

INFORMAL STATISTICAL INFERENCE REVISITED

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Several years ago, we characterized “informal statistical inference” as a claim that went beyond the data, using the data as evidence and acknowledging uncertainty (Makar & Rubin, 2009). This characterization was intentionally ambiguous in order to provide a context for ongoing research on statistical reasoning (especially with younger learners and those without formal statistical training) and to provoke discussion among researchers whose uses of the term differed from one another. Since then, research about informal statistical inference has proliferated, from work with young children to tertiary settings. In this paper, we review recent research on informal statistical inference, investigating issues and new questions that have emerged around looking “beyond the data”, using “data as evidence” and “articulating uncertainty”.

Several years ago, statistics education researchers acknowledged that many of the big ideas underpinning statistical inference were similar whether they were formalised (e.g., hypothesis testing) or informal (e.g., everyday predictions). In the past 5-10 years, researchers have contributed a great deal to inform the teaching and learning of informal statistical inference. Many of these studies have increased the emphasis on relevant and contextual use of statistics, recognising that “a focus on an interesting problem and engaging context may influence students’ inclination to look beyond the data they have” (Paparistodemou & Meletiou-Mavrotheris, 2010, p. 4). The logic of formal statistical inference and its underlying statistical concepts, tools and procedures appear either highly abstract or counter-intuitive to students when presented formally for the first time. By introducing these ideas earlier with stronger ties to informal reasoning, there is promising evidence that students may be better prepared to make sense of statistical inference when they later encounter hypothesis testing. Informal statistical inference can create greater coherence in the statistics curriculum (Bakker & Derry, 2011) and references to it have appeared in new mathematics and statistics curricula in some countries (e.g., Australian Curriculum Assessment and Reporting Authority, 2012; Ministry of Education (NZ), 2007; Common Core State Standards (US), 2010), although often only implicitly or in non-mathematics curriculum areas (Watson & Neal, 2012).

The Fifth International Forum for Research on Statistical Reasoning, Thinking and Literacy (SRTL5) was dedicated to understanding informal statistical inference from primary through tertiary levels, in the workplace and in teacher education; recent and future SRTL forums have continued a focus on informal statistical inference (e.g., in relation to reasoning about context, SRTL6; samples and sampling, SRTL7; uncertainty, SRTL8; and models, SRTL9). Although the ideals of informal statistical inference were embraced, descriptions differed, often describing inference using key statistical concepts needed to understand it (e.g., aggregate, sampling, and sample size) (Pfannkuch, 2006; Rubin, Hammerman, & Konold, 2006; Zeiffler, Garfield, delMas, & Reading, 2008). However, these descriptions neglected informal statistical inference with younger children or those without a secure background in descriptive statistics. To be more inclusive, we created a framework to capture informal statistical inference broadly as having three key characteristics:

- A generalisation or claim that extended *beyond the data*;
- Use of the *data as evidence*;
- Expression with *probabilistic* (non-deterministic) *language*. (Makar & Rubin, 2009)

In this paper, we review recent findings that have revealed new understandings about informal statistical inference using these characteristics as a framework. Within each characteristic, we discuss new research with young children (age 4-10), middle/secondary school learners (over 10 years old) and university students; we close with recommendations for future research.

GENERALIZATIONS BEYOND THE DATA

Because prediction is an everyday activity, generalising “beyond the data” has relevance for all ages. A key distinction made by this characteristic is that statistical inference must extend from data at hand to a larger population, another sample, or a mechanism/process/context that generated the sample. This characteristic contrasts statistical inference with descriptive statistics, which describes available data.

Young Children

Our original framework was partially motivated to understand informal statistical inference with children making claims about some “wider universe” (Ben-Zvi, Gil & Apel, 2007). Research on informal statistical inference with children under age 10 is promising but uncommon. English (2014) reported on young children engaged in data modelling, using data they had to draw broader conclusions. Papanastasiou and Meletiou-Mavrotheris (2010) involved children aged 9 with data they collected about themselves. They reported three ways that the children drew conclusions using (1) *data-based argumentation*, where students drew conclusions about their own school based on data they had; (2) *data-based argumentation and generalisation*, which extended this by drawing conclusions beyond the data to the general context (e.g., people in the town or human behaviour); and (3) *data argumentation and chance*, in which students’ additionally articulated uncertainties in their conclusions beyond the data. These studies suggest how young children can benefit from activities involving informal statistical inference with additional support, acknowledging that children “possess many conceptual resources that, with appropriately designed and implemented learning experiences, can be bootstrapped toward sophisticated forms of reasoning not typically seen in the early grades” (English, 2014, p. 68).

