THE INTERSECTION OF STATISTICAL LITERACY, REASONING, AND THINKING

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Statistical literacy, reasoning, and thinking are three cognitive processes recognized as learning objectives for teaching statistics. Some researchers acknowledge that these processes share certain elements; however, it is not clear just what those shared elements are. In this article, we present the results of a study that allowed us to identify indicators associated with four big statistical ideas at the intersection of the three processes: data, representation, variability, and sampling. According to the literature, each idea is treated differently depending on the process. These four big ideas can be reference points around which we can move from one process to another, depending on how they are handled in teaching. These findings contribute to our understanding of the similarities and differences among concepts fundamental to achieving these three learning objectives.

INTRODUCTION

The emergence of statistical literacy (SL), statistical reasoning (SR), and statistical thinking (ST) began with the firm intention to reorient statistics education, despite a lack of clarity in their definitions and in their implications for teaching and assessment. Hence, different researchers have contributed to research on the development of the processes and their implications. In the research literature we find definitions to distinguish among these processes (e.g., Ben-Zvi & Garfield, 2004; delMas, 2002) as well as specific characteristics or components that allow us to understand a particular process in greater detail (e.g., Gal, 2002; Garfield, 2002; Wild & Pfannkuch, 1999). There are also studies related to the design and implementation of instruments to assess achievement of representative elements of these processes (e.g., Callingham & Watson, 2005; Garfield et al., 2003; Gómez-Blancarte et al., 2021; Sabbag et al., 2018).

Despite the wide range of research on SL, SR, and ST, there is still a lack of consensus on the elements that distinguish them or the elements that they share. There are two theoretical representations of these three processes (Figure 1(a), Figure 1(b)) and another that is empirical (Figure 1(c)). The first representation shows three intersecting sets (Figure 1(a)) that inform us about elements that overlap among the three processes and distinctive elements. The second (Figure 1(b)) shows one set with two subsets. In this case, SR and ST are subsets of SL and the former two share certain elements (see delMas, 2002; Garfield & Ben-Zvi, 2008). Empirically, a unidimensional representation is shown in Figure 1(c). In this case, during the teaching of elements of SL, SR, and ST, teachers do not perceive distinctions among the processes (see Gómez-Blancarte et al., 2021). This last representation may exemplify delMas' (2002) comment that "from an instructional perspective, the overlap suggests that a single instructional activity can have the potential to develop more than one of these outcomes" (p. 3).

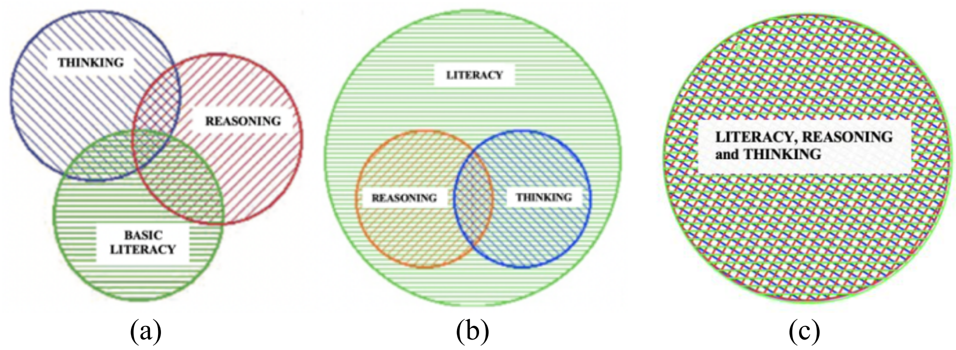

Figure 1. Representations of the literacy, reasoning, and statistical thinking processes

In any of the three representations, we can see that these three processes have more overlapping similarities than differences. But what are those overlapping similarities? To try to answer this question, in this article we present a study that informs us about some of the statistical ideas that are important for developing SL, SR, and ST.

