process is equally likely (Batanero, Henry, & Parzysz, 2005) and, based on this view, many authors have derived useful examples for instruction from the seemingly simple random dependent situations involved in games of chance (Gal, 2005). However, although equiprobability is clear when tossing a symmetric coin or a die, this assumption is rarely possible to apply to functional or everyday situations (Gal, 2005). If we agree that one central goal of teaching probability is to develop rational and conscientious citizens, there is a strong need to develop our understanding of learning and teaching of probability in situations that go beyond "idealistic" game-like situations. We need to look at how students develop an understanding of situations where randomness appears naturally in everyday, realistic situations.

Fischbein (1975) emphasizes the need to offer students opportunities to experience randomness concretely in the learning of probability. Working practically with random experimentations provides students an opportunity to sense random behavior and, challenges them to make predictions and verifications of probabilities. Shaughnessy (2003) stresses Fischbein's suggestion further, recommending that the teaching of probability not only should involve data-experimentation, but also should actually start from there. Probability questions should be generated from data sets. Experimentation-based teaching shifts the perspective from the learning of merely technical tools and artefacts of statistics and probability toward a holistic, process-oriented perspective (Makar & Rubin,

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2009). In an experimentation-based teaching situation, students may become engaged in formulating statistical and probabilistic questions, collecting data, analyzing data, and drawing data-based conclusions and inferences (Paparistodemou & Meletiou-Mavrotheris, 2008).

Even if experimentation-based random situations seem to offer certain promises for the learning of probability, research has shown that this does not happen automatically. Makar and Rubin (2009) report that frequency data are often disregarded by students (seven to nine years old). Regardless of sample size, they found that students had difficulty connecting data to conclusions (Makar & Rubin, 2009, p. 95). Pratt, Johnston-Wilder, Ainley, and Mason (2008) also highlight students' difficulties in interpreting information in a data sample as they challenged students aged ten to eleven years to infer, from data, the unknown configuration of a virtual die (See also Stohl & Tarr, 2002). Previous research has also highlighted the importance of being aware of how an overemphasis on deterministic ways of reasoning may inhibit students' ability to develop sound data-based reasoning in probability-laden situations (Jones, Langrall, & Mooney, 2007).

The present study is part of a larger research program, which has the overall aim of increasing our understanding of students' ways of trying to make sense of randomness, brought to the fore in new and novel types of teaching situations that are intended to promote knowledge of probability that is functional for every-day life. From this overall perspective, the research program aims particularly at developing task design principles of experimentation-based outdoor situations where randomness cannot be regulated by the assumption of an equiprobable underlying sample space. The current study examines how students try to make sense of real-world, random dependent situations, by basing analysis on how the students balance between deterministic and probabilistic ways of reasoning.

The study looks at an episode of probability teaching located in an outdoor setting. In an outdoor setting, students can break with common classroom patterns that may limit their participation and creativity in mathematics learning activities (Nilsson, Sollervall, & Milrad, 2009) and be offered certain opportunities to build their mathematical understanding in connection to realistic, real-world phenomena (Broda, 2002; Fyhn, 2006; Kennard, 2007). The activity under investigation involves students creating their own frequency data by planting seeds of sunflowers and marking on a diagram the number of seeds that grow. One might suppose that experimentation in an outdoor nature-oriented activity would raise certain questions about the interplay between deterministic and probabilistic reasoning in relation to how students see data as useful evidence in making probability-based predictions: although nature has its own logic, it is far too complex to allow us to make completely certain estimates regarding its future. From this background, the following research question is posed: How can aspects of deterministic reasoning influence the ways students make self-generated frequency data available for reflection and use the data as evidence in making probabilistic predictions of random dependent ecological situations located outdoors?

2. ANALYTICAL APPROACH

2.1. CONTEXTUALIZATION

The analytical approach of the present study is grounded in a social constructivist perspective on mathematical learning and understanding. How students develop an understanding of a task or a phenomenon is considered through a process of contextualization (Halldén, 1999).

