INTRODUCTORY STATISTICS STUDENTS' CONCEPTUAL UNDERSTANDING OF VARIATION AND MEASURES OF VARIATION IN A DISTRIBUTION

Rachel Chaphalkar and Cindy Leary
Department of Mathematical Sciences, University of Montana, Missoula, USA
rachel.chaphalkar@gmail.com

The Guidelines for Assessment and Instruction in Statistics Education College Report (Aliaga, et. al., 2010) encourage a focus on conceptual understanding and statistical thinking in introductory statistics courses. One of the main components of Wild and Pfannkuch's (1999) model of statistical thinking is consideration of variation. The concept of variation is extremely important when students learn about sampling methods, probability, distributions, and sampling distributions (Shaughnessy, 2007). Previous studies have focused on either students' conceptual understanding about variation in a distribution (Cooper & Shore, 2008) or measures of variation (delMas & Liu, 2005; Turegun, 2011). In this observational study, students responded to conceptual questions asking them to compare the variability in histograms in general or using numerical measures. In this paper, we will compare students' ability to reason conceptually about variation and with measures of variation in a distribution.

INTRODUCTION

Wild and Pfannkuch (1999) emphasize the importance of variation in statistics as: "variation is omnipresent; variation can have serious practical consequences; and statistics give[s] us a means of understanding a variation-beset world" (p. 235). Being able to reason about variation is critical to statistical thinking, yet existing literature shows that students have misconceptions about variation (e.g., Reading, 2004; Torok & Watson, 1999).

One of the complicated aspects about variation is how it should be measured. In college level introductory statistics, we discuss three measures of variation, range, interquartile range (IQR), and standard deviation. We also note that the range and standard deviation are sensitive to outliers and skewness, while the IQR is resistant to outliers and skewness. While the range and IQR are typically well-understood, students tend to have difficulty understanding standard deviation. The standard deviation of a distribution can be thought of as the approximate average deviation from the mean. In relation to graphical displays, students learn to read and create a histogram. However, they often struggle to understand the connection between a specific measure of variation and a particular histogram. The purpose of this exploratory study is to find out how students' ability to reason conceptually about variation in a distribution compares with being able to reason about measures of variation. Results from this preliminary study will help to set the framework for a more in-depth study in a large lecture introductory statistics course on how students conceptualize variability

LITERATURE REVIEW

Several studies on students' reasoning about distributions and their variability are found in the existing literature. Watson's (2009) study focuses on primary and secondary students' reasoning about variation in the context of a distribution. Through student constructions of graphical representations of three different datasets of varying degrees of difficulty, Watson found that students acknowledged variation before they acknowledged expectation in a distribution (2009). In Cooper and Shore (2008), researchers identified student misconceptions in introductory college statistics courses about center and variability when data are presented in histograms and stem and leaf plots, such as that students tended to incorrectly identify bell shaped histograms with more variable bars as having greater variability or any two histograms with the same range to have the same amount of variability regardless of other features of the distribution. Furthermore, students in higher level statistics courses did not perform significantly better on a four-item assessment than the students in the lower level courses. The researchers recommend the use of histograms to help students learn to make "valid comparisons between shape and relative variability" (p.11).

In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona, USA.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org [© 2014 ISI/IASE]

Two studies focused on students' reasoning about measures of variation, including standard deviation, range, and interquartile range. delMas and Liu (2005) conducted an exploratory study of students' conceptions of standard deviation using an interactive computer environment and histograms, where the histogram bars were able to be moved around in order to find the arrangements with the largest and smallest standard deviations possible. After a period of exploration, participants answered a series of questions where they were asked to decide which of two histograms had a larger standard deviation and explain their answer. Only a few students had erroneous ideas while completing the exploration which were then corrected by the program (delMas & Liu, 2005). In Turegun's (2011) doctoral dissertation, the focus was on students' conceptual understanding of range, interquartile range and standard deviation at the community college level.