METHOD

Our review of literature on SL, SR, and ST for earlier studies (Chávez Aguilar, 2020; Gómez-Blancarte et al., 2021; Gómez-Blancarte et al., 2022) allowed us to identify topics important for all three processes. We found that these shared topics broadly include ideas related to *data*, *representation*, *variability*, and *sampling*. For this paper, both the literature already consulted for those studies and new publications were reviewed in a search for additional information on how these four ideas play a role in the three processes.

Our understanding of SL, SR, and ST was based mainly on traditional references, such as Gal (2002), Ben-Zvi and Garfield (2004), and Wild and Pfannkuch (1999). To expand the information available, we conducted a new search trying, as far as possible, to locate the most recent references. Because this review focused on looking for references to support the proposal that the four ideas (*data*, *representation*, *variability*, *sampling*) form part of the foundation for these processes, the selection of new literature was based on two criteria: (a) articles that focus on, or make reference to, one of the processes and (b) articles that mention at least one of the four statistical ideas (*data, representation, variability*, *sampling*) as a key element in its description of the process. The resources consulted were obtained through the Google search engine and specialized search engines (e.g., Eri PLUS, IOP Science). The search included keywords, including statistical education, statistical literacy, statistical reasoning, statistical thinking, data, representation, variability, and sampling). Of the papers found, preference was given to those written in the last five years, from 2017 to 2022.

Despite space limitations, we try to mention at least two references for each idea: one traditional and the other more recent. When this is not possible, we mention the ones we found, regardless of whether they are traditional or recent. The following section presents the results of this review. Although not exhaustive, the review is certainly relevant and sufficiently representative to allow us to discuss the sense in which the ideas of *data, representation, variability*, and *sampling* are important for the three processes.

FINDINGS

Statistical Literacy

For SL, we include four references (Gal, 2002; Gehrke et al., 2021; Gould, 2017; Pereira et al., 2020) that mention these four ideas as elements involved in SL. We found that Gould (2017) relied on the notions in Gal (2002) to propose an updated definition of ST, one that considers data science and so includes the term "data literacy." Gehrke et al. (2021) used this term later and identified the competencies needed for a "science of statistical literacy." Pereira et al. (2020) referred to SL following Gal and proposed the idea of variability as an essential concept for its development.

- Data: Developing SL involves developing competence in various aspects of dealing with data, including recognizing the need for data (especially empirical) and considering the origin of the data, how they were produced, how they are analyzed, how they relate to the conclusions reached, whether the instruments used to generate the data (tests, questionnaires, interviews) are reliable or accurate, and whether the information reported is appropriate for, and supported by, the data (Gal, 2002; Gehrke et al., 2021; Gould, 2017).
- Representation: Tasks for developing SL include tasks for developing knowledge about how to create descriptive representations of data (lists, tables, indexes, schedules, charts, graphical displays) and how to identify, interpret, and use the information given in those representations (numbers or percentages) (Gal, 2002; Gould, 2017). In addition, SL requires "understanding how representations in computers can vary and why data must sometimes be altered before analysis" (Gould, 2017, p. 22).
- Variability: Important for developing SL is the basic understanding of notions of probability in relation to variability because people are expected to understand "at least intuitively, the idea of a chance variability in (random) phenomena" (Gal, 2002, p.10). Variability is seen as an essential concept for the development of SL since it "is part of the quotidian and noticing it and analyzing it statistically makes one's perception of the world more critical, helping his formation development and his social participation" (Pereira et al., 2020, p. 2).

- Sampling: The development of SL requires understanding sampling as an instrument for obtaining information about a population. People must understand, at least intuitively, the logic of sampling when interacting with surveys and polls and must also be aware of the advantages of random sampling and the disadvantages of convenience sampling. Finally, it is imperative to know how to analyze and interpret data appropriately based on their origin (Gal, 2002; Gould, 2017).

