COMBINING NONPARAMETRIC INFERENCES USING DATA DEPTH, BOOTSTRAP AND CONFIDENCE DISTRIBUTION

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For the purpose of combining inferences from several nonparametric studies for a common hypothesis, we develop a new methodology using the concepts of data depth and confidence distribution (CD). In recent years, the concept of CD has attracted renewed interest and has shown high potential to be an effective tool in statistical inference. In this project, we use the concept of CD, coupled with data depth, to develop a new approach for combining the test results from several independent studies for a common multivariate nonparametric hypothesis. Specifically, in each study, we apply data depth and bootstraps to obtain a p-value function for the common hypothesis. The p-value functions are then combined under the framework of combining confidence distributions. The method will be illustrated using simulations and aircraft landing performance data.

BACKGROUND

A confidence distribution (CD) is a sample-dependent distribution function that can be used to estimate parameters of interest. It is a purely frequentist concept yet can be viewed as a "distribution estimator" of the parameter of interest. Examples of CDs include Efron's bootstrap distribution and Fraser's significance function (also referred to as p-value function). In this project, we use the concept of CD, coupled with data depth, to develop a new approach for combining the test results from several independent studies for a common multivariate nonparametric hypothesis. This approach has several advantages. First, it allows us to resample directly from the empirical distribution, rather than from the estimated population distribution satisfying the null constraints. Second, it enables us to obtain test results directly without having to construct an explicit test statistic and then establish or approximate its sampling distribution. The proposed method provides a valid inference approach for a broad class of testing problems involving multiple studies where the parameters of interest can be either finite or infinite dimensional.

The powerful computers have now made automatic data collection an operational routine in many domains. The collected data are often intensely studied by researchers to extract useful patterns or inferences for the purpose of scientific discoveries or marketing values. Motivated by competition (for example, Walmart vs Target in competing for consumers), there are often multiple studies from different sources on the same target hypotheses or parameters. Naturally, by pulling together the findings from the individual studies, one would expect to draw a more effective overall conclusion about the common hypotheses or parameters.

This note describes a new nonparametric approach for synergizing findings from different studies for a common hypothesis, by using confidence distribution, data depth and bootstrap.

P-VALUE FUNCTION BY BOOTSTRAP AND DATA DEPTH

We use a simple hypothesis testing problem to walk through the approach of using data depth and bootstrap to obtain a p-value. This approach has been introduced in (Liu ad Singh, 1997). Let $\{X_1, \ldots, X_n\}$ be a random sample from a d-dimensional distribution F_θ, where θ is the parameter of interest. We consider the following simple test of hypotheses,

$$H : \theta = \theta_0 \ vs \ K : \theta \neq \theta_0 . \tag{1}$$

Let $\hat{\theta}$ be the sample estimate of θ, and θ^* the corresponding bootstrap estimate of θ. We can repeat the bootstrap procedure k times to obtain k bootstrap estimates: namely, the first bootstrap sample $\{X^*_{1,1}, \ldots, X^*_{1,n}\}$ yields θ^*_1, …, and the k-th bootstrap sample $\{X^*_{k,1}, \ldots, X^*_{k,n}\}$ yields θ^*_k.

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Now, we can define a p-value for the test (1) as

$$p_n \equiv P^*_{G^*}\{\theta^* : D_{G^*}(\theta^*) \le D_{G^*}(\theta_0)\}. \qquad (2)$$

In other words, p_n is the proportion of θ^*s in the cloud of θ^*s which are less central (or more outlying) than θ_0. Here G^* is the bootstrap distribution for θ^*, and $D(\cdot)$ is a data depth function. Fuller discussions on data depth can be found in Liu, Parelius and Singh (1999) or Zuo and Serfling (2000).

CONFIDENCE DISTRIBUTION (CD)

Recently, there is a strong renewed interest in distributional inferences, including confidence distribution, generalized fiducial inference, objective Bayes, belief function (e.g., see the review (Xie and Singh, 2013) and the relevant references there). CD is a key part of recent developments on distributional inferences. The idea of the CD approach is to use a sample-dependent distribution (or density) function to estimate the parameter of interest. Fiducial inference, bootstrap and Bayesian/Objective Bayesian approaches can be treated as two convenient mechanisms for producing CDs. The p-value p_n in (2) can be justified as a CD.

COMBINING INFERENCE USING CDS

Assume that $X_{1,1}, X_{1,2}, \ldots, X_{1,n_1}\, i.i.d. \sim F_1,\ \theta_1$ is a functional of F_1, $\cdots$, $X_{k,1}, X_{k,2}, \ldots, X_{k,n_k}\, i.i.d. \sim F_k,\ \ \theta_k$ is a functional of F_k. Assume that $\theta \equiv \theta_1 = \theta_2 = \ldots = \theta_k$.

Step 1. Obtain a p-value function from each of the studies for testing $H: \theta = \theta_0$ *vs* $K: \theta \ne \theta_0$.

$$P_{i,n_i}(\theta_0) = P_{G^*_{i,n_i}}\{\vartheta : D(G^*_{i,n_i}; \vartheta) \le D(G^*_{i,n_i}; \theta_0)\}$$

where G^* is the bootstrap distribution and D is a data depth function.

Step 2. Combine p-value functions from all the studies

$$p_c(\theta) = \Phi\Big(\frac{1}{\sqrt{\sum_{i=1}^{k} w_i^2}}\big(w_1\Phi^{-1}(p_1(\theta)) + \ldots + w_k\Phi^{-1}(p_k(\theta))\big)\Big)$$

where $p_i(\theta)$ is the p-value function for the i^{th} study, $\Phi(\cdot)$ the cdf of a standard normal distribution, and w_i is a combining weight.

Step 3. Make inference based on the combined p-value function $p_c(\theta)$.

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