COMPARATIVE ANALYSIS OF THE PROBABILITY MEANING IN THE CURRICULUM PROVIDED AND OFFICIAL OF A STATISTIC CLASS

Beatriz Adriana Rodríguez González[1], Judith Alejandra Hernández Sánchez[2], Gabriela Noemí Figueroa Ibarra[1], Eduardo Briceño Solís[2], and DarlyKú Euán[2]
[1]Polytechnic University of Zacatecas, México
[2]Autonomous University of Zacatecas, México
brodriguez@upz.edu.mx

This study compares the meanings of probability found in the enacted curriculum of a professor's class and the meanings found in the official curriculum for a statistics course in a business degree program at the Polytechnic University of Zacatecas. The method used is the first organizer of didactic analysis, content analysis. The interest of this paper lies in the importance that didactics of statistics currently have in the field of research, and in particular, the concept of probability, in which the main difficulties in solving problems related to the topic have been highlighted.

INTRODUCTION

Research in probability and statistics education is visible in the numerous papers presented at conferences such as the Congress of the European Society for Research in Mathematics Education (CERME) or the International Conference on Teaching Statistics (ICOTS) (see, e.g., Batanero et al., 2016 for numerous citations to this work). Ruiz et al., (2021, p. 69) mention that the scientific community needs to strengthen research and improve teaching activity for the teaching and learning of this particular topic.

According to Batanero and colleagues (2016), the multifaceted nature of probability is not found in other areas and creates special challenges for teachers and students in teaching probability. This multifaceted vision of probability is evidenced through its referents (intuitive, Laplacian, frequentist, subjective, and mathematical as proposed in Batanero, 2005) that can make sense and acquire functionality through a diversity of contexts, situations, and phenomena. Rico (2012) proposes that conceptual referents, phenomenology, and registers of representation are the three components that provide evidence of the multiplicity of meanings associated with a mathematical concept. We are interested in those components in relation to the concept of probability.

Meanings associated with a mathematical school concept can be transformed (omitted, modified, enriched, enhanced) depending upon the contexts and situations in which they are encountered. Such contexts and situations are proposed in a written or official curriculum and should be learned by students. The model of Eichler and Erens (2014) provides evidence of the processes that usually transform the curriculum and hence the meanings when the curriculum is enacted. Some studies such as Dolores and García (2012) present comparative studies of curriculum where different meanings of mathematical content have been reported from these transformations. According to Hernández et al. (2020), these types of studies ratify the weak connection between the different types of curricula. It is in these differences, and the transformation of a curriculum model, where the problem of the present research is located. In the present work, we identify the meanings associated with the definition of probability.

This research presents relationships and differences between the meanings of probability identified in the official curriculum that is taught in a statistics course that is part of an international business class degree program. To achieve the objective, we apply conceptual analysis to identify different meanings of probability using the three components of Rico (2012). The information obtained from the study is useful for comparing the taught (or enacted) and official curriculum in a statistics class. The first phase to identify meanings is the conceptual analysis, and the meanings are organized in a second phase. The conceptual analysis entails identifying the meanings of intuitive probability from five historical documents that reference games of chance. The second phase allows the meanings of intuitive probability in the business program to be organized and compared with enacted curriculum from observations of a professor´s class.

THEORETICAL FRAMEWORK

The notion of the meaning of a mathematical school concept is proposed by Rico (2012), who considers three components as central to establishing meanings that can be studied: conceptual structure, representations systems, and phenomenology.

Rico (2013) mentions that content analysis techniques can be used to discover the internal structure of communication. This analysis incorporates a reductive dimension by breaking down the content into simpler units that can be used to determine themes and identify categories. The application of content analysis to educational research helps to discover speech patterns, contrast hypotheses, and infer interpretive meanings from a text (Rico, 2013). In general, the procedure to conduct a content analysis follows certain stages as follows (Rico, 2013, p. 17).

- Determine the corpus of content (text, speech, written production) to analyze.
- Specify the unit of analysis: word (noun, verb, or adjective), sentence, or paragraph.
- Locate or infer units of analysis in the text.
- Name, define, and interpret the categories considered, avoiding the category of "others" as possible to avoid uncertainties.
- Encode and quantify the analysis units using mean frequencies or ranges. The analysis units can be assigned by a predetermined category system or inferred through such a category system.
- Find relationships among codings to establish and interpret categories, considering their assigned units of analysis.
- Relate the content analysis process with the question being investigated and with the intervening agents (speaker/written or listener/reader).

METHODOLOGY

This research is part of a qualitative study of a descriptive nature, and the method adopted is content analysis (Rico, 2013). The phases used for this research are the ones proposed by Rico and Fernández-Cano (2013). They are presented below in the context of this study.

1) Sources of information used for the content analysis phase of this study are listed in Table 1. The numbers that accompany the author names were used in coding the units of analysis that are presented in step 3. Even though the content analysis does not specify the number of documents or analysis fragments to be used in analysis, the authors decided to take a primary source (Hacking, 1975), two papers, a book chapter, and a book, principally, for the content and historical coincidences that were delimited there.