Statistical Reasoning

The five references (Ben-Zvi & Garfield, 2004; delMas, 2004; Garfield & Ben-Zvi, 2008; Garfield & Chance, 2000; Jones et al., 2004) used for SR consider the ideas of *data*, *representation*, *variability*, and *sampling* as key concepts for statistical reasoning. Jones et al. (2004), for instance, state that it is possible to observe recurring patterns or levels of statistical reasoning when students deal with concepts such as sampling, variation, and organizing and representing data, among others.

- Data: SR involves interpreting statistical results based on a set of data, representations of data, or statistical summaries of data, in order to make sense of statistical information by combining the ideas of data and chance (Ben-Zvi & Garfield, 2004; Garfield & Chance, 2000). These ideas related to data arise when asking if a study is observational or experimental, if the conclusions are appropriate for the type of study, and if the methods adopted are adequate to avoid biased data (Garfield & Ben-Zvi, 2008).
- Representation: As mentioned, SR "involves making interpretations based on sets of data, representations of data, or statistical summaries of data" (Ben-Zvi & Garfield, 2004, p. 7), but also an understanding of real data in a context, using distinct representations to explore trends and patterns (delMas, 2004).
- Variability: To reason statistically, one must recognize and understand that data vary, an element of the idea of variability (Garfield & Ben-Zvi, 2008). This entails using cognitive models that incorporate decision-making, prediction, and inference, and taking the existence of variation into account when exploring data (Jones et al., 2004).
- Sampling: According to delMas (2004), promoting students' SR requires that students gain "familiarity and understanding with concepts that are difficult to experience in everyday life (e.g., the sampling distribution of a statistic)" (p. 92). Experiences can range from questioning how the data were produced, how the sample was obtained, and how different samples vary, to more sophisticated ideas such as sampling distributions and the central limit theorem (Garfield & Ben-Zvi, 2008; Jones et al., 2004).

Statistical Thinking

Six references (Ben-Zvi & Garfield, 2004; Garfield et al., 2015; Garfield & Franklin, 2011; Masjudin et al., 2020; Pfannkuch & Wild, 2004; Wild & Pfannkuch, 1999) report on ST in relation to these four key ideas. The research by Masjudin et al. (2020) offers a narrower view of ST compared to the one proposed by Wild and Pfannkuch (1999) because it addresses only the processes of describing, organizing, representing, analyzing, and interpreting data.

- Data: According to Masjudin et al. (2020), "a person's ability to understand data is called statistical thinking ability" (p.1). In ST, the idea of data refers to one of the types that are fundamental to statistical thinking. Understanding data involves recognizing that the collection and analysis of appropriate data are necessary to make reliable judgments about real situations (Pfannkuch & Wild, 2004).
- Representation: When students can represent data in tables, diagrams, or graphs and explain the process carried out to elaborate those representations, they show high ST ability (Masjudin et al., 2020) because ST implies a process of *transnumeration*, which occurs when "the data that have been collected are transformed from raw data into multiple graphical representations, statistical summaries, and so forth, in a search to obtain meaning from the data" (Pfannkuch & Wild, 2004, p. 18). In this sense, "[s]tudents who were handling multiple representations in a meaningful and creative way, and were using graphs to search for patterns and to convey ideas—coupled with a critical attitude—were considered to be thinking statistically" (Pfannkuch & Wild, 2004, p. 34).
- Variability: The idea of variability is fundamental for ST and specifically, the consideration of variation (Wild & Pfannkuch, 1999). It deals with three principal conceptions: "variation is

omnipresent; variation can have serious practical consequences; and statistics give us a means of understanding a variation-beset world" (Wild & Pfannkuch, 1999, p. 235). Garfield and Franklin (2011) point out that a learning outcome of ST demands recognizing the importance of examining, and attempting to explain, variability.

