HOW POSITIVE IS "POSITIVE"? TEACHERS' ORIENTATIONS REGARDING TEACHING STATISTICS COMPARED TO TEACHING FRACTIONS

Sarah Scheuerer[1], Frank Reinhold[2], and Kristina Reiss[1]
[1]Technical University of Munich, Germany
[2]Freiburg University of Education, Germany
sarah.scheuerer@tum.de

For teaching, not only teachers' knowledge but also their motivational and emotional orientations are important: they influence teaching practices and student achievement. In statistics education, previous research suggests that teachers have positive orientations, but the exact meaning of 'positive' is unclear. A comparison with other mathematical topics might provide clarity here. We therefore asked teachers about their orientations regarding teaching statistics and compared them with fractions. Additionally, we performed cluster analyses to identify teachers' profiles. The results showed significantly poorer orientations regarding statistics compared to fractions. Cluster analyses revealed three similar profiles in both topics, but the assignment of teachers to the clusters was topic-dependent. Because orientations are assumed acquirable, our findings demonstrate the need to strengthen teachers' orientations regarding statistics during teacher education.

INTRODUCTION

In order to fulfill the personal requirements for successfully coping with professional demands, teachers need professional competence, which includes not only cognitive aspects (i.e., knowledge) but also affective aspects (Baumert & Kunter, 2013). Affective aspects refer to "a wide range of beliefs, feelings, and moods" (McLeod, 1992, p. 576) and therefore also include teachers' motivational and emotional orientations (Hannula, 2011), which influence teaching practices (Kunter et al., 2008; Reinhold et al., 2021) and, relatedly, impact student achievement (Ulug et al., 2011). In the field of statistics, however, there is little research on teachers' orientations, and these studies have often investigated prospective teachers rather than in-service teachers, which is unfortunate because they are not able to provide information about teachers' orientations in relation to their teaching experience (Eichler & Zapata-Cardona, 2016).

Prospective teachers have generally reported *positive* (i.e., above the scale mean) orientations regarding statistics (Estrada & Batanero, 2008; Hannigan et al., 2013; Nasser, 2004; Zientek et al., 2011). These studies, however, did not relate their results to other mathematical subfields, although teachers' orientations seem to differ with respect to different content areas within a subject: in a survey regarding prospective teachers' self-efficacy when teaching various mathematics subfields, more than half of the participants stated that they were least self-effective in teaching statistics (Lovett & Lee, 2017). So how positive are teachers' 'positive' orientations regarding statistics really? To clarify the exact meaning of 'positive,' a systematic comparison with teachers' orientations regarding other content areas taught at a similar grade level seems essential.

In addition to this consideration, a further question is whether teachers' individual orientations are consistent across different mathematical content areas, e.g.: Are teachers with high anxiety and low self-efficacy regarding statistics equally anxious and low self-effective regarding other content areas, such as fractions? One approach to answer such questions is to determine teachers' orientation profiles for different content areas using cluster analyses, which build differentiable groups of teachers that share similar orientations. These orientation profiles allow a detailed look at the individual facets of teachers' motivational and emotional orientations and thus reveal similarities and differences across content areas. Moreover, considering teachers' orientation profiles regarding statistics seems relevant, as these findings could be used, for example, to predict the effectiveness of teacher training in statistics (for similar analyses see Reinhold et al., 2021).

Overall, for a more precise understanding and classification of teachers' orientations regarding statistics, further investigations and comparisons to mathematical content areas are required. For example, such findings may help teacher educators to address certain content areas during mathematics teacher training to a greater or lesser extent. For this reason, in the present study, we examine in-service teachers' motivational and emotional orientations regarding teaching statistics as well as fractions. The content area of fractions is appropriate for comparison because both statistics (e.g., frequencies) and

fractions (e.g., expanding/reducing) are taught in early secondary education, and can therefore be assumed to have a comparable level of difficulty. We address the following research questions: (a) *Do in-service teachers' motivational and emotional orientations regarding teaching statistics differ from those regarding teaching fractions?* Based on the results of Lovett and Lee (2017), our hypothesis was that there are differences in these orientations to the disadvantage of the content area statistics. In addition, unlike statistics, the content area of fractions has long been part of mathematics curricula and may therefore be more familiar to teachers. () *Which groups of teachers with similar motivational and emotional orientations regarding teaching statistics exist, and are they comparable to the groups that have similar orientations regarding teaching fractions?* To answer this question, we chose a person-oriented exploratory statistical approach with cluster analyses on teachers' self-reported orientations.

