DEVELOPING STATISTICAL KNOWLEDGE FOR TEACHING OF VARIABILITY THROUGH PROFESSIONAL DEVELOPMENT

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This paper reports on a professional development programme focusing on statistical literacy, more specifically the development of in-service teachers' awareness of and reasoning about variability in multiple trials under uncertainty, and their ability to transfer their understandings to related tasks. Variability and uncertainty are key concepts in statistics, but are under-emphasised in many school curricula. These topics formed part of the focus of the intervention. Analysis of post intervention tasks revealed a growth in teachers' levels of reasoning about variability and their ability to transfer these competencies to related tasks. The results emphasise the value of well-designed learning experiences and rich discussions in teacher professional development programmes in statistical literacy.

INTRODUCTION

Statistical literacy is a basic requirement for citizens in our information-based society and has to be developed at school (Watson, 2006). For teachers to be able to develop the statistical literacy of their learners, their own statistical thinking, knowledge of statistics and how to teach it, need to be on par (Burgess, 2011; Wessels, 2009). Many teachers all over the world did not study statistics during their initial education and may therefore not have well-developed identities as statistics teachers, lacking the knowledge, skills and/or confidence to teach statistics and probability effectively.

A number of questions therefore arise: How can statistical knowledge for teaching (SKT) be characterised? How can the development of teachers' statistical knowledge be supported? Since the inclusion of data handling and probability in the school curriculum in South Africa, concerted efforts by the Department of Education, university mathematics education departments and the national statistics office, Statistics South Africa (StatsSA) have contributed to an improvement of mathematics teachers' statistical knowledge (Zewotir & North, 2011) through professional development programmes. These efforts have largely been focused on Grade 10-12 teachers, with the result that teachers in lower grades have not had as many opportunities for professional development in statistics and probability (Wessels & Nieuwoudt, 2011). The need for a professional development programme for grade 8 and 9 teachers based on research resulted in the development of the research project described here.

STATISTICAL KNOWLEDGE FOR TEACHING

Different models for statistical knowledge for teaching (SKT) have been developed (including Burgess, 2011; Godino et al, 2011; Groth, 2007; Gonzales, 2012; Wassong & Biehler, 2010), based among others on the model of mathematical knowledge for teaching (MKT) (Ball, Thames & Phelps, 2008), the statistical thinking model of Wild and Pfannkuch (1999) and the GAISE framework (Franklin et al, 2007). This paper takes as point of departure for SKT the MKT model of Ball et al. (2008) which is based on Schulman's seven knowledge categories. Two domains, content knowledge and pedagogical content knowledge, form the backbone of the MKT model. Content knowledge is divided into common content knowledge, specialised content knowledge and knowledge at the mathematical horizon, whereas pedagogical content knowledge includes knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum. Burgess (2011) emphasises the connectedness between these different knowledge variables and the fact that they do not act in isolation from each other. Wassong and Biehler (2010) extended this model to include technological content knowledge as well as technological pedagogical content knowledge.

According to Pfannkuch (2008) the development of SKT goes hand in hand with the development of statistical thinking. The components of statistical thinking are described by Wild and Pfannkuch (1999) as (a) recognising the need for data, (b) considering of variation in data, (c)

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transnumeration of data to engender better understanding, (d) reasoning with statistical models, and (e) synthesising contextual knowledge and statistical knowledge. In the professional development of teachers described in this paper, the focus was on three of these thinking elements: variability, reasoning with models, and transnumeration.

Variability is the quality of an entity to vary, also under uncertainty. Variability and uncertainty is furthermore closely related (Wild & Pfannkuch, 1999). Variability is fundamental to statistical thinking and reasoning and the presence of variability in the world necessitated the development of statistical methods to make sense of data (Cobb & Moore, 1997; Franklin et al., 2005; Shaughnessy, 2007; Wild & Pfannkuch, 1999). It is self-evident therefore that the development of statistical thinking and reasoning should be one of the major goals in statistics education and should take into account the omnipresence of variability in data (Franklin et al., 2005). Teachers need to recognise the fact that variability plays a crucial role in handling data and specific specialised content knowledge about variability that is needed includes sampling reasoning, sampling variability, inferential reasoning and the consideration of sources of variability (Pfannkuch, 2008).

