HIGH SCHOOL TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF VARIABILITY

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This research sought to explore teachers' pedagogical content knowledge of the concept of variability. Twelve mathematics high school teachers were tested on their knowledge of the concept of variability. Subjects were then asked to react when presented with scenarios describing students' strategies, solutions and misconceptions when faced with a task based on the concept of variability. Outcomes of this study uncovered interesting teaching interventions that could prove useful to teachers faced with such scenarios. Results of both teachers' tests and interviews revealed that teachers had difficulties and misconceptions related to the concept of variability. Furthermore, teachers' reactions to some scenarios highlighted the influence of content knowledge of the concept of variability on the pedagogical content knowledge related to this concept.

INTRODUCTION

Generally speaking, statistics is taught within the school mathematics program. In this study, we have focused on the concept of variability, a key concept for the development of statistical thinking in so far as we define statistics as the science of natural and social events' variability in the world around us (Wozniak, 2005).

Acknowledging an event's variability means recognizing that the results are subject to variation, understanding that they are unpredictable, taking into account the sampling fluctuations, and letting go of certainty to enter the world of uncertainty. By giving up certainty, one can then use statistical methods to partially control uncertainty in order to estimate, predict, and make decisions within acceptable risk. This is the true issue of statistical reasoning (Vergne, 2004).

In order to develop students' statistical thinking, the concept of variability must be taught. It becomes therefore appropriate to verify teachers' knowledge of this concept. Since the end of the last century, studies on the teaching of statistics have multiplied. Some studies include the concept of variability but the study subjects were generally students (Reading & Shaughnessy, 2004; Shaughnessy, Ciancetta & Canada, 2004). Little attention has been given as to how high school mathematics teachers cover the concept of variability (Dabos, 2011). We have therefore explored ways of answering the following question: Given the teachers' content knowledge, what is their pedagogical content knowledge with regards to the concept of variability?

CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE

Two aspects of teachers' knowledge need to be addressed: content knowledge and pedagogical content knowledge. Shulman (1988) defines content knowledge as how a specialist in a specific field understands a related subject matter. Pedagogical content knowledge refers to the ability to introduce and explain a topic for others to understand. This type of knowledge goes beyond content knowledge and focuses on a different dimension; knowing the content in order to teach it. Pedagogical content knowledge includes the teacher's understanding of what makes it easy or not for a student to learn a specific content. Teachers refer to their own strategy, their most frequent mistakes and their possible interventions to help students with misconceptions and difficulties. So now, how can we evaluate pedagogical content knowledge?

According to Vergnaud (2002), there are two forms of acquired knowledge: predicative and operative. Predicative knowledge is defined as the written knowledge as found in textbooks for instance. It states the properties and relations between objects of thought. Operative knowledge is called upon to act in a situation. Vergnaud in a later work (2006) suggested that most of our acquired knowledge is implicitly and even unconsciously operative. Sometimes, an impressive gap exists between what a person does in a given situation and his or her ability to comment on it. In the present case, for example, a person may find it difficult to describe a priori inaccurate notions related to the concept of variability, but this doesn't mean that he or she has no knowledge on the

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subject. If he or she can identify this knowledge in action, we may then say that he or she has operative knowledge (Vergnaud, 2002).

METHOD

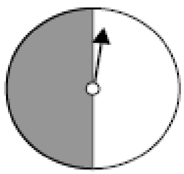
The research described in this article involved twelve Québec high school mathematics teachers (Vermette, 2013). The experiment was done in two phases. First, teachers had to answer a questionnaire of 6 questions involving the concept of variability. This first phase allowed the evaluation of teachers' content knowledge of the concept of variability. It was followed by an interview where teachers were presented with teaching simulations. The objective of the second phase was to observe the teachers pedagogical content knowledge in action. The simulations involved a pedagogical task related to each item of the previous questionnaire. In his or her day to day work, a teacher must accomplish different tasks; some of which qualify as pedagogical tasks as they call upon pedagogical content knowledge. Our study targeted the following pedagogical tasks: analyzing problems, and analyzing students' solutions and strategies as well as the teachers' responses.

Below are two scenarios pertaining to pedagogical tasks presented to teachers who were asked to consider students' answers and reasoning and suggest a pedagogical approach. They are preceded by the related questionnaire's item on variability.

Question 1

The content question (adapted from Watson, Kelly, Callingham & Shaughnessy, 2003):

A teacher gives a wheel, identical to one in the picture below, to each student and asks them to do 5 series of 50 spins. For each series, they must count the number of times the arrow stops in the shaded area. You have already done the experiment. Write a list of numbers that show the number of shaded areas obtained. Why have you chosen these numbers?



