TEACHING PROBABILITY: USING LEVELS OF DIALOGUE AND PROPORTIONAL REASONING

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Understanding probability and conditional probability are important aspects of a statistical literacy program. Unfortunately many students find these difficult concepts to grasp. Enhancing students' understandings of probability is explored from two perspectives. The first is based on the notion that students are better able to construct their understanding of probability when teachers use a hierarchy of dialogue questions from descriptive to the abstract, depending on the students' understanding and responses. The second approach to enhance students' understanding of probability is to extend their understanding of proportional reasoning and their ability to conceptualise how the same numerical value can be expressed in different forms.

Probability and conditional probability continue to be identified as "hard" concepts to teach as part of a statistical literacy program of study in schools and even at the University level (Tarr & Jones, 1997; Watson & Kelly, 2007). It is not, however, the intention in this paper to investigate directly the teaching of probability or conditional probability in a school setting (see Shaughnessy, Chance, & Kranendonk, 2009, for examples of this). Rather the intention in this paper is to investigate why students' learning of probability and conditional probability is often a complex procedure for them to grasp. This investigation will focus on the hypothesis that the effective teaching of probability requires teachers to perceive the issue from at least two theoretical perspectives; one that is from a linguistic orientation and one that is from a proportional reason orientation. Both of these perspectives will be reviewed.

LINGUISTIC ORIENTATION

Typically when teachers teach probability to their students they usually give a working definition, such as probability is the chance or possibility that an event will happen (Tarr & Jones, 1997). Words, such as probability and likelihood are however inferred and what understandings the students do have about those words are often developed in a non-mathematical context. Until the words are also understood in a mathematical sense the students often struggle with the mathematical concepts associated with those words (Ellis & Shintani, 2014).

The issue is the teacher who is teaching statistics has often internalised his/her thinking and understandings associated with the terms (such as probability) to such an extent that they assume that a brief introduction to the words and their concepts in sufficient. Getting "down" to the students' level of understanding of the concepts associated with probability is not always easy and it is made more difficult when the concept is an abstract. For example one of my adult research students achieved an output from the SPSS statistical package that read: F (1, 215) = 24.2, Sig .000. The student was concerned that the significants between the two groups was zero and her intervention had not worked. From a cognitive learning perceptive this example supports the notion that student's lack of understanding of statistical literacy is typically associated with a lack of immediate recognition of the vocabulary, concepts, and the instructional cues that needed to be triggered before comprehension is achieved (Watson & Kelly, 2007; Hay, 2010). This is specially the case where the context of the learning is unfamiliar to the students and there is a need to enhance the students' motivation and level of self-efficacy, interest and engagement that directly and indirectly influences students' ability to master the content associated with statistical literacy (Carmichael, Callingham, Watson, & Hay, 2010; Hay, Callingham, & Carmichael, in press).

Therefore, when teaching the concepts associated with probability the teacher has to work at the formation of the students' new reasoning about the meaning and application of probability. Such constructed knowledge is typically developed through classroom modelling, supported practice and dialogue between the teacher and the students (Guk & Kellogg, 2007). How this supported dialogue is developed by teachers has been researched by Hay et al. (2007) and Hay,

In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona, USA.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

Callingham and Wright (2013) using a hierarchical teacher questioning model, initially conceptualised by Blank (2002).

The four basic levels of questions and interactions are outlined in terms of their complexity in Table 1. Students who have limited mastery of the lower levels of complexity will generally have difficulty with the more advanced levels. Teachers can use these four levels to introduce and review topics for discussion and learning, and to go backwards or forwards across the levels depending on the responses provided by the student. The following is an example of dealing with probability based on an activity using 10 balls in an urn, of which 3 balls are red and 7 are blue.

Table 1. Four levels of language dialogue, related to a probability task

Level	Task	Example of Teacher Dialogue
1	Characteristics	What do you see? How many balls are there in the urn? How many are red?
2	Classifying	Group the balls by colour. If I take one red ball how many are left? How many red balls are left? How many blue balls are left?
3	Reordering	Re-tell me the story of what is happening. When I have 10 balls, 5 red and 5 blue, the change getting a red ball from the 10 balls is 5 out of 10,
4	Abstract/infer	Re-tell me the story in mathematical notation. Explain what you think the word probability means

PROPORTIONAL REASONING ORIENTATION

Imbedded with difficulties in understanding probability are students' difficulties with proportional reasoning, with difficulties in proportional reasoning having an ongoing negative influence on students' ability to reason mathematically (Boyer, Levine, & Huttenlocher, 2008). Proportional reasoning involves individuals thinking about relationships and making comparisons of quantities and values (Van de Walle & Lovin, 2006). Proportional reasoning is particular relevant in developing students' knowledge, understandings and applications of ratios, rates and rational number and so fractions, decimals and percentage. Proportional reasoning therefore permeates much of the reasoning associated with statistical literacy. An example of this linkage is illustrated in Table 2.

Table 2 Example of a probability activity and proportional reasoning

A plane has rows numbered from 1 to 30, with 6 seats per row, 3 on each side of the isle. Seats in each row are labelled A through F. Use this information to answer these questions.

	Question	Answer
1.	How many seats are in the airplane?	*180 seats*
2.	What are your chances of sitting in row number 7?	*6/180, or 1/30*
3.	What are your chances of sitting in a window seat? *Note there are two window seats per aisle, for a total of 60 window seats.*	*60/180, or 1/3*
4.	What are your chances of sitting in an "A" seat? *Note there are 30 A seats*	*30/180, or 1/6.*
5.	What are your chances of sitting in an even-numbered row? *Note of the 30 rows, 15 are even-numbered,*	*15/30, or 1/2.*

At the core of the students' understanding of the Table 2 statistical literacy task is the students' understanding of proportional reasoning and their ability to understand the ratio between the specific question asked and the total number of outcomes, which is either the total number of seats or rows. It is the students' ability to conceptualise their answer as a fraction, decimal, or percentage that is critical in proportional reasoning. For example, the probability of sitting in seat 7A is 1/180, or 0.00555. In statistical literacy the ratio is often presented as a decimal to illustrate if the probability of an event is great or small. At the start of this paper an example was provided of the graduate student who did not understand what a significant difference of .000 meant. In reality for this student the difficulty in understanding was more a difficulty in understanding proportional reasoning and how a numerical value can be expressed as a ratio, a fraction, or a decimal. Again the critical issue is spending time with students on proportional reasoning tasks and providing them with meaningful experiences so they are comfortable with expressing the same numerical value in different forms.

In conclusion, helping students to understand the concept of probability can be enhanced using two teaching practices. One is incorporating a hierarchy of questions to help students contrast a stronger and greater understanding of the topic. The second is to highlight and revise proportional reasoning so that the students understand how a similar numerical value can be represented in different ways. Both of these strategies are evidence based, but in a busy classroom the teacher of statistics may incorrectly assume that these strategies are not that necessary, where the evidence is the strategies need to permeate much of the instruction associated with statistical literacy.

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