MEANINGS OF PROBABILITY IN SPANISH CURRICULUM FOR PRIMARY SCHOOL

Emilse Gómez-Torres[1] and J. Miguel Contreras[2]
[1]Universidad Nacional de Colombia, Colombia
[2]Universidad de Granada, Spain
egomezt@unal.edu.co

Recent curricula in many countries suggest that children are introduced to chance and probability in early years of schooling. Probability has had various meanings throughout history, most of which are complementary; however their epistemological differences have been a source of philosophical debates and cognitive conflicts. In this paper we analyze the meanings suggested for probability in the primary school Spanish curricular guidelines. Using ideas from the onto-semiotic approach, we identify the mathematical objects (problems, concepts, propositions and procedures) suggested in these curricular documents for different meanings of probability. We finally establish a reference meaning that may be useful to understand and analyze teaching practices, as well as to predict and help overcome children's possible learning conflicts.

INTRODUCTION

The role of probability in the school curricula and its relation to statistics is a current topic of debate, as is visible from the Joint ICMI/IASE study (Borovcnik, 2011; Burrill, & Biehler, 2011). Children are surrounded by randomness in their personal and school lives, and this implies their need to understand random phenomena in order to be ready to make adequate decisions when confronted with uncertainty. This need was recognized, among others, by educational authorities in Australia, Brazil, South Africa, Uganda, United Kingdom and United States (Jones, Langrall, & Mooney, 2007; Batanero, Burril, & Reading, 2011), where probability is included in the curricula from primary education to high school and at university level.

The aim of this paper is to analyze the probability content in the Spanish curricular guidelines for primary school, found in the official guidelines (MEC, 2006), which are compulsory at a national level. We base our work on the onto-semiotic approach (OSA) that we briefly summarize below. We identify the main probabilistic objects and the probability meanings suggested in these guidelines. Finally, we conclude with some suggestions for teaching and remarks on the possible applications of our analyses.

BASIC CONCEPTS OF ONTO-SEMIOTIC APPROACH

In this theoretical framework, the *meaning of any mathematical object* (such as probability) can be considered from institutional or personal points of view and is defined as the set of actions carried out by people or institutions when facing characteristic problem situations related to that particular mathematical object (Godino, 2003). Here institution refers to a group of people that share tools, rules and actions; for example, a school or an educational teaching level.

The meanings for a particular object (such as probability) are characterized by the following primary entities (Godino, 2003): *problem situations* (extra- or intra-mathematical applications) where the object is used; *language* (terms, expressions, notations, graphs) that represent the data problem or that serve to operate with the object; *concepts* implicitly or explicitly used to solve the problems; *propositions* (properties or attributes of the concepts); *procedures* (operations, algorithms, techniques) applied to solve the problem; and *arguments* (such as deductive or inductive reasoning) that serve to validate and explain the solutions.

According to Godino (2009) the identification of the above entities in teaching materials or in teaching guidelines helps us understand what mathematics we expect the students to learn. Below we follow this suggestion and analyze these entities in the official guidelines for teaching probability included in the Spanish curriculum (MEC, 2006).

MEANINGS OF PROBABILITY AT PRIMARY SCHOOL

The following meanings of probability (Batanero, 2005) will be used in our analysis:

- *Intuitive meaning.* At primary school we can use children's intuitive ideas related to chance and probability and use qualitative expressions (probable, unlikely, feasible) to express their

In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona, USA.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

degrees of belief in the occurrence of random events. Some results deserve more confidence than others, even when we cannot predict which of them will happen in a particular trial.

- *Classical meaning.* Probability is conceived as the proportion between the number of cases favorable to an event and the number of possible cases; this approach is popular in teaching and probabilities for single events are easy to compute in chance games common in a child's life. However when dealing with compound events, children need combinatorial reasoning that is difficult for them; moreover it is difficult to apply probability outside games of chance.
- *Frequentist meaning.* Probability is defined as the limit of relative frequencies of an event when an experiment is repeated a large number of times. This approach is now common, given the availability of computers and simulators to reproduce random experiments and quickly show the effect of sample size and stochastic convergence (Batanero, Henry, & Parzysz, 2005).
- *Subjective meaning.* In this approach probability is a personal degree of belief and can be updated with new information via Bayes theorem. Godino, Batanero and Cañizares (1987) suggest it is possible to introduce the subjective meaning of probability at primary school in an intuitive way. The idea is to assign qualitative probabilities to events or by locating these events on a probability scale and later revising these probabilities after new experiences.

PROBABILITY IN GUIDELINES IN THE SPAIN MATHEMATICS CURRICULUM

The primary school in Spain is divided into cycles: First (6-8 year olds), second (8-10 year olds) and third (10-12 year olds) cycles. The mathematics curriculum is organized around four main content areas (MEC, 2006): Numbers and operations; magnitudes and measurement; geometry; data handling, chance and probability, which are explicitly included in each educational cycle. In Tables 1 and 2 we present the mathematical objects linked to each probability meaning that are explicit or implicit in curricula guidelines.