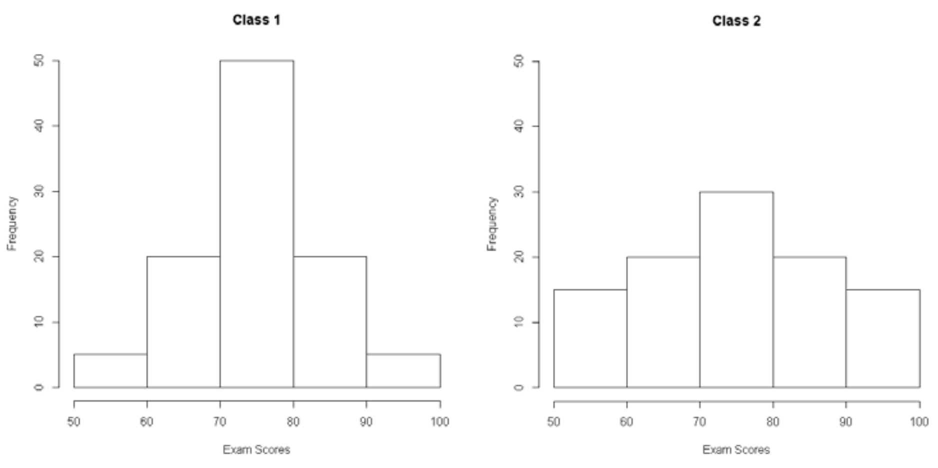
None of these studies have addressed how students' reasoning about variability connects with their reasoning about measures of variability and so we consider the following research question (as part of a larger study being conducted by the first author): Is there a relationship between a students' conceptual understanding of variability in a histogram and their ability to numerically assess the standard deviation of a histogram?

METHODOLOGY

In order to answer the research question, an exploratory observational study using mixed-methods was conducted. All students who took an introductory statistics course in the spring of 2013 at a medium-sized pacific northwestern university were invited to participate. Of the approximately 500 students enrolled in the course, 53 students chose to participate in the study (signed consent form) and took both the exam and a short online survey.

The survey and exam questions are included in Figures 1 and 2. The exam questions were designed so that there would be one correct answer. The online survey question was designed so that students were unable to use the range to determine variability and did not need to worry about histograms with different frequencies. Students were asked to choose which class had more variability (Class 1/Class 2/They are equal/I'm not sure) and briefly explain their answers. This question was adapted from Cooper and Shore (2010). The researchers think that the best answer to this question is "I'm not sure" since it is not difficult to choose values for the two classes so that class 1 has a larger standard deviation and IQR than Class 2 or vice-versa. However, since assuming a random distribution of data will typically lead to a larger standard deviation and IQR in Class 2, this is also an acceptable answer.

Consider the distributions of exam scores for two different classes.



Without making any calculations, which class has more variability?

Figure 1. Online Survey Question

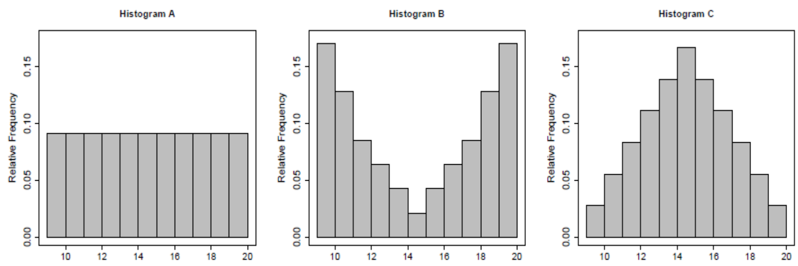
The exam questions were given approximately one month before the online survey.

RESULTS AND DISCUSSION

Exam Question

A larger percentage of students were able to identify the histogram with the smallest standard deviation (C) than the graph with the largest standard deviation (B). Only 40% of students were able to identify both correctly, as seen in Table 1.

(3 pts) 3. Shown below are three different histograms.



- Which distribution has the largest standard deviation? (Circle One) A B C
- Which distribution has the smallest standard deviation? (Circle One) A B C

Figure 2. Exam Question

Table 1. Two-way table of student exam answers.

	Smallest Correct	Smallest Incorrect
Largest Correct	21	2
Largest Incorrect	17	13

Online Survey Question

In answering the online survey question, no students chose "I'm not sure," 66% chose Class 2, 8% said that there was equal variability, and 26% chose Class 1 as having more variability. However, not all explanations were consistent with the answer chosen, and so students' explanations in combination with their answers were coded in the following way. In order for students to have a correct answer, they had to choose class 2 and explain in clear and correct terms why they believe this distribution has more variability. Students with partially correct explanations answered class 2 but provided an explanation that was somewhat ambiguous or they answered that the distributions had the same amount of variability due to having the same range. Students with incorrect explanations either answered class 1 or provided an incorrect explanation. Students with correct explanations (49%) gave reasons such as that the scores were more spread out or more scores were further from the mean, had a larger IQR or standard deviation in class 2. These explanations are generally consistent with previous research (delMas & Lui, 2005). Students with partially correct explanations (13%) attempted to use the empirical rule or range for their determination. Students with incorrect reasons (38%) often wrote that the bar heights in class 1 were more variable than the bar heights in class 2, or gave other reasons that were incorrect. Both researchers coded all explanations based on their correctness. Initially, inter-rater reliability was 75%, however all disagreements were resolved through discussion to arrive at the final code for each response. Only 30% of explanations included a measure of variability (range, IQR, standard deviation) and of these, only 44% used the measure correctly.