METHOD
Sample and Procedure
Data collection occurred during a four-hour professional teacher development program in Medellín, Colombia, taught by the first author of this paper. A total of $N = 88$ Colombian in-service mathematics teachers (34 female, 53 male, 1 did not specify) participated in the training. The sample consisted of teachers from private and public schools, in urban as well as rural areas. Prior to the training, teachers took part in the paper-based cross-sectional study voluntarily, without reimbursement, and with their own consent.

Instruments
We used the *BeSt Teacher* framework, a validated instrument developed to assess teachers' professional competence in descriptive statistics (Schumacher, 2017). We modified the original scales of motivational and emotional orientations to assess orientations regarding statistics (*s*) more generally, and to assess orientations regarding fractions (*f*). In addition, the original German items were translated into Spanish, the native language in Colombia. Teachers' motivational orientations were assessed in terms of their *self-efficacy* regarding content in statistics and fractions at the level of early secondary education. The scales were introduced by the stimulus "How confident do you feel in *statistics / fractions*? Please estimate in advance how confident you are in being able to solve tasks on the following topics." Each scale consisted of seven items (e.g., *statistics*: "tasks concerning absolute and relative frequencies"; *fractions*: "tasks regarding expanding and reducing fractions"). Items had to be rated on 4-point Likert-scales (1 = 'unconfident' to 4 = 'confident'). The reliabilities of the scales, measured with Cronbach's α, were high: $α_s = 0.91$ and $α_f = 0.93$. Teachers' emotional orientations were assessed in terms of the *joy* and *anxiety* they feel regarding teaching statistics and fractions. The scales were introduced by the stimulus "How do you feel about teaching *statistics / fractions*? Please indicate how much you agree with the following statements." Each scale consisted of four items (e.g., *joy*: "In general, I enjoy teaching *statistics / fractions*."; *anxiety*: "When teaching *statistics / fractions*, I am tense and nervous in general."). Items had to be rated on 4-point Likert-scales (1 = 'I totally disagree' to 4 = 'I totally agree'). The reliabilities were again high: $α_s = 0.94$ and $α_f = 0.95$ for *joy*; $α_s = 0.78$, $α_f = 0.80$ for *anxiety*.

Data and Statistical Analysis
All analyses were conducted in *R*. For answering the first research question, we compared teachers' motivational and emotional orientations regarding teaching statistics and fractions. For this purpose, we performed paired sample *t*-tests with participants' individual mean scores for the Likert-scales of the three scales *self-efficacy*, *joy*, and *anxiety*, and calculated corresponding effect sizes. For answering the second research question, we partitioned teachers into groups that share comparable motivational and emotional orientations regarding statistics, on the one hand, and regarding fractions, on the other. For this purpose, we conducted two hierarchical cluster analyses (one for each content area) with the *Euclidean* distance measure and the *Ward* clustering method on the standardized mean scores for the Likert-scales of the three scales under study to find the best cluster solutions in terms of (a) the number of clusters and (b) the assignment of teachers to clusters. The best cluster solution for each content area was determined using the *NbClust* package (Charrad et al., 2014), which evaluates the validity of cluster solutions using 30 validation indices.

RESULTS

We compared teachers' motivational and emotional orientations regarding teaching statistics and fractions and expected differences to the disadvantage of statistics. Paired sample t-tests showed that teachers reported, in fact, to be significantly less self-effective ($M_s = 2.97$, $M_f = 3.44$), $t(85) = 7.57$, $p < .001$; less joyful ($M_s = 3.18$, $M_f = 3.37$), $t(84) = 2.64$, $p = .01$; and more anxious ($M_s = 2.11$, $M_f = 1.71$), $t(84) = -4.97$, $p < .001$, regarding teaching statistics compared to teaching fractions. The effect sizes were of small (*joy*: Cohen's $d = 0.29$), medium (*anxiety*: $d = -0.54$), and large magnitude (*self-efficacy*: $d = 0.82$), according to Cohen's classification.

