STATISTICAL KNOWLEDGE FOR TEACHING: ELEMENTARY PRESERVICE TEACHERS

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A major component of statistical thinking deals with the omnipresence of variability in data. Advances in technology allow for the development of tasks that can engage students more readily in data analysis so that they come to see this variability as early as the elementary grades. Yet how do we help prepare elementary preservice teachers (PSTs) to understand variability in data for themselves and to consider the statistical thinking of children? This paper will share tasks that were designed for a statistics course for elementary PSTs. These tasks make use of several forms of technology such as Tinkerplots® and Interactive Whiteboards (IWBs), and have the intent to develop PSTs' statistical knowledge for teaching. Preliminary data analysis reveals that these tasks provided PSTs with a conceptual way of appropriately attending to measures of variability in a manner that the knowledge of procedures could not.

INTRODUCTION

What is the statistical knowledge needed for teaching in grades K-8? Burgess (2009) notes, "There is an extensive research literature on teacher knowledge needed for teaching mathematics. Statistics education research has a much shorter history and its literature pertaining to teacher knowledge is relatively scarce" (p. 18). Further, Sanchez, Silva and Coutinho (2011) believe there is an urgent need for studies of teachers' professional knowledge for teaching variation. Building upon recent work with middle school students in exploring measures of center and given the American Statistical Association's (ASA) endorsement of the Guidelines for Assessment and Instruction in Statistics Education (GAISE) report (Franklin et al., 2005), as well as the more recent Common Core State Standards for Mathematics (CCSSM) in the US (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), we are redesigning our course in statistics and probability for elementary/middle school preservice teachers (PSTs). We are reworking aspects of the course to better prepare our PSTs for the demands of teaching the fundamental concepts of statistics as well as helping PSTs to think about the statistical reasoning of students in grades K-8. This paper will focus on describing tasks developed for the course that focused on developing the PSTs' understanding regarding measures of variability.

FRAMEWORKS

In his paper conceptualizing statistical knowledge for teaching (SKT), Groth (2007) indicated there is sufficient overlap from the work describing the mathematical knowledge for teaching to inform the study of SKT. With this in mind, Groth (2012) presented a model for investigating SKT based on Ball, Thames, and Phelps' (2008) model of Mathematical Knowledge for Teaching. Our primary concern has focused on the aspects of the model that pertain to subject matter knowledge. Table 1 provides descriptions for these content-related aspects that Groth (2012) adapted for statistics.

Table 1 *SKT framework from Groth (2012) p. 23-24*

Aspect of Subject Matter Knowledge	**Definition adapted by Groth (2012) for SKT**
Common Content Knowledge (CCK)	Statistical knowledge needed for general purposes, e.g., measures of center and spread.
Specialized Content Knowledge (SCK)	Statistical knowledge needed specifically for teaching, e.g., transitional displays such as hat plots (Watson, 2008).
Horizon Content Knowledge (HCK)	Knowledge of how the statistics being taught relates to statistics that the students will learn in the future.

In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona, USA.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

In addition to these aspects of subject matter knowledge, we used the GAISE report (Franklin et al., 2005) to look at more detailed information on what the PSTs' level of understanding needed to be when they completed their statistical course of study. At the heart of the GAISE report is the framework that identifies four process components, along with two aspects of variability, and classifies student progression across three levels (A, B, and C). The process components and aspects of variability are:

I. Formulate questions
II. Collect data
III. Analyze data
IV. Interpret results
V. Nature of variability
VI. Focus on variability

The levels A, B, and C are intended to show what appropriate grade-level understandings are for K-5, 6-8, and 9-12 respectively. However, as in the van Hiele levels for geometry (van Hiele, 1959), it is quite often the case that students do not exhibit understandings appropriate to their age group due to a lack of *experience* with statistics. Thus, when reflecting on our elementary PSTs' performance at the outset of the class we found a non-trivial portion of our classes performing at level A. Our PSTs have some statistical background, but it is chiefly based on a few high school course units related to statistics embedded within other mathematics courses, such as algebra. It is not typical for our elementary PSTs to have successfully completed an AP statistics course in their high school program. Teaching probability and statistics to elementary and middle school children requires all three aspects of subject matter knowledge which suggests, given HCK, that PSTs should have a minimal requirement of performing within level C, preferably surpassing it. We endeavored to design tasks to move the PSTs' understanding towards that level.