Context, in the present meaning of contextualization, does not refer to the spatiotemporal setting of the learning activity but rather to a mental device, shaped by personal interpretations of the activity (Cobb, 1986). Connected to this, the notion of context emphasizes principles of guiding and framing, which alerts us to how different knowledge elements make the activation of other elements either more or less likely (cf. Shelton, 2004). Content-related principles and ideas are brought about and assimilated on the basis of how they fit into the construction of a network of interpretations (Nilsson, 2009b), of which situational and social elements are a part (Janvier, 1989). Sense-making, in terms of rendering a phenomenon or task intelligible and plausible, thus involves creating consistency and coherence in personal contexts of interpretations, that is, in the way the phenomenon is experienced by the learner (Caravita & Halldén, 1994). From this contextual view, context is not perceived as an entity only influencing conceptual understanding, instead, conceptual principles are integrated into the context and constitute central parts of a learner's way of contextualizing a task or a phenomenon (Nilsson, 2009a).

2.2. DETERMINISTIC AND PROBABILISTIC REASONING

To be able to make sound predictions in situations involving chance one must be able to balance deterministic and probabilistic considerations (Hacking, 2006; Prodromou & Pratt, 2006). For instance, assigning 1/6 as the probability of getting a six in the throwing of a die depends on the interpretation that the physical shape of the die determines an equal chance for each side of the die. The situation of throwing a matchbox up in the air can also be considered. The probability for landing on a particular outcome is not easy to estimate a priori in the same way as for the symmetric die. However, it can still be assumed that the shape of the box will give preference to the box landing on one of its two large areas. Based on the Piaget and Inhelder (1975) constructivist ideas that the understanding of chance and probability assumes operational thinking, Prodromou and Pratt (2006) stress the importance of developing an understanding of causality in the learning of probability. Understanding randomness and probability is, according to Prodromou and Pratt, about understanding that causality has limited explanatory power at the micro-level but can be harnessed to understand what regulates global, long-term patterns of probability distributions. Hence, probabilistic reasoning and modeling often need elements of deterministic considerations. However, research has found that people often over-attribute deterministic elements to situations involving chance (Jones et al., 2007).

Stohl (2005) argues that one reason that the classical interpretation has reached such a high status in the mathematics classroom is that the approach connects very well to the general deterministic character of school mathematics. The classical approach offers a way to determine the probability of an event in advance by calculation principles, leading to a single answer. Stohl's argument is consistent with the findings of Fischbein, Pampu, and Minzat (1975). However, of certain relevance for the present study, they broaden the perspective beyond the mathematics classroom, stressing that concepts, such as chance, uncertainty and random variation become inhibited as the child "is in the habit (inculcated by instruction in physics, chemistry, mathematics, and even in history and geography) of seeking causal relations which can justify univocal explanations" (p. 73).

Seeking causal relationships is central to deterministic reasoning. One interpretation of the classic heuristic of *representativeness* (Kahneman & Tversky, 1972) is that it implies elements of causal relations. In applying this heuristic to a random dependent situation, people consider the antecedent to determine the consequent (Fischbein & Schnarch, 1997). The binary experiment of flipping a coin will serve as an illustration. Assume the series HHTHHH is produced. Adopting representativeness, people claim that tails (T) is more likely than heads (H) to come up next, since the former is less frequent and recent than the latter, according to the parent population of a 50-50 chance between landing on tails or heads. The outcomes of the experiment are not considered to be of an equiprobable kind. The sample, the frequency information, is not ignored. In a deterministic sense, prior data are considered to cause tails to be favored over heads.

In Iversen and Nilsson (2007), students (aged 14-15) were asked about the probability that marbles would end up in boxes when dropped from the top of a bifurcated tree-diagram structure in an ICT (information and communication technologies) environment called Flexitree (Figure 1). The situation called for reflection on the product law of probability. However, on several occasions students gave scant consideration to the principles of the product law. One type of contextualization that was singled out and used to explain the reason for this behavior was the *main-road approach*. In this way of thinking, students developed a contextualization of a Flexitree system according to features of a practical situation, in which they found it difficult to neglect physical and geometrical concerns when explaining the distribution of marbles within a Flexitree system. The main idea of this way of thinking is that the students did not perceive the bifurcations as purely random. They considered it to be more 'natural' for a marble to continue in a direction already taken instead of changing its direction. Their reasoning was built on deterministic elements such as the physical and mechanical features of the slopes and, particularly, how these distances would promote the extremes because it would be easier for a marble to continue to the side instead of 'turning back' to the middle.