Middle/secondary School

Research on informal statistical inference has been most prevalent at this age, particularly involving technology-rich settings in which students are making inferences from samples. Ben-Zvi engaged middle school students in looking at samples of data from students in their grade with Tinkerplots, then making informal statistical inferences about the larger cohort of students (Ben-Zvi, Makar, Bakker, & Aridor, 2011). Using a “growing samples” design, students were exposed to the effect of sample size on their confidence in conclusions about the population. Arnold and Pfannkuch (2010) used hands-on activities that morphed into simulations to help secondary students make informal statistical inferences in comparing groups when distributions overlapped. By focusing on the visual rather than the numerical, students used imagery and gesture to gain insights into sampling variability but did not take their eyes off the data when making an inference (Wild, Pfannkuch, Regan & Horton, 2010). These studies illustrate how introducing middle and secondary students to informal statistical inference can provide rich conceptual experiences with complex statistical ideas at an informal level.

University

There has been increasing use of informal statistical inference at the university level to bridge descriptive and inferential statistics and advance Cobb’s (2007) call to increase a focus on statistical reasoning in introductory statistics. Because formal statistical inference is so challenging, the introduction of informal statistical inference at the tertiary level provides new ways to support students’ conceptual development over this difficult terrain towards hypothesis testing. For example, Trumpower (2013) argued that providing university statistics students with opportunities to carry out informal ANOVA tests (where students assessed relative differences within and between groups without statistical measures) improved their conceptual understanding. New visions of introductory statistics have often used a problem-based approach that encourages students to look “beyond the data” from the beginning of the course. While the interpretation of “informal” varies across these courses (Rubin, 2013), the application of informal statistical inference supports an increasing emphasis on student reasoning (Cobb, 2007). Some cases have made informal statistical inference central in the course, like the CATALST course, which initiated students into inferential processes through modelling with simulations prior to introducing formal concepts and tools (Garfield, delMas, & Zieffler, 2010).

DATA AS EVIDENCE

This characteristic recognises that not all inferences are statistical, but to be a statistical inference, it must be based on data rather than anecdote. However, the rigour of the data will depend on the audience and/or person making the inference. For example, children would likely rely on non-random samples (e.g., data from their own class), while older students may use experimental design to improve the quality of the data and validity of the inferences. In this section, we explore recent research on students' use of data as evidence in informal statistical inference.

Young Children

McPhee and Makar (2014) noted challenges in shifting young children (aged 4-6) from relying solely on personal anecdote as evidence for making informal statistical inferences. Following a series of familiar activities that emphasised the role of data in making predictions, many of the children in their study were able to use data they collected for their predictions in a statistical investigation, rather than basing their predictions only on personal experience. Fielding-Wells (Fielding-Wells & Makar, 2013) designed a statistical inquiry in which young students (aged 7-8) sought to design the best card for a game of Addition Bingo (Allmond, Wells, & Makar, 2010). Children initially relied on personal beliefs about lucky numbers and equiprobability; but when outcomes from the game conflicted with expectations, students increasingly relied on data from the game to improve and justify their predictions about the outcomes. Fielding-Wells (2010) also engaged older students (aged 9-10) in statistical inquiry and noted their inexperience in envisioning relevant evidence was an obstacle to their planning. Explicit teaching on seeking links among *question-evidence-conclusion* helped the students to plan for quality evidence. These studies suggest that young children are able to plan, predict and make informal statistical inferences with targeted support, particularly when the context of the problem is familiar.