CONTEXT

The context of our study was the course, *Probability and Statistics for Elementary/Middle School Teachers,* a content course designed for future elementary/middle school teachers. It is one of three required mathematics content courses for all undergraduates in elementary education at a doctoral-granting university in the U.S. Midwest. The class of 12 students was taught during the seven-week Summer term in 2013, with it meeting four days a week for 100 minutes. The technology environment in which the class was held included several interactive whiteboards (IWBs) that were utilized throughout the term. Students were expected to bring their laptop computers and graphing calculators to each session. The Web 2.0 tool Google Documents was used daily. The instructor (one of the co-authors) would post the content focus of each session, course announcements, and indicate which student would write up a summary of key ideas for that day's session on these documents. PSTs would use the same Google document to raise homework questions with their classmates. PSTs were encouraged to respond to the posted questions at the beginning of class on the Google document itself; those that were left with no response were used as initial class discussion points. PSTs were assigned to write daily session summaries and these were posted on a class Wikispaces page.

The initial redesign of current tasks, the incorporation of new tasks, and the sequencing of topics were conducted in order to study what tasks may facilitate the development of the PSTs' statistical knowledge for teaching, and to begin to study the learning trajectories of major statistical concepts. Some of the newly created tasks evolved around the use of TinkerPlots® (Konold & Miller, 2012), statistical software designed for data analysis and probability explorations for upper elementary and middle school students. This paper will focus on the tasks related to developing the PSTs' understanding of measures of variability, in particular, the mean absolute and standard deviation.

METHODS

The work described here is a part of a larger design experiment conducted throughout the semester consisting of cycles and iterations of developing rich tasks that made strategic use of digital tools to engage the PSTs in statistical and probabilistic reasoning. The intent of the design experiment (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) was to be innovative in the way we incorporated the major ideas from the GAISE document (Franklin et al., 2005), the CCSSM for grades K-8 (NGACBP & CCSSO, 2010), as well as various digital technologies into the content course through specially designed or modified tasks. Tasks were created in light of these documents and tools with the intention of developing major statistical ideas such as variability. These tasks were then implemented in the classroom by the authors.

To help understand the impact of the designed tasks on student understanding, we collected four different types of data on the thinking of the PTSs. First were the instructor's lesson notes created in preparation for class and then modified afterwards. The second data source included classroom observations where field notes were taken during the implementation of the tasks. The third was the class work that PSTs generated consisting of assignments, quizzes, exams, and projects. Much of the time these work items had a technological tool component. This digital work was collected through Dropbox, software that allows one to store data and provide access to many people at one time. The final pieces of data on the PSTs' thinking were student interviews. The PSTs (who volunteered) participated in a 20-30 minute interview with one of the researchers shortly following the completion of the course. The purpose behind collection of the interview data was to provide PSTs with the opportunity to explain their thinking on particular tasks and to comment on affordances and constraints of the tasks and technological tools utilized in the course.

With regard to data analysis, this paper presents preliminary findings as we are only in the beginning stages of analysis. The qualitative analysis of the data will include in-depth descriptions of coursework, analysis of themes within individual PSTs' work, analysis of themes across their work, and interpretation of these themes. A constant comparative method, where multiple researchers read, code critical points, discuss, and refine codes, will be used to reduce threats to validity (Creswell, 2007; Strauss & Corbin, 1990). This paper will provide preliminary findings from tasks and class sessions examining variability in data, specifically that of making sense of the absolute mean and standard deviation measures. Our chief data sources for this discussion will be in-class assessments, instructor lesson notes, and post-class interviews.

BACKGROUND AND DESCRIPTION OF TASKS

PSTs had been involved in several small in-class "statistical investigations" that made use of the first four process components outlined in the GAISE document (Franklin et al., 2005). They had posed questions to investigate, collected relevant data, organized and analyzed the data, and interpreted their results. As we cycled and recycled through these components in the investigation process, new ideas would be introduced such as variable types (e.g. numerical or categorical), ways in which to display the data, types of questions to ask children based upon constructed displays, and ways in which to describe, analyze, and summarize the data. In one of these cycles, questions and tasks were introduced that focused on the spread or distribution of data.