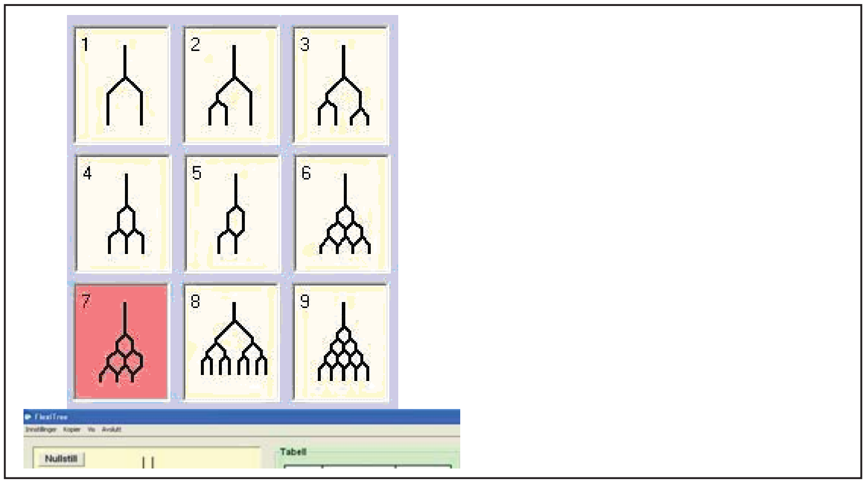

Figure 1. Screenshot of the Flexitree environment, showing its nine available systems

When inclined to the *outcome approach* (Konold, 1989), people also make predictions that are often based on causal factors, and tend to assign numbers as ‘probabilities’ on the basis of the strength of the perceived causal relationship. If the strength is sufficient for a certain outcome, the outcome-oriented person would expect it to happen (Pratt, 1998). The outcome approach disregards frequency information. People using this approach do not see the result of a single trial as one of many such trials in an experiment, but rather regard the result in isolation. For an irregularly-shaped bone problem, Konold (1989) found that even when a summary of the results of 1,000 trials was shown, some students still preferred to base their predictions on a visual inspection of the bone rather than on the available data. Konold speculated that students considered properties of the bone to be a more stable source of evidence when compared to frequency data, which can fluctuate from sample to sample.

A notion of control is often attributed to deterministic thinking. Many of us have tried to direct the outcome of a die by throwing it in a certain way or from different heights. In Fischbein, Nello, and Marino (1991), students (grades one to five) were asked to compare the probability of throwing a “5” three times either by throwing one die three times or by simultaneously throwing three dice. It was found that several students distinguished between the situations. Although both situations were suggested as offering the highest chance by different students, the most common prediction was that “by successively throwing the die, they have a higher chance to obtain the expected result” (Fischbein et al., 1991, p. 529). On the basis of follow-up interviews, it was argued that such a prediction was based on a belief that the individual can control the outcome of a throw. The researchers then concluded that such a belief is incompatible with the notion of *independence*, i.e. that the probability of each number on the dice remains constant. Pratt (1998) investigated (aged 10-11) students’ resources for understanding short-term random behavior and found that students are able to perceive such behavior as impossible to control. Pratt (1998) describes the main point of this kind of thinking as “if I believe that I can direct the outcome through my own physical actions, then I am unlikely to regard the phenomenon as stochastic” (p. 144).

As Piaget and Inhelder (1975) and Prodromou and Pratt (2006) argue, it is not the determinism as such that is the problem of developing sound predictions in situations of uncertainty. Looking for patterns, preferences and underlying regulations is part of modeling uncertain outcomes by probabilistic means. However, research indicates that people often over-attribute the deterministic in favor of probability-related concepts such as data, uncertainty and random variation. As Stohl (2005) and Fischbein et al. (1975) highlight, a conflict between determinism and probability might appear when the teaching of probability is connected tightly to situations in which concepts and principles from scientific subject areas such as physics, biology and chemistry may be brought to the fore. However, such probability teaching has not been systematically explored and, therefore, the present study aims to investigate the learning of probability when the teaching is located in an outdoor, ecology-laden situation that offers students the opportunity to generate and reflect on frequencies for making predictions about random dependent outcomes. Although the activity was designed to promote learning in relation to a frequentist interpretation of probability, students' actual learning and understanding of this interpretation are not the primary focus of the present analysis. Instead, the analysis is focused on what may be prioritized as determining factors in an outdoor, ecology-laden learning environment and how these factors influence students' tendency to ignore frequency data for judging the outcome of a random event.