Table 1. Probability problems, concepts and properties in the primary school curriculum

		Cycle	1st	2nd	3rd
Sit. Probl.		*Intuitive:* Quantifying uncertain events; expressing degrees of belief	x	x	x
		Frequentist: Forecasting trends from data, in random phenomena		x	x
		Classical: Assessing the likelihood for given outcomes in games of chance		x	x
Concepts		*Intuitive:* chance, variability; event, certain and impossible; possibility	x	x	x
		Classical: game of chance; favorable/possible cases; probability; fair game		x	x
		Frequentist: population, attribute, frequency (absolute, relative), probability estimated value, simulation		x	x
		Subjective: uncertain event; probability as personal degree of belief	x	x	
Properties	intuitive	Unpredictable outcome	x	x	x
		Possible event: any outcome in a random experiment	x	x	x
		Impossible event: never happens	x	x	x
		Certain event: always happens	x	x	x
		Possibility can be compared	x	x	x
	classical	Finite and numerable number of outcomes		x	x
		Equiprobability of outcomes			x
		Favorable cases: cases we are interested in		x	x
		Possible cases: all outcomes		x	x
		Probability only depends on number of outcomes		x	x
		Laplace rule			x
	frequentist	Population: similar elements that differ in observable attributes	x	x	x
		Attributes can (or not) be equiprobable	x	x	x
		Probability: objective, hypothetical, unknown value			x
		Simulation: replace one experiment by another			x
	sub.	Uncertain event: unpredictable, even having some additional information		x	x
		Probability: conditioned by knowledge		x	x

This curriculum suggests developing the children's skills to compute and interpret probabilities and in the appropriate use of everyday probabilistic language, as well as some specialized language. We can also notice that some probabilistic objects are included in all the cycles, although their complexity increases. We found problem situations common to the three

cycles: a) expressing degrees of belief about uncertain events (intuitive meaning); b) analyzing results in simple games of chance (classical meaning), with no consideration, for example, of unfair games; and c) when the collection of data is included, the description of trend from data (frequentist meaning) is implicitly suggested. The complexity level of these problems increases with the cycles, without excessive formalization. In the first and second cycles, the contents are related to the intuitive meaning of probability and some statistical content is linked to the frequentist meaning; and, in the third cycle the content is related to the classical and frequentist meanings.

The detailed description of probability in the Spanish intended curricula for primary school (Tables 1 and 2) suggests that some probability content implicit in the guidelines should be included in the education of teachers with little probability experience before they teach these topics. Moreover this analysis also may help teachers organize lessons in probability in line with the current requirement. For example, a teacher could design an exercise for a specific cycle, where the relationship between the assigned probability of an event with classical or frequentist approach is highlighted. In the same way the exercise may emphasize different concepts, properties and procedures (e.g. the probability of the impossible event is zero, since in the classical meaning there are no favorable events and in the frequentist meaning there are no observed data with the given conditions).

Table 2. Probability procedures in the primary school curriculum

	Cycle	1st	2nd	3rd
intuitive	Recognizing random events	x	x	
	Interpreting degrees of possibility or belief		x	
	Qualitative evaluation of possibilities		x	
	Qualitative comparison of probabilities		x	
	Recognizing certain, impossible and possible events	x		
classical	Analyzing different games of chance	x	x	x
	Listing (counting) favorable / possible cases with tree diagrams or combinatorial procedures		x	x
	Distinguishing favorable and unfavorable cases	x	x	x
	Recognizing equiprobable elemental events		x	x
	Comparing probabilities with proportional reasoning			x
	Assigning probability to compound events with Laplace rule			x
frequentist	Listing or identifying attributes in a population	x	x	
	Computing absolute frequencies from observations or data	x	x	x
	Computing or graphically representing a distribution	x	x	x
	Reading one and two way tables (compound experiments)		x	
	Estimating probability from repeated trials			x
	Recognizing approximate nature of this estimation			x
	Simulating a random experiment with technology			x
su.	Analyzing experiments where probability depends on personal information			x

This study of the curricular guidelines also included a textbook analysis (Gómez, Ortiz, Batanero, & Contreras, 2013; Gómez, Ortiz, & Gea, in press) where some strengths and weaknesses of the curricula in the textbooks and ways to relate probability to other content, such as data handling, numbers and operations were identified.

CONCLUSIONS

As a result of our analysis we realized that the intended probability curriculum in Spain is in line with the suggestions to improve probability literacy by Gal (2005). We identified in the Spanish guidelines the basic elements of probability-related knowledge described by this author (fundamental probabilistic ideas, understanding of probabilities, probabilistic language and applications to different contexts), as well as promotion of positive dispositional elements (beliefs, attitudes, and personal feelings regarding uncertainty and risk).

We showed that the Spanish curriculum includes topics related to the intuitive meaning of probability in first cycle (6-8 years old); connects statistics and probability and introduces subjective and frequentist meanings in the second cycle (8-10 years old); and partially formalizes

the classical and frequentist meanings in the third cycle (10-12 years old). It is expected that children are introduced to all these meanings, at least to a basic extent. The application to common life situations illustrates to the children the interdisciplinary nature and relevance of probabilistic thinking and language, in public and private contexts.

Using tools from the ontosemiotic approach, we identified the probabilistic objects (problems, concepts, procedures, properties) included in the guidelines by cycles to inform teachers about the expected learning progression for these objects for each probability meaning implicit in the guidelines. This information can help teachers planning their teaching, as well as predicting and solving possible learning conflicts in the children. The teacher should search for adequate teaching strategies to help children progress from an intuitive meaning to the classical and frequentist meanings of probability.

Furthermore, this analysis is useful for teacher educators responsible for the preparation of prospective teachers regarding the mathematical and pedagogical knowledge needed in teaching at primary school. It may also help teachers to avoid promoting common heuristic, biases and misconceptions, such as equiprobability in their future students.

We hope this paper contributes to the improvement of the teaching of probability in primary school and will facilitate the teachers' work in the classroom. Furthermore, we hope that teacher educators recognize that probability and its teaching should be included in the education of prospective teachers.

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