As an initial step in one of these cycles, PSTs explored data plots initially using the divider tools in TinkerPlots®. The use of the divider tools was not prescribed, but was a student-generated idea originating through exploration within TinkerPlots®. Divider tools allowed the PSTs to divide the data set with visually shaded regions, either by equal count or equal width. The equal count option set a stage for discussing a box plot. While discussing features of the box plot, PSTs were presented with two sets of data where the mean, median, and mode of both sets were identical, yet the data had very different distributions. Box plots of the two data sets nicely presented these different distributions by accentuating some clear visual differences. The idea of the median was shown to be a key feature in a box plot, with PSTs finding the median of a data set and then upper and lower "medians" for the upper and lower halves of the data. While discussing how the data related to the median in the box plot, PSTs were asked if perhaps another measure of spread other than the median could be used to provide "differences from a center". They chose the mean and so we began to investigate the idea of the mean absolute deviation.

This discussion set the stage for the heart of the task which followed. PSTs constructed a dot plot of the data set under investigation. Using the IWB pen and building upon an activity described by Lee, Hollebrands, and Wilson (2010), we drew horizontal lines on the projected display, representing the distance of each point from the mean. Then we discussed that a measure that could be used to provide an "average" of all of these distances from the mean was labeled the mean absolute deviation. In a different color, we partitioned the distribution into sections of this distance from the mean, noting that the "longer" mean absolute deviation represented a greater spread or variability in the data.

In the next session of class, we built upon this geometrical connection to measures of spread by taking each line drawn from the mean to the individual data points and drawing squares on all those lengths, thus showing the squares of the distances. The intent of drawing the squares was essentially to note their sizes and to see how a combined size of all of the squares compared to the overall spread of the data. From this geometrical base, PSTs used the list feature of their calculators to move to a more numerical approach, finding the "areas" of the squares, finding the mean value based on the sums of the squares, eventually arriving at the length of the side of the mean square and naming this length the standard deviation. We see this task as attending to all three aspects of subject matter knowledge from Table 1. It addresses a conceptual understanding of measures of spread, it provides a model for this understanding, and it sets students up to describe and interpret spread within data. These attend to CCK, SCK, and HCK respectively.

RESULTS

In the analysis of our collected data we found evidence that our students progressed in their SKT about ideas of variability because of various elements within this sequence of tasks. We focus on data provided by a quiz item that asked PSTs to provide an appropriate display for a given data set and then to choose an appropriate standard deviation measure from a given list. They were to justify this choice without calculating the standard deviation directly, relying on their conceptual understanding of standard deviation. PSTs all had access to laptops during the assessment and were responding to the question in the TinkerPlots® environment. Class results show seven of the 12 PSTs used either hat plots, dividers and/or lines/squares in their description of determining a standard deviation. While only slightly greater than half of the students, this kind of thinking had not been used by PSTs in previous semesters and demonstrates a component of SCK; the technology allowed us to present the ideas of standard deviation in a visual, geometric fashion and for the PSTs to access this thinking during the quiz.

During interviews conducted at the end of the course, PSTs were asked to reflect on the same quiz item. We discuss two particular contrasting examples involving Ann and Lisa (pseudonyms). When asked to reflect on her thinking about this assessment item Ann mentioned both the divider tool and the IWB activity as contributing to her understanding of variability and her ability to justify claims about it illustrating both her understanding of the concept (CCK) and her ability to interpret and connect (HCK). She said, "I used the divider tool to show the percentage of points that were within a certain range of the mean which helped me justify that there was a clump. But then it also showed me the percentages that were outside of that range which also helped me justify that there was some significant deviation from the mean." When asked what tasks or activities from the course helped her be able to understand and answer this type of question, Ann referred to the IWB task where the mean deviations were drawn out and physically squared by saying, "And I think at one point we turned those lines into squares to show squaring them and then we found the mean of those to find the variance. And for some reason just visualizing all the squares really helped me see exactly what it was that we were trying to find." When asked why it helped her to visualize, she expanded on her response by saying, "Um, I think just because the lines showed the actual deviation instead of just saying it deviated from the mean by seven units, it was a lot easier to see that this is a long line on this side and that these are short lines over here but together they showed the same number." This task of physically drawing the mean deviations and physically creating the squares provided Ann with a visual model that she referred to later and served as a way to connect ideas of deviation and mean. This along with the divider tools gave her language that she could then use to describe variability in a data set.