3. METHOD

The outdoor teaching activity of the present study took place at Torslunda research station, which is part of the Swedish University of Agriculture. Schools located close to the research station visit and use the garden at the research station to learn about nature and to use real-world situations in the learning of mathematics. Hence, the 12 students (aged 8-9) selected to participate in the study were familiar with outdoor teaching at the research station. This familiarity was appropriate for the reported study since an entirely new situation might have influenced the students to investigate aspects of the outdoor setting, which would have been too far from the intentions of the teaching situation. The students had, however, not previously been taught the content of the activity.

The entire teaching consisted of two activities. In the first activity, the students planted sunflowers. Four weeks later, they counted the number of growing sunflowers and worked with questions connected to their observations.

The overall objective of the teaching was to engage students in interpreting data and reasoning about probability in connection to the whole process of generating, observing, counting and interpreting data from real-life experiences.

Activity 1 – Planting seeds The teacher of the class and the head of the research station led the planting activity. The teacher took field notes during the activity and distributed them to the rest of the research team. The twelve students planted 15 seeds each, i.e., a total of 180 seeds. From 180 observations, the regularities of a random experiment were expected to emerge. Each student was given her own square meter, within which she was asked to distribute her 15 seeds. The distance between any two neighboring seeds was thus about 25 centimeters. To stimulate reflection on random variation, the seeds were prepared to keep too high a proportion of seeds from growing. The motive for this was that if 14-15 seeds grew, this could imply to the students that there was no random behavior involved and that everything was predetermined: the seeds simply grow!

Activity 2 – Collecting frequencies and the 'sunflower lottery' In the second activity the students counted the growing sunflowers and marked the number with a sticker in a large pre-constructed diagram (Figure 2). For instance, there were two squares that each had 11 seeds growing.

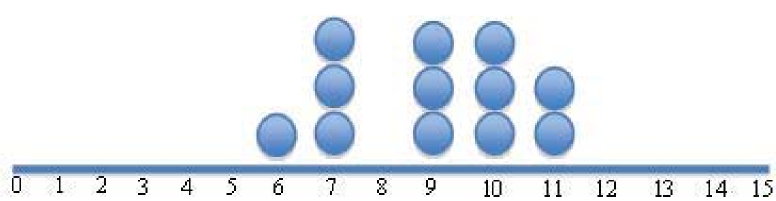

Figure 2. The observed distribution of growing seeds per square meter

When all students had finished marking their observations in the diagram, the teacher of the class assembled the students and their discussion was video-taped. In this discussion, the teacher encouraged the students to reflect briefly on the diagram. They were also asked about how and why they thought some seeds grew while others did not. In the discussion, several students pointed to ecological aspects such as the need for water and the sun's energy. One student articulated the need to give 'love' to the plants. None of the students referred to issues of randomness, data or probability.

The class was randomly divided into three groups. As one of the 12 students from the first activity was absent, one group (Group A) consisted of only three students. The other two groups (Groups B and C) each consisted of four students. All three groups were audio-taped, and Groups A and B were also video-taped. All students' names are fictitious.

The whole activity took place outdoors and the group tables were placed to make it easy to switch between group and whole class discussion. During this time the author of the paper acted as the teacher of the class. The (Figure 2) diagram was placed beside the author, visible to all students. It is the 35 minutes of this part of the second activity that form the main focus of the analysis.

Makar and Rubin (2009) stress the importance of posing a driving question to encourage students' investigation of empirical data. In the present case the driving question concerned the form of a sunflower lottery. In the lottery, each group received a (fake) 1,000 Swedish kronor (SEK) bill to bet and was read the following statement:

> Say you take any seed in the basket and put it in the soil. Do you think it will grow or not grow? Bet your 1,000 SEK on the alternative the group decides on.

The sunflower lottery concerns the single outcome of whether a seed will grow or not. It is not about *predicting* the long-term behavior of frequencies. Instead, the situation is about *interpreting* frequencies of a self-gathered sample and to use the relative frequency of the sample as evidence of a degree of certainty for the outcome prior to a single trial.

4. RESULTS AND ANALYSIS

The class is referred to as a whole when several students respond at the same time.

4.1. INTRODUCING RANDOMNESS AND CHANCE IN CLASS DISCUSSION

The author began by drawing attention to the fact that not all the seeds were growing. He asked if anyone was able to tell in advance how many seeds would grow. Virtually all students answered that they had not been able to. No one said they could, but some remained silent. The author continued, saying that this is the case with many phenomena and that we do not know for sure how they will behave:

Author: A seed may grow, but a seed may not grow. When we can't decide something with certainty what…is there somebody who has heard the word 'randomness'?