The pedagogical content question:

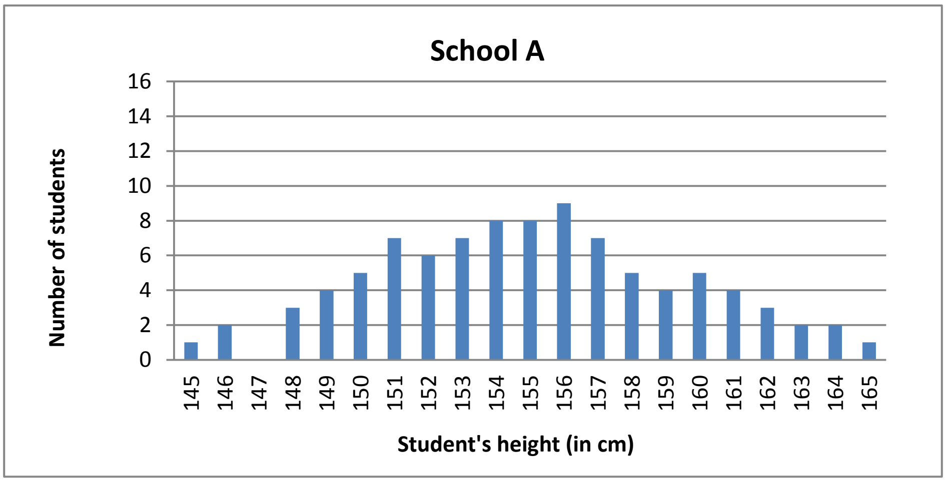
A student finds the experiment too long and decides to turn the wheel 5 times and multiply the result by 10. He does the same for each series. What do you think of his strategy? How would you respond?

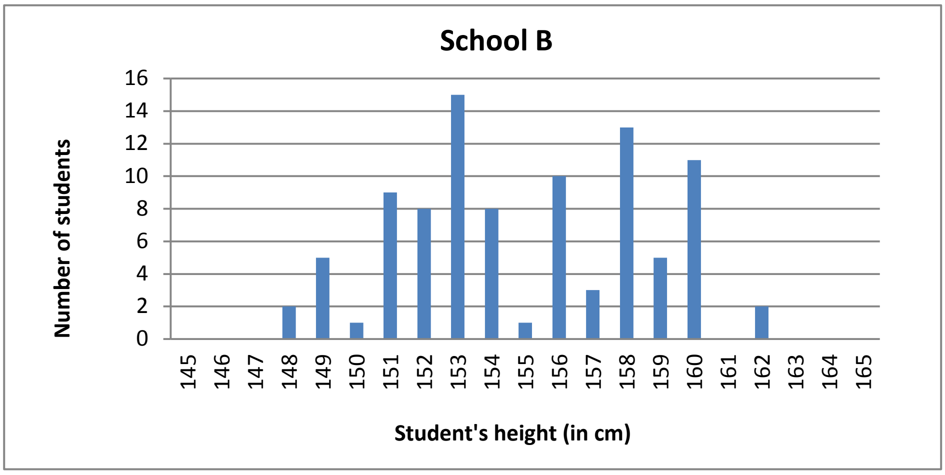
This situation highlights the concept of variability in a probabilistic sampling context. The task given to the teacher is based on a student's misconception which doesn't take into account the size of the sample as if it had no influence on the variability of the results. By thinking this way, the student assumes that the results would be the same for each of the 10 repetitions. According to Reading and Shaughnessy (2004), by the end of high school, some students use this proportional reasoning to link samples' and the population's proportions. Here, a student obtaining 4 shaded areas in 5 rotations would deduct that he could then get 40 in 50 rotations. However, as the sample size increases, the features of a random sample resemble the statistical features of the population. Therefore, the variability of a size 5 sample is greater than a size 50 sample. It is also important to point out that the student's strategy makes it impossible to obtain the value corresponding to the theoretical probability of 25.

Question 2

The content question (adapted from Canada, 2004):

The charts below describe the height, in centimeters, of 1st year high school (grade 7) students from two different schools. Each school has 93 students. Which chart shows the greatest variability in 1st year high school student's height? Explain your choice.





The pedagogical content question:

Although they reasoned differently, two students come to the same conclusion for this question; school B's graph shows a greater variability. The first student justifies his or her reasoning with the fact that school B's bar chart has an oscillating pattern. The second student finds school A's bar chart almost symmetrical and concludes that school B's chart shows a greater variability. What do you think of the students' answers? Which reasoning do you favor? How would you respond to each student?

In this situation, the concept of variability is shown through the data dispersion over the two bar charts. Again, the task given to the teacher stems from the students' misconceptions. Both reasoning highlighted in this problem are based on the works of Cooper and Shore (2008), delMas and Liu (2005) and Meletiou-Mavrotheris and Lee (2005) who demonstrated that, when interpreting the variability of data distribution on a bar chart or a histogram, some students were influenced by aspects related to the shape of the distribution. The first student's answer is influenced by the variation in height of school B's bars. This student's reasoning refers to the frequency variability and not to the subjects' height variability. As for the second student, he is influenced by the symmetry of school A's chart. The distribution's symmetry is not a variability indicator.