Reasoning with statistical models refers to an understanding of the structural organisation of qualitative and quantitative data in plots such as two-way tables, dot plots, bar graphs, histograms, box plots, scatter plots and reasoning about the information contained in these plots. Pfannkuch (2008) emphasises that teachers not only need to know how students build the intertwined concepts of aggregate-based reasoning and reasoning with different plots, but need to build these concepts themselves through experience.

Transnumeration refers to the changing of representations to engender understanding (Wild & Pfannkuch, 1999). To scaffold learning teachers need insight into learners' intuitive representations, how to encourage learners to consider other representations and how to make transitions from one representation to another (Pfannkuch, 2008).

PROFESSIONAL DEVELOPMENT

Effective professional development in statistics has to be rooted in a professional learning community in which teachers participate in authentic learning activities, such as experiencing the whole statistical investigation process, in "… simulated classroom settings, reflecting on and studying the theoretical basis or rationale for the teaching method from a learner and teacher perspective, observing demonstrations by experts, the teacher educators, and having time to learn in and from practice . . ." (Pfannkuch, 2008).

In our technology driven society, teachers must be prepared to incorporate technology effectively in the teaching and learning of statistics. The implication again is that teachers themselves need to use technology in informal inferential reasoning and the solving of statistical problems in sequenced instructional tasks, just as their learners would have to do (Meletiou-Mavrotheris, 2003; Chance, Ben-Zvi, Garfield & Medina, 2007).

RESEARCH FOCUS

The research project focused on the development of Grade 8 to 12 mathematics teachers' statistical thinking and statistical knowledge for the teaching, including SKT of variability.

METHOD

Research Context and Participants

In the first phase of the study 90 grade 8 to 12 teachers were profiled to determine their statistical knowledge for teaching and their beliefs about statistics in everyday life to inform the second phase, a series of professional development workshops aimed at the development of their statistical thinking and statistical knowledge for teaching (Wessels & Nieuwoudt, 2011). The research in the professional development phase of the project was qualitative in nature.

Fourteen volunteer teachers regularly attended the series of 8 workshops based mainly on research by Canada (2004) with pre-service teachers and a professional development model for in-service teachers developed by Watson (1998). None of these teachers have been involved in any activities on variation in a repeated sampling context prior to the professional development workshops (Wessels & Nieuwoudt, 2013).

Workshops included a combination of hands-on activities and discussions aimed at the development of teachers' own statistical thinking as well as the development of their statistical content knowledge, their statistical pedagogical content knowledge and their knowledge of connections between topics in the curriculum. Reflection on the content and their learning experiences thereof was an important ingredient of each workshop. This focus on discussion and reflection fostered the development of teachers' statistical language and therefore their statistical argumentation and communication skills.

The topics and contexts of the workshops (Table 1) are not all explicitly included in the data handling strand of the South African mathematics curriculum, but were included because they are fundamental to the discipline of statistics. The workshop content therefore had connections with the curriculum, but was not limited to the curriculum. Variability for example is fundamental to statistics (Franklin et al, 2007; Moore & Cobb, 1997; Shaughnessy, 2007) but is not mentioned at all in the South African curriculum (Wessels & Nieuwoudt, 2013). The three statistical thinking elements mentioned above differed in prominence in the eight workshops (Table 1).

Table 1: Workshop content

Workshop	Focus	Contexts	Statistical thinking elements
1	The statistical investigation cycle – categorical data	Data, graphs, sampling	Variability, Transnumeration, Reasoning with statistical models
2	The statistical investigation cycle – numerical data; Sources of variability	Data, graphs, sampling	Variability, Transnumeration, Reasoning with statistical models
3	Variability Identifying trends in data	Data, graphs	Variability, Transnumeration, Reasoning with statistical models
4	Chance – chance language, chance measurement, general links with everyday life	Probability, risk, fairness, bias	Variability
5	Sampling from known and unknown populations; Variability in chance context	Data, graphs, sampling	Variability, Reasoning with statistical models
6	The statistical investigation cycle, terminology, graphs, etc.	All	Variability Reasoning with statistical models
7	Informal inferential reasoning	Data, graphs	Variability, Reasoning with statistical models
8	The use of technology for developing statistical thinking and reasoning	Data, graphs	Variability, Transnumeration , Reasoning with statistical models

Before starting on the topic of variability in a repeated sampling context participating teachers completed a number of tasks to establish their statistical content and pedagogical content knowledge on variability. The survey tasks included the sampling of known and unknown populations and probability sampling (candy jars). Post-survey questions on variability in a repeated sampling context again included the sampling of known and unknown populations and probability sampling, but different contexts were used (spinners, movie starting times, dice, rainfall, class scores) to determine if teachers could transfer their understanding of variability to these new tasks.