Albin and Anna, members of Group A, answered that they have heard about randomness and the author asked them to elaborate on this:

Anna: Anything can happen.
Albin: Yes, anything can happen.
Author: Can you [turns to Albin] give an example of something around us that could be random, that anything can happen?
Albin: Yes, a soccer game. If a team…if a team is much better than the other one it's only random which team wins. Even the bad team might win.

Even though the other students said they understood what Albin was expressing, they sat quietly when the author asked them for another example. To stimulate the discussion, he asked the class to consider the throwing of a rounded-back thumbtack.

Author: If I throw it [the thumbtack] up in the air, is there someone who can tell if it will land with the tip upwards or with the tip [angled] downwards...
Class: No!
Author: …with certainty?
Class: [Shaking their heads].
Author: No, there isn't. But, can we look at the thumbtack and guess what we believe?
Albin: In that case the heavier part will be down [hits the table with his hand].

Author: What would you guess? [Asking Anna in Group A]
Albin: The tip maybe…
Author: The tip upwards?
Anna: I believe the other…yes, the tip upwards, I believe that too.

The episode ended with several other students calling out that they believed the tip would land upwards. What the episode disclosed is that several of the students were able to make sense of how a phenomenon that is afflicted with uncertainty behaves. Stimulated by the interaction with the author, they used words connected to uncertainty such as “maybe” and “believe” when they talked about the outcomes of throwing a thumbtack. The episode also shows evidence that the students connected deterministic and probabilistic reasoning in their context of interpreting the outcomes of the thumbtack. On the one hand, they considered that the shape [determinism] of the thumbtack makes it more likely that the tip will land up. On the other hand, however, they believed – that is, they cannot say for sure [randomness] – that this is what will happen. Albin offered another example of this issue when he said that the result of a soccer game cannot be predicted with certainty even if one team is the obvious favorite to win.

4.2. THE SUNFLOWER LOTTERY AND THE EMERGENCE OF AN ECOLOGY CONTEXTUALIZATION

The author introduced the sunflower lottery by stressing that you cannot, in the same manner as with a thumbtack or a coin, look at the seed of a sunflower and decide which alternative to bet on. This was said with the purpose of directing the students’ attention towards the experiment they conducted and the statistical information it gave:

Author: We look here…we can use this experiment. We have a garden over here and we also have this table [pointing to the diagram]. So, we can use this experiment, which we have done here with Torsten [head of the research station] to also, maybe, be able to reason our way,…what we should bet the thousand kronor on?

The students then began to work on the task in their respective groups. In two of the groups, A and C, the discussion immediately turned to deterministic considerations, regarding biological needs in an ecology context. In Group A, Albin introduced the biological conditions as an explanation for why they should bet their money on the outcome that the seed will grow:

Albin: I actually think that, if it [the seed] is there by itself, then it takes all the sun, all the nutrients, all the water itself [Albin looks at Anna, who nods]. If there are a lot [of seeds] sharing, then everybody has to share [Anna is nodding], but if it’s there alone it gets everything, so I actually think it will grow.

Albin asked the author, “Do we only put one seed in the soil?” and the author answered “Yes”. The group discussion continued.

Anna: We [at home] usually do tomatoes, and first we take one [seed] each and put it into a little pot and all of them grow so…
Albin: But like, one, if there’s one, it’ll get all the water, all the oxygen, everything.
Anna: Yes.
Albin: If there are more [seeds], like it was over there [pointing to the experiment],…
Anna: Yes.
Albin: …then they have to share…
Anna: Yes.
Albin: and then…
Anna: There’ll be no…
Albin: There’ll be less.
Anna: Yes.

Albin: It's the same as if it had been people…
Anna: Yes, exactly.
Albin: …if there are a lot of [people] you have to share.

Sara, the third student of the group, sat quietly until Albin's last comment. It was difficult to hear all of what she said, but it concerned her experience of being used to getting less of something if she has to share it with her siblings.

The author interrupted the group discussions and initiated a whole class discussion. All groups decided to bet their 1,000 SEK that the seed will grow.