The previous tasks showed whether the teachers participating in teaching simulations were able to recognize, in action, misconceptions linked to the study of the concept of variability and to identify how they would have responded. The goal was to observe their acquired pedagogical content knowledge of the variability concept.

RESULTS

The research identified three types of teaching intervention. The first one, *explanation*, refers to the reasoning that allows one to clarify the concept and answer by the same time the question. In this case, pupils are not engaged in a reflection, research and validation of their knowledge. The second one, *confrontation*, points out the erroneous reasoning of pupils and presents conditions allowing them to doubt their reasoning and the resulting answer. It raises a cognitive conflict that forces pupils to question their conceptions and to rectify them. The third one, *experimentation*, came up, particularly in probabilistic scenarios, and consists in engaging pupils in one or more experimentations from which they should be able to reassess their representations.

Question 1

For question 1, eight teachers identified the issue of sample size and suggested a response. Of those eight subjects, six had given a varied list of values representing the number of shaded areas obtained from the different samples. The other two teachers gave a list based solely on the expected value corresponding to the theoretical probability. Reading and Shaughnessy (2004) had observed that for high school students, learning probability within the school mathematics program influences their use of the variability concept.

- Experimentation

The two teachers who did not include the sampling fluctuation in their response to the pedagogical question did recognize that the student's reasoning may distort reality. They suggested that the student try different size series and compare the results to see the differences between the series.

- Explanation

One teacher explained how the sampling size affected the sampling fluctuations.

- Confrontation

- Extreme case: Three teachers presented a student with an extreme result: associating two results of 0 shaded areas obtained by turning the wheel 5 or 50 times.
- Special case: Two teachers responded by exaggerating the student's reasoning. For example, they asked the student to apply the same method but this time by turning the wheel three times instead of 5. The percentage of the shaded areas obtained will distance itself from the value corresponding to the theoretical probability of 50% as the student will only be able to obtain one shaded area out of two or vice-versa.
- Analogous case: One teacher suggested considering the experiment in a different context to prevent the occurrence of an event when results are transferred from a small sample to a larger one by proportional reasoning.

> "*If I were to roll a six-sided dice 5 times I would not obtain the six possible results. Now if I multiply my results by 10 it means that I could only obtain five different results in my experiment. While this is not impossible it is highly improbable i.e. 50 rolls would produce each possible result at least once.*"

Question 2

For question 2, seven out of the twelve teachers identified both students' mistakes and suggested a response. These teachers had answered the content question correctly.

- Explanation

Four teachers explained the problem by opposing the variability in sizes and frequencies to illustrate that in this case the problem needs to be solved horizontally and not vertically.

- Confrontation

- Transition to numbers: One teacher suggested tabulating the values so the shape of the distribution wouldn't influence the students.
- Counter-example: Two teachers gave the students a counter-example. For instance, one teacher suggested a symmetrical distribution showing a low variability despite wide differences in the bars' heights.

> "*If 14 students are 153 cm tall, 14 students 155 cm tall and 2 students 154 cm tall, you obtain high and low bars and the distribution is symmetrical. Does the students' height vary greatly? Not really, they all measure almost the same.*"

Most teachers who were not able to offer a response had not answered the content knowledge question correctly. Some teachers had difficulty to refuse inaccurate reasoning as they had also given the same answers.

CONCLUSION

The suggested pedagogical situations allowed a better understanding of the subjects' pedagogical knowledge in action. Some responses were more creative while still offering the right conditions for students to realize their mistakes.

The overall results of our study show that teacher's pedagogical content knowledge depends on their content knowledge even though good content knowledge does not guarantee good pedagogical content knowledge. This study also showed that the reasoning previously observed in pupils and university students were equally observed in high school teachers.

This research highlights the importance of studying teachers' content knowledge and pedagogical content knowledge. The pedagogical questions used in this research allow a look at the teaching of the concept of variability, especially by pinpointing and documenting possible responses based either on various strategies including explaining, experimenting or confronting, for teachers who must deal with students' reasoning and answers. The outcomes of this research should be taken into considerations for future teacher training.

Knowledge of conceptions relative to a particular concept allows teachers to more appropriately plan their lessons. The various conceptions of variability reviewed in this research will be useful for training pre-service and in service teachers while helping them to recognise erroneous or inadequate conceptions and prepare them to counter these misconceptions with adequate interventions.

It is necessary to implement a method to evaluate teachers' pedagogical content knowledge and to see how it is used in action. Simulations are to be considered as they let the subjects respond spontaneously to classroom-like situations. Simulations are also easier to implement than classroom observation where it becomes difficult to target the study of a concept taught over many years. It goes without saying that researching students' learning is necessary as it is the foundation for the creation of pedagogical situation.

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