In addition, we used exploratory cluster analyses to identify differentiable groups of teachers that share comparable motivational and emotional orientations regarding statistics, on the one hand, and fractions, on the other, and then compared these group profiles. The analyses revealed three similar group profiles of similar size in both content areas: delighted teachers, positively oriented teachers, and moderately oriented teachers (see Figure 1).
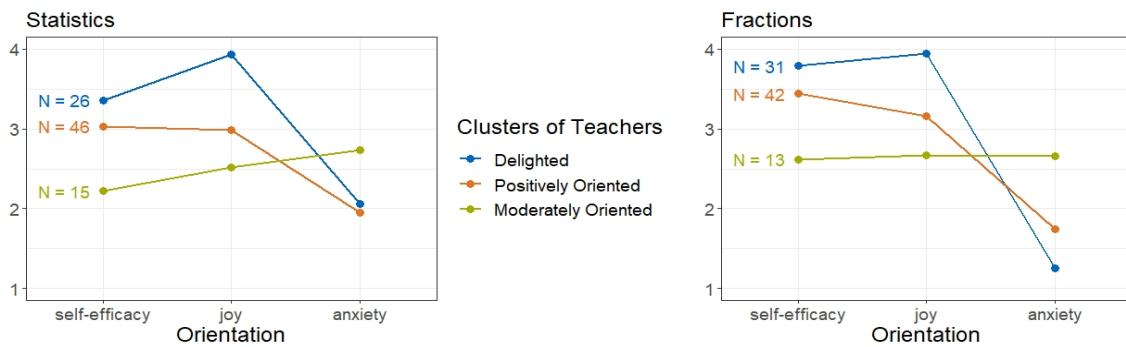

Figure 1. Group profiles and cluster means for the three teacher orientations found

However, despite similar group profiles, the assignment of teachers to the clusters was not the same for both content areas: about half of the teachers were assigned to different clusters, with all nine possible combinations of cluster assignments represented (see Figure 2). Thus, the group profiles found for the content area statistics are not consistent for the content area fractions.
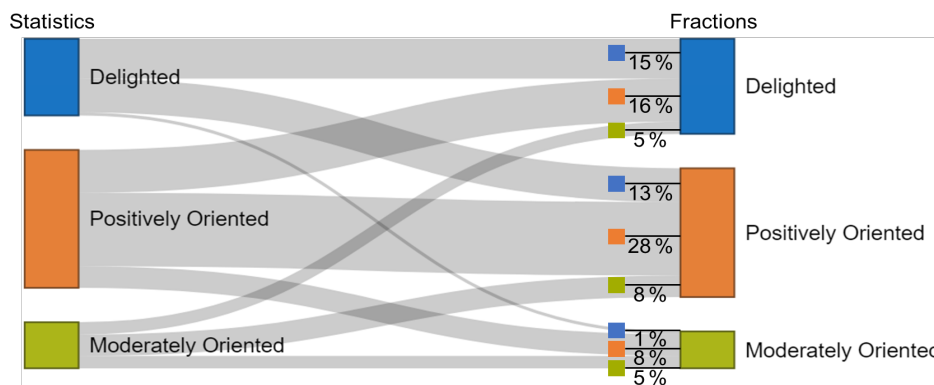

Figure 2. Sankey diagram showing the assignments of teachers to group profiles for the content areas of *statistics* (left side) and *fractions* (right side)

DISCUSSION

Our study provides valuable insights into the previously little studied motivational and emotional orientations of *in-service* teachers regarding their teaching of statistics. The results showed that teachers' orientations (a) can substantially differ across mathematical content areas and (b) were poorer for the content area of statistics compared to fractions. These findings underscore the need for further research to investigate the reasons for these poorer orientations in statistics and to find ways to improve the situation. For example, Lovett and Lee (2017) found that the lack of (pedagogical) content knowledge in statistics is responsible for prospective teachers' lower self-efficacy regarding teaching

statistics. Because orientations are assumed acquirable, our findings might suggest that the content area of statistics is not given enough emphasis in teacher education. However, in order to clarify that the comparison with fractions is not an exception, further studies should compare orientations regarding teaching statistics with orientations regarding teaching other mathematical content areas, or even regarding teaching mathematics as a whole. This could add value to the ongoing discussion of statistics as a separate discipline from mathematics.

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