Exam and Survey Questions

Figure 3 gives the results of each of the exam questions compared to the coded responses of the conceptual explanations from the online survey. A surprising number of students correctly answering the exam question correct later provided an incorrect explanation to the online survey conceptual question (over 40% on the smallest standard deviation question.) In addition, more than 60% of students who got the smallest standard deviation question wrong on the exam later provided a correct explanation to the online survey question. One hypothesis is that students who got the exam question wrong may have taken time during the survey to look up their answer or has remembered the problem from the exam, while students that got the exam question correct thought they knew the correct reasoning. Ultimately, there does not seem to be a strong relationship between students' answers to these two questions.

Limitations of this study include the small number of participants, a potentially biased sample, and the exploratory nature of this study.

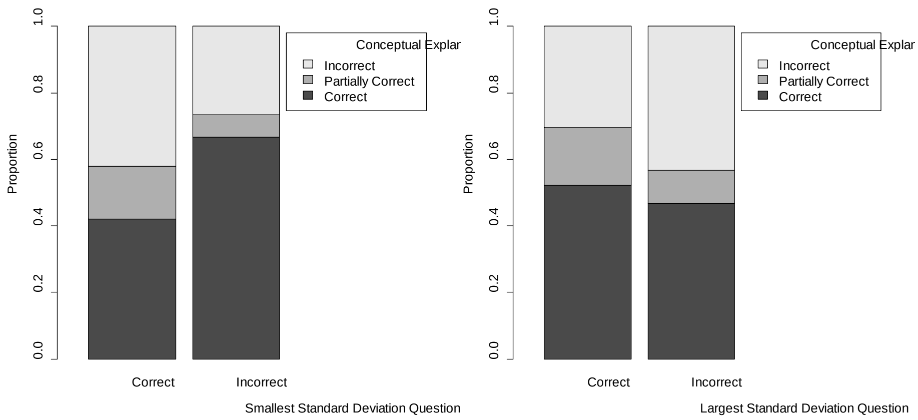


Figure 3. Test Questions compared to online survey question

CONCLUSION

It is surprising that students tend to not connect measures of variation with the concept of variation. In addition to being able to find values for the measures, students should be able to reason about the measures from a given graph. Greater class time needs to be taken to ensure that students have an understanding of the measures of variability in a college-level course and how to apply them in a conceptual manner or possibly using an interactive activity (e.g., see delMas & Liu, 2005) could be used to address all measures of variation.

REFERENCES

Aliaga, M., Cobb, G., Cuff, C., Garfield, J., Gould, R., Lock, R., ... & Witmer, J. (2010). *Guidelines for assessment and instruction in statistics education: College report*. American Statistical Association. Online: http://www.amstat.org/education/gaise/GAISECollege.htm.

Cooper, L. L., & Shore, F. S. (2008). Students' misconceptions in interpreting center and variability of data represented via histograms and stem-and-leaf plots. *Journal of Statistics Education, 16*(2), 1-13.

Cooper, L. L., & Shore, F. S. (2010). The effects of data and graph type on concepts and visualizations of variability. *Journal of Statistics Education, 18*(2), 1-16.

delMas, R., & Liu, Y. (2005). Exploring students' conceptions of the standard deviation. *Statistics Education Research Journal, 4*(1), 55-82.

Reading, C. (2004). Student description of variation while working with weather data. *Statistics Education Research Journal, 3*(2), 84-105, http://www.stat.auckland.ac.nz/serj

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In F. K. Lester (Ed.), *Second handbook of research on the teaching and learning of mathematics* (pp. 957-1009). United States of America: Information Age Publishing, Inc.

Turegun, M. (2011). A model for developing and assessing community college students' conceptions of the range, interquartile range, and standard deviation. Unpublished doctoral dissertation.

Torok, R., & Watson, J. (2000). Development of the concept of statistical variation: An exploratory study. *Mathematics Education Research Journal, 12*(2), 147-169.

Watson, J. M. (2009). The influence of variation and expectation on the developing awareness of distribution. *Statistics Educations Research Journal, 8*(1), 32-61.

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223-265.