Author: What do you say…what do you say over there [talking to Group A], you've bet that the seed will grow, why did you choose that?
Albin: If it's like it is there [pointing to the experiment], then they all have to share the water and such.
Author: Yes.
Albin: But if it's alone, it can have everything to itself.
Author: Yes.
Albin: And then it'll have the most, and then it can grow.

In an attempt to challenge Albin's way of reasoning, the author suggested a change in the formulation of the task.

Author: But if we say that it's exactly the same conditions as over here [pointing to the experiment]. Do you think it'll grow anyway?
Albin: Yes!

Then the author turned to Group B, which was just repeating what they were saying in the group – that the seed would grow – as they are a lucky group. When the author then turned to Group C, Alice repeated much of Albin's line of reasoning.

Alice: The others, those over there [referring to the experiment], they have to share everything, but that one doesn't…[difficult to hear what she is saying], it's only one, it'll have all the water itself.

Carl, a member of Group B, reacted:

Carl: It [the seed] can also have too much water… [difficult to hear]…so it'll be bad because of that.
Author: Yes, it can also be worse.

Unfortunately, the author did not make use of Carl's input as an opportunity to initiate a discussion about the randomness of the experiment and the deterministic conditions the students referenced. Such a discussion could have encouraged the students to raise doubts about their deterministic position and stimulated them to be more open to other ways of thinking about the situation. It could have made it possible for the author to turn the discussion, in a natural way, towards the statistical information of the experiment. What happened instead was that the author more or less recapitulated and forced the students to attend to the diagram. However, in light of students' previous interpretation of the situation and the discussion that had taken place, there is every reason to believe that the final part of the discussion was unclear to the students. The discussion did contain a number of key aspects of a frequency perspective of probability, but it was essentially the author who did the talking and the students were only active to a limited degree. For this reason, the rest of the discussion is left out of the present paper and the analysis focuses on what is outlined above.

Even though only a limited number of students expressed their thoughts explicitly, it becomes clear in the group discussions and the whole class discussion that none of the students explicitly used the data from the experiment to determine their bets. Although the author, when introducing the sunflower lottery, explicitly encouraged the students to attend to the experiment and the diagram, the students, with Albin's reasoning leading the way, appeared interested only in finding arguments for their betting in biological, causal relationships, which they were able to exert some control over and find meaning in (cf. Pratt, 1998).

4.3. THE PRINCIPLE OF 'SHARING'

Taking a close look at Albin, we see that he elaborated his ecology context by developing the conceptual principle of *sharing*. This principle involves and operates on deterministic, ecology-oriented features of the situation. It builds on the idea that there is a limited or given number of pre-conditions and materials that are to be distributed over a set of elements or individuals of a population (e.g., seeds or people), and that the amount of material for each element depends on the size of the set of elements. It does not really occur explicitly in the activity, but it is reasonable to believe that this sharing is considered to be of an equal kind, similar to the mathematical operation of division: every element in a set gets the same conditions. There are three interrelated aspects in the situation that help in the understanding of the development of this contextualization and of how it helps in explaining why the students did not bring frequency information into reflection to account for their betting in the lottery. The analysis, at times, contrasts the interaction within the sunflower lottery with the discussion of the thumbtack.

The first aspect relates to the teaching activity as a whole. The class teacher's intention was not just about the learning of chance and probability. Much of the previous discussion was centered on plants' conditions for growing. These issues were also highly emphasized when the teacher of the class summarized and discussed the experiment right after the students marked the number of plants observed in a diagram. Moreover, like when Anna described her experience of planting tomatoes, most of the students brought to the situation similar experiences from home, regarding growing conditions for vegetables. Hence, they seemed to be very influenced by how the teacher and the leader of the research station framed the situation. The discussion about the thumbtack situation did not seem to have any real impact on the discussions about the sunflower lottery. In the thumbtack situation, while the discussion did not turn into explicit reflection on frequency information, several students showed an understanding of random variation. In the sunflower situation, none of the students reflected on random variation in a real substantial manner. It was only Group B that expressed some vague ideas about luck. The point here is that if the students had allocated the sunflower lottery in a more probabilistic context, like they allocated the outcomes of the thumbtack, they might have been more inclined to appeal to frequency information. However, this did not happen. Instead, and connected to the social turn of contextualization (Nilsson, 2009b), the students' previous experiences of planting and the teacher's way of framing the situation are considered crucial to how the students developed their deterministically-oriented sharing context, which, in turn, kept them away from using frequency information of the experiment as evidence of a degree of certainty for predicting the outcome prior to the planting of a single seed.