Middle/secondary School

Building students' awareness of sampling variability using data as evidence in context has been an important focus of research on informal statistical inference at this age. Langrall (2010) argued that making informal statistical inferences and taking a critical stance towards data enables students "to *interact with* data rather than merely apply arithmetical procedures to *act on* data. Knowledge of the data context can encourage and support such interactions" (p. 6). However, she reported that middle school students were unaccustomed to connecting their knowledge of the context to their work with data in school. Ben-Zvi and Aridor-Berger's (2012) research focused on students' integration of the context and data worlds. Students in their study initially regarded data and context separately, but through inquiry were increasingly able to integrate them. Prodromou (2012) reported on two studies in which middle school students used simulations to reason about connections between their data and the wider context they were investigating. As students collected data, they progressively used their samples as evidence to adjust their informal statistical inferences about the problem context. Jacob and Doerr (2013) report that even if secondary students correctly interpret and calculate likelihoods from a sampling distribution, they may struggle with trusting samples to make informal statistical inferences. The level of uncertainty attributed to data from a single sample may prevent them from using their data as evidence. They argued that students need more experience working with comparing a single sample to a sampling distribution in hands-on investigations to gain confidence in what can be concluded from data in a single sample.

University

Pratt and his colleagues (Pratt et al., 2011) argued that in everyday settings such as interpreting graphs in the newspaper, the resources drawn on are typically not mathematical/statistical but rather based on affective responses and personal experiences. On the other hand, in pedagogic and enquiry settings use of mathematical/statistical evidence typically plays a central role in making decisions in lieu of affective and personal resources. They argue that in making data-based decisions in risk-based contexts, affective and personal resources need to be coordinated with statistical evidence rather than suppressed. For example, in authentic inquiry, there is a need to evaluate the quality of the evidence, requiring an understanding of the context.

ARTICULATING UNCERTAINTY

Statistical inference relies on the idea that estimates and predictions beyond the data can never be completely certain. Although the language of informal inferential reasoning is an important scaffold to developing statistical reasoning (Reading, 2009), this area of research has received the least emphasis.

Young Children

Little research has focused on young children's expression of uncertainty when making informal statistical inferences. Because uncertainty is grounded in variability, awareness of variation when working with young children is a critical first step. Watson and English (2013) embedded informal statistical inference in measurement activities, as children often expect all measurements to be precise and accurate. They argued that the variability in repeated measures supported children to recognise and articulate inherent uncertainty in measurement data, emphasised by asking children to comment on the confidence in their estimates. The children in their study (aged 8-9) estimated arm spans based on repeated measures they collected, making informal inferences about the mechanism that produced their measurements (and variation).

Middle/secondary School

The middle school students in a study by Ben-Zvi initially oscillated between expressing inferences with complete certainty and complete uncertainty (Ben-Zvi et al., 2011). For example, they expressed one conclusion deterministically ("Boys like computers more than girls", p. 5), but when asked if a second sample might change their conclusion, they said, "It could be completely different!" Their language became more balanced as they worked with the investigation further. While expressing uncertainty is often qualitative in informal settings, in later schooling students increasingly quantify uncertainties in informal statistical inferences through confidence levels and margins of error (e.g., Dierdorff, Bakker, van Maanen & Eijkelhof, 2012).

University

Little direct research has been conducted on adults' articulation of uncertainty in the context of making an informal statistical inference; however at this age the meaning of "articulation of uncertainty" may shift as more formal statistical tools and processes are encountered, even if statistical tests are only informally carried out (Biehler, Frischemeier, & Podworny, 2013).

DISCUSSION

In this paper, we introduced new literature that highlights opportunities and challenges in introducing informal statistical inference prior to formal hypothesis testing. Informal statistical inference has potential to develop stronger statistical reasoning at all levels. At the tertiary level, research has been useful in describing students' reasoning at this level, but more research is needed to provide evidence of whether and how informal statistical inference improves the transition to formal statistical inference as has been hypothesised. This could motivate a broader shift beyond the pockets of innovation described in the studies reviewed here to using informal statistical inference in university statistics courses.

Much of the new literature on informal statistical inference has aimed at middle and high school, as curricular documents are increasingly acknowledging (explicitly or implicitly) the opportunity it has for making statistics more relevant. A renewed focus on reasoning about data in context and exposure to sampling variability has been a key feature of new research. Several cases of innovation provided insights into new approaches to teaching with informal statistical inference, particularly in using technologies. Increasing a research focus on theorising informal statistical inference for a broader educational audience could improve its adoption in schools.

New research working with students prior to learning descriptive statistics emphasises a potential for informal statistical inference to introduce children to the power of making predictions from data from an early age. These studies suggest the importance of engaging young children in familiar contexts to develop early informal experiences with statistical concepts such as variability, uncertainty, and using data as evidence rather than only anecdotes in making predictions. Further research is needed to build a credible foundation from which curriculum can be developed.

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