- Sampling: ST implies an "understanding of the nature of sampling, how we make inferences from samples to populations" (Ben-Zvi & Garfield, 2004, p.7). "[R]easoning about samples and sampling variability appears to provide a pathway for developing expert statistical thinking regarding statistical inference" (Garfield et al., 2015, p. 329).

CONCLUSIONS AND IMPLICATIONS

We have shown how the statistical ideas of *data*, *representation*, *variability*, and *sampling* are important for the cognitive processes of SL, SR, and ST. According to the literature reviewed, each concept is addressed with certain similarities among these three processes, as shown below.

- The idea of data in SL is associated mainly with understanding the provenance and production of, and the need for, data. In SR, this involves analyzing the data to make interpretations, propose and test conjectures, and make decisions, as well as to identify the collection methods. ST encompasses recognizing that real data are necessary for answering statistical questions, judging situations, and solving problems. Also, ST demands adequate collection and understanding of data.
- Representations can be graphical or of other kinds (e.g., symbolic, numeric), and the process of transnumeration is important for working with representations. In SL, the idea of representation refers to identifying, interpreting, and using information presented mainly in tables, summaries, and graphs. In SR, exploratory data analysis is used to plot data, find patterns, and interpret them in context. Data representation is related with the *transnumeration* process for ST, and transnumeration is one of the types of thinking fundamental to statistical thinking.
- In SL, an intuitive understanding of variability is used to recognize that events vary and are unpredictable. SR involves understanding the idea of variability that can be explained by the sampling technique in which data might vary, to some extent, in a predictable way, and by examining how data are spread out on a graph. ST involves recognizing variability as one of the fundamental types of thinking and is considered to be an omnipresent entity. ST also includes considering variability in the sampling design and seeking to explain the variability within the context of the statistical research being carried out.
- The idea of sampling in SL is based on the need to answer the questions of why data are needed and how they are produced, so there must be at least an intuitive understanding of the sampling process, of the advantages of taking a random sample, and of the disadvantages of convenience sampling. In SR, the concepts of sample size, random process, and the relationship between sample and population are important. An understanding of the nature of sampling, how inferences are made from a sample to the population, and how causal relationships are established from the sample to the population are all part of ST.

However, from a theoretical point of view, it seems that what makes the difference between one process and another is the vision of learning outcomes in each one. Chance (2002) states that SL "can be narrowly viewed as understanding and interpreting statistical information presented, for example in the media," and SR "as working through the tools and concepts learned in the course" (p. 4). ST requires that "the statistical thinker is able to move beyond what is taught in the course, to spontaneously question and investigate the issues and data involved in a specific context" (Chance, 2002, p. 4). When teaching statistics, it is important to distinguish among the different types of learning outcomes so that "educators reflect on the content and instructional methods they are using" (Garfield & Franklin, 2011, p. 135).

This study specifies three implications for statistics education:

- First, while SL, SR, and ST may be defined distinctly, teaching students to develop SL, SR, and ST and evaluating students' engagement in the processes overlap (see Chance, 2002; Gómez-Blancarte et al., 2021). The present study shows how this overlap may occur because the ideas of statistics are intertwined among the three processes analyzed. Moreover, relationships can be observed among the ideas themselves.
- Second, realizing that the concepts of *data*, *representation*, *variability*, and *sampling* are inherent in all three processes, and that they are considered fundamental for developing an understanding of

statistics (see Burrill & Biehler, 2011), should stimulate researchers and educators to pay greater attention to the teaching of these concepts as cross-cutting ideas around which the teaching of statistics can be organized.

- Third, considering the overlap among the three processes and the fact that in practice, teachers do not seem to distinguish among them, we believe that more ethnographic research is needed to explore how curriculum materials and the teaching practices of teachers are organized around these processes in different levels of the educational system.

In addition to the SL, SR, and ST processes, new ones are emerging (e.g., data literacy, data science, civic statistics). Therefore, we must contemplate the implications of the existence of this variety of processes for the teaching of statistics, and how to best convey them in the classroom context.

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