The second aspect of explanation also relates to social or semiotic issues, rather than to students' conceptual limitations regarding probability. The students stressed one specific outcome: that the seed will grow. In addition, they did not include frequency information in their reasoning. This is very much in resonance with the outcome approach (Konold, 1989). Applying an outcome-oriented approach to the present situation, however, is not very strange as this is greatly stimulated by the task design. That is, in comparison to the questions raised in connection to the thumbtack situation, the task (deciding whether or not a seed will grow) does not, in itself, invite reflection on random variation. The task concerns deciding on one of two outcomes, without any reflection on degrees of certainty. On account of this, the students' focus may have turned to the mission of finding enough support to decide on the growing of a seed and, based on their sharing contextualization, the students found this support in relation to ecological conditions. However, that the students were asked to decide on one specific outcome does not mean that they by default had to ignore frequency information in making their decision. This brings us to the third aspect of explanation.

The third aspect concerns how the sunflower lottery seemed to change the conditions of the experimental situation for the students and, thereby, did stimulate their sharing contextualization. It is argued above that the sharing context connects to deterministic, biological features of an ecology context. Arguably, this context also incorporates aspects of the concept of *independence*, related to the perceived changed conditions of the random situations. In the experiment the students planted a total of 180 seeds. Hence, the system of investigation involved 180 seeds in the same area. In the lottery, however, the question was not about planting many seeds but just one seed, and the students interpreted that this makes a difference. Above all, Albin, and the students who connected to his line of reasoning, considered it difficult to regard individual seeds as independent random observations when there are several seeds planted in the same area. The sharing of growing conditions was perceived to depend on the size of the sample. The students had difficulty ignoring the fact that the size of the planting area was rather large (the distance between neighboring seeds was about 25 cm) and considering the conditions of water, nutrients, the sun's energy and so on to be the same, regardless of whether 180 seeds were planted or just one. Instead, they repeatedly returned to the notion of sharing, which implied that they considered the (very) limited share of pre-conditions available for distribution among the seeds planted. It is in this sense that their ecology contextualization involves elements of dependence: that elements in a sample are perceived to affect the conditions of each other's possibility of occurring.

5. DISCUSSION

The current study takes up the challenge by Shaughnessy (2003) to investigate experimentation-based environments for the learning of probability. An outdoor sunflower experiment was designed in order to call for reflection on probability based on empirical data. As nature has its own logic, it is far too complex to allow completely certain estimates of its future. Based on this assumption, the analysis aimed at investigating how deterministic reasoning may influence students to use data as evidence in making probabilistic predictions of real-world, outdoor, random dependent situations. The analysis not only shows how the students ignored, or did not bring into reflection, sample information in the experiment; it also shows how deterministic features dominated their reasoning and how this happened within the frame of an ecology context, centering on a principle of *sharing*.

Before further elaboration on the results of the analysis, a reflection on the trustfulness and generalizability of the study is presented. The analysis basically draws on only one specific student, Albin. However, the teacher regarded Albin as performing above average during the ordinary teaching of mathematics. Hence, if Albin found it difficult to see empirical data as useful evidence in making probability predictions, there is good reason to believe that many of the other students would do so as well. Moreover, the analysis is grounded in previous studies that have also identified how deterministic reasoning can influence probabilistic reasoning. On account of this, the results of the analysis are argued to be trustful and theoretically generalizable.

Makar and Rubin (2009) show that students do not always see statistical information as useful evidence for getting a picture of a population or making probability predictions regarding random situations. But is it really the case that the students do not see the usefulness of statistical information, or is it simply that the information is not activated in the context of their interpretations? That students are not seeing frequency information as useful evidence implies that they make a conscious choice to reject such information, in favor of other kinds of information. The present study does not show any indications of such conscious acts. Instead, it is claimed that the information is not available for reflection to them, by reason of the way they allocate their perception of the situation to an ecology context, in which the biological laws of nature and sharing are stressed. Speaking in terms of contextual dominancy (Nilsson, 2009a), students who showed evidence of this ecology-oriented contextualization did not establish or activate any reason to question this contextualization or complement it with frequency information.