The thinking of another student, Lisa, serves as an interesting contrasting case that demonstrates the importance of tasks that engage students with statistical concepts. On the same assessment item, Lisa provided an incorrect response that began with a statement of what the procedure is for finding the standard deviation (SD) saying, "The SD is a measure of spread from the mean of a data set. If we were able to find the squares of each data value's deviation from the mean, then find the mean of those squared deviations (the variance), and take the square root of the variance, we could calculate the actual SD for this data set." Lisa had a correct understanding of the procedures necessary for computation (CCK) that categorizes her understanding of spread according to GAISE at level B, but the rest of her response demonstrated a lack of a conceptual understanding of SD, illustrating that she had not reached GAISE level C. To finish her response, Lisa created a hat plot and reasoned incorrectly that the boxes in the hat plot were illustrating the size of one standard deviation and stated that most of data values had a deviation of about 1.

Lisa serves as an interesting contrasting case to Ann because when asked about the most influential parts of the course on her thinking about this task Lisa identified the tables in the text that organized the procedures for computing SD. She said it was influential because, "Well, I actually got to carry it out. You started with the raw set of data and you squared each number and you added them up and you found, um, the variance of that and you do the square root of that and so then you were able to compare your results from that with your table to what they had already kind of walked you through and shown you how to do it." Lisa had a clear procedural understanding of SD but did not rely on the conceptual building blocks that the tasks were designed to create. Lisa acknowledged that Tinkerplots'® tasks and tools were beneficial to get students to visualize and think conceptually (SCK), but Lisa didn't rely on them herself. Lisa's own words could be used to summarize the necessity of tasks that provide students with tools to talk about variability and spread conceptually, "I knew what standard deviation was [procedurally], it was just hard to try to use that to justify what it was [conceptually]."

CONCLUSION

As noted earlier, we are in the beginning phases of our data analysis, but are encouraged by the findings presented above where PSTs are beginning to develop a more meaningful understanding of mean absolute and standard deviation. With the new content expectation of mean absolute deviation for 7th grade as suggested by the CCSSM (NGACBP & CCSSO, 2010), we believe that activities that focus on a more conceptual, geometric approach to this measure of spread based upon a data set from a particular context would be far more beneficial than those that would emphasize merely the computational aspect for finding a number, void of meaning for the student. This contextual understanding of these measures connects back to the SKT. It isn't sufficient for PSTs to merely be able to find measures of spread by selecting the appropriate keys on a graphing calculator; they need to have a sense of what the measures tell them about a data set as well as a means of conveying that sense in a conceptual manner. The TinkerPlots® software provided several means of thinking about spread with the hat plots and the divider tools. The IWB provided a means of making a geometric connection to measures of spread by drawing in lines and squares. We see this as all part of developing the specialized content knowledge of teaching statistics to young children.

REFERENCES

Ball, L. D., Thames, H. M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? [Research]. *Journal of Teacher Education, 59*, 389-407.

Burgess, T. (2009). Statistical knowledge for teaching: Exploring it in the classroom. *For the Learning of Mathematics, 29*(3), 18-21.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Creswell, J. W. (2007). *Qualitative inquiry and research design choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2005). *Guidelines for assessment and instruction in statistics education (GAISE) report: A Pre-K-12 curriculum framework.* Alexandria, VA: American Statistical Association.

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. *Journal for Research in Mathematics Education, 38*(5), 427-437. doi: 10.2307/30034960

Groth, R. E. (2012). The role of writing prompts in a statistical knowledge for teaching course. *Mathematics Teacher Educator, 1*(1), 23-40.

Konold, C., & Miller, C. D. (2012). *TinkerPlots: Dynamic data exploration* (Version 2.1). [Computer Software] Emeryville, CA: Key Curriculum Press.

Lee, H., Hollebrands, K. & Wilson, P.H. (2010). *Preparing to teach mathematics with technology*. Dubuque, IA: Kendall Hunt Publishing Company.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common Core State Standards Mathematics*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Sanchez, E., Silva, C., & Coutinho, C. (2011). Teachers' understanding of variation. In C. Batanero, G. Burrill, & C. Reading (Eds). *Teaching statistics in school mathematics – Challenges for teaching and teacher education* (pp. 211-221). New York: Springer.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research.* Newbury Park, CA: Sage Publications.

van Hiele, P. M. (1959). *Development and learning process: A study of some aspects of Piaget's psychology in relation with the didactics of mathematics*. Utrecht: University of Utrecht.

Watson, J. M. (2008). Exploring beginning inference with novice grade 7 students. *Statistics Education Research Journal*, *7*(2), 59-82.