Analysis

Teachers' responses to the pre- and post-survey tasks on variability were analysed using the hierarchical model of four types of reasoning developed by Reading and Shaughnessy (2000). The four types are iconic, additive, proportional or distributional reasoning and show increasing levels of sophistication. Iconic reasoning relies on physical circumstances or personal stories to predict sample proportions; additive reasoning focuses on frequencies, but not absolute frequencies; proportional reasoning explicitly refers to connections between sample proportions and population proportions; and finally distributional reasoning takes into account both centres and spreads, explicitly mentioning variation about the expected value (Shaughnessy, 2007).

RESULTS AND DISCUSSION

Before exposure to tasks in a repeated sampling context, teachers' reasons for expected variation displayed 25% iconic reasoning giving vague reasons or referring to physical circumstances (Wessels & Nieuwoudt, 2013). Of the responses 27.5% showed additive reasoning focusing on absolute, not relative frequencies and not referring to the proportions of the population. Proportional reasoning was evident in 45% of responses with reasons implicitly or explicitly attending to sample proportions, population proportions, probabilities or percentages. Only 1 response out of 40 (2.5%) demonstrated distributional reasoning with reasoning about centres as well as variation around the centres.

After the intervention tasks in a variety of contexts were used to assess whether teachers had gained a better understanding of variability through exposure to activities and tasks on variability during the professional development workshops. Only 8 teachers attended the pen-ultimate workshop when the post-survey on variability was administered. From the analysis it is clear that teachers' awareness and understanding of variation in multiple trials grew during the intervention, and that 6 out of 8 teachers participating in the post-intervention survey could transfer this improved understanding to related tasks. Analysis also showed that teachers found it easier to describe their expectations and reasoning in tasks about repeated trials in contexts about spinners and dice than in tasks where they had to come up with a graph of repeated trials or had to compare variability across distributions. Most teachers still displayed some difficulty with the language of variability when explaining their thinking, with the transnumeration of the data and with reasoning about the data with different kinds of representations.

The fact that several teachers motivated their expectations of and reasoning about variation by referring to their own experiences during the workshops, was encouraging. Teacher comments in their reflections on the whole intervention included references to the improvement of their specialized statistical content knowledge, their statistical knowledge of content and students, statistical knowledge of content and teaching, knowledge of the use of statistical software for the development of statistical thinking, as well as knowledge of the statistics and probability curriculum. All the teachers declared that their confidence in the teaching of statistics grew and that they have developed a better understanding of the differences between statistics and mathematics.

Reflection on the content and their learning experiences thereof played an important role in this process. The fact that these experiences and discussion were embedded in the supportive community of learning that developed between the teachers created a climate of trust needed for effective learning.

IMPLICATIONS FOR TEACHER PROFESSIONAL DEVELOPMENT

Professional development of teachers should be embedded in a professional learning community and should give ample opportunity for teachers to reflect on their own learning experiences and share their thinking with colleagues. Participation in the same kind of activities as their students is fundamental to the development of statistical thinking of teachers and should be a fundamental element in professional development initiatives. Activities and discussions should not only focus on content, but on how statistical thinking of students can be developed and how the understanding of statistical concepts can be scaffolded.

A recurring theme in teachers' reflections after the series of workshops was the need for further support in the form of follow-up workshops on content as well as regular ongoing sessions on the use of technology to develop statistical thinking. Professional development should ideally entail a regular and longer term involvement with teachers.

CONCLUSION

The sequence of professional development workshops proved to be a worthwhile starting point for the development of teachers' SKT and statistical thinking. Involvement in a professional learning community and participation in activities doing statistics to develop statistical thinking are crucial for the success of such professional development initiatives.

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