An alternative explanation could be that frequency data were actually used by the students, but did not become the focus of discussion. It could be the case that the data they collected predisposed them at an implicit level towards predicting that a single seed will grow. The deterministic references were then added to support or strengthen the explanation of this choice. As argued above, there is little in the data that support such an interpretation of the students' reasoning. However, the tension between these two alternative interpretations, and the structure of the activity in its whole, raise the question of a follow-up study where the seeds are prepared so that the proportion of seeds that do grow will be much less than the proportion of seeds that do not grow. In such a situation, will the students still stress physical and biological factors, or will they recognize the need to use the frequency data for their predictions?

Makar and Rubin (2009) also point to the importance of implementing a driving question in the teaching of making inferences from data. The present study does not question this. However, what the present study does highlight is the need to be alert to how a driving question may influence students' reasoning. Above all, students' outcome-oriented way of reasoning (Konold, 1989) can be regarded as a product of the task design. The task did not invite or challenge the students to reflect on random variation or degrees of certainty. The students were asked to decide on a particular outcome, regardless of whether they only found it to occur with a likelihood of, for instance, 70%. However, regardless of the fact that they were asked to decide on just one outcome, they could have used frequency information for motivating their choice; but this did not happen. Nevertheless, the study emphasizes the importance of studying further the role of task formulation in students' ways of contextualizing a random situation in general and an ecology-laden situation in particular.

Task construction may be crucial to the forming of the sharing context as well. Particularly, some students stated that the sunflower lottery changed the underlying growing conditions of the experiment. In addition to the analysis already done on this issue, there are similarities to the findings of Fischbein et al. (1991) regarding how Albin and his followers compared the situation of planting 180 seeds to the situation of planting just one seed. Fischbein et al. found that students encounter difficulties seeing the similar mathematical structure in throwing one die three times successively and throwing three dice simultaneously, and concluded that such a belief is incompatible with a notion of independence, that the probability of the outcomes of each die stays constant regardless of the throwing situation. In the sunflower situation, the students were never asked a question that challenged them on this issue, but from the group discussions and the whole class discussion there is good reason to believe that they would have regarded the two situations – the planting of 180 seeds simultaneously and the planting of 180 seeds in successive order – as different regarding the likelihood that the seeds would grow. This would be similar to the findings of Fischbein et al. (1991). However, in Fischbein et al., students often explained the difference between the two situations by referring to a belief that they were more able to control (cf. Pratt, 1998) the throwing of a single die. Hence, the single outcome was at the forefront of the students' explanations. In the present case it is argued that the single planting of a seed is more implicitly dealt with. Based on the sharing context, issues of dependency came to the fore as the students perceived that the seeds influence each other's living conditions in the simultaneous planting, and thereby decreases the likelihood that the seeds will grow. Speaking in terms of Konold (1989), the *strength of the causal relationship* is perceived to depend on the number of elements in a population and on the number of conditions that the elements have to share. This may demonstrate a new form of deterministic reasoning, which affects students' ways of valuing issues of dependence and independence in a randomized experiment.

Based on the analysis, the current study stresses a need for research on the learning of probability to broaden the range of methodological approaches. In particular, the study highlights the challenges teachers and students may encounter when a probability teaching situation includes the investigation of a random experiment that spans a longer time-period. Traditionally, much of the methodology has concerned the throwing of dice or coins, pulling marbles from urns, reflecting on the outcomes of spinners and so on. These classic random situations could be considered 'demarcated', time-independent random systems. Of course these situations are repeatable, but the thing is that you can investigate them at the moment, based on what you perceive in the actual situation. If we look at the world around us, this is not often the case in random dependent processes. Like the stock market, weather forecasts, medical treatments and the planting of seeds, you cannot determine the probability in advance. There is a time span involved and, with it, a complex system of variables included, whose behavior you are not able to perceive information about at the moment. This was the case with the amount of nutrients mixture, water and energy from the sun involved in the planting of seeds. Hence, to make probabilistic sense of such situations there is a certain need for statistical information. However, as the analysis discloses, learners do not automatically perceive this need. Particularly, the analysis raises questions about the way such a situation should be introduced and framed in a sequence of teaching and what specific questions should be included to drive students' reasoning and encourage them to develop probabilistically oriented contextualizations of the situation. Hence, the present study points to a need for further research to investigate more 'open', time-extended, real-world random situations to increase our understanding of the instructional challenges involved in the teaching of probability and probability literacy (Gal, 2005), and of the role of empirical data in making probability predictions in random experiments (Makar & Rubin, 2009).

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