Table 1. Sources of information for historical development

Authors	Year	Title	Document
01 Luis F. Restrep B. y Julián González L.	2003	*La historia de la probabilidad*	Paper
02 Carmen Batanero	2005	*Significados de la probabilidad en la educación secundaria*	Paper
03 Ian Hacking	1975	*The emergence of probability*	Book
04 Gregoria Mateos y Aparicio Morales	2002	*Historia de la probabilidad (desde sus orígenes hasta Laplace) y su relación con la Historia de la Teoría de Decisión*	Chapter
05 Leticia Mayer Celis	2015	*Rutas de incertidumbre. Ideas alternativas sobre la genesis de la probabilidad, siglos XVI y XVII*	Book

2) Name, define, and decode the categories considered. The categories for analyzing meanings in the historical development of probability are those proposed by Rico (2013, p. 18): conceptual structure, representations systems, and phenomenology. The conceptual structure category considers the historical time and the population (who communicates and to whom they

communicate) of the time. This category also includes formal and structural elements of the content, including concepts, definitions, and procedures, along with formal structure, which provides reference to the content used. The representational category includes the graphic and symbolic notations and sign systems involved. The phenomenological category addresses the phenomena that give rise to the concepts, the contexts in which they are used, and those situations in which they occur and in which they are applied that give meaning to the content under study. In addition, a cognitive classification of the contents will be addressed, considering the conceptual field that refers to the substance of knowledge: what is it composed of? Also addressed is the procedural field that includes the procedures and modes of action with respect to knowledge (Cañadas et al., 2018).

3) Code the units of analysis. An identification code is used. For example, for the code of SI_01_83_01, SI corresponds to the acronyms of intuitive meaning, 01 corresponds to the author referred to in Table 1, 83 is the page number where the citation is found in the text, and 01 is the number of the paragraph.
4) Relate the codes to each other and interpret the established categories, considering their units of analysis. To relate to each other, the categories are the three components of meaning that have been mentioned above.
5) Finally, according to Makón (2004), through comparison, what is sought is to identify explanations for the occurrence or non-occurrence of a phenomenon, considering whether these occurrences are different or similar. This situation allows us to compare the meaning of intuitive probability found in the business course study plan (the official curriculum) and in the speech of a teacher teaching the course (the enacted curriculum). Comparative analysis (Makón, 2004) of the official and enacted curricula will be used to compare: (a) components of the meaning of a mathematical concept (Rico, 2012); (b) applicability, which is the part of the curriculum that corresponds to intuitive probability and the taught curriculum; and (c) the link with empirical evidence. The corpus of data to be analyzed is the syllabus and observations of one teacher's class.

COMPARATIVE ANALYSIS

Historically, the development of intuitive probability did not reach the level of structure of meanings in step 1 of analysis. Within a contemporary typology of probabilities, intuitive probability precedes other types. Terms included in texts for a game of one dice and a game with two dice arise from the ideas of Pascal due to his constant companion, the knight Meré. Meré was famous for his fondness of games of chance. Meré claims that the behavior of the dice game is different when using one die or two dice. Pascal doubts Meré’s claim and tries to prove his disagreement along with the French jurist Pierre Fermat. The two find that if results are equiprobable, then there is a proportional relationship between moves. Thus, the concept of equiprobability arises as part of step 2 of analysis. Historically, this is when the theory of probability finally begins to be formalized.

In correspondence between Fermat and Pascal, we identified numeral terms, letters, and fractions as well as the introduction of new concepts: mathematical expectation, possibilities, combinatorial, fair game, frequency, and sample space. These concepts set the tone for the emergence of intuitive probability; however, with the background described, the concept of probability has not been formally defined.

In terms of representation systems, use of ordinary language is identified in text. In non-mathematical contexts, words such as chance or combination are used on a daily basis and are identified in the correspondence between Fermat and Pascal. The latter tried to solve the problems posed by the knight Meré and supported by arithmetic.

Few situations are found in a mathematical context. Undoubtedly, the situation that prevails is a non-mathematical context, namely games of chance such as dice or card games. A player's experience is taken into account.

An analysis of unit 3 from the official business statistics curriculum appears in Table 2, which exemplifies the information recorded for the official curriculum. The analysis shows that three of the five meanings of probability have been observed: Laplacian, frequentist, and subjective. The analysis also reveals the meanings of probability included in the official curriculum for a statistics course in the bachelor’s degree in the International Business Program at the Polytechnic University of Zacatecas (UPZ). According to Batanero (2005), the frequentist meaning has a wide field of applications, which may enrich the phenomenology of the concept of probability and thereby give meaning to this content in the context of student training.

Table 2. Registration form and example of filling in information about the program

Code: LNI_PyE_U2_p8		Analysis unit • Explain the methods for calculating probability • Probability approximation using relative frequencies		
Reference meaning: *Frequency*				
Concept structure		Representation systems	Phenomenology	
Concept field concepts: *Relative probability frequencies* Outcome: *Methods for calculating probabilities*	Procedural field reasoning: *Approximate probabilities* Abilities: *Explain methods for its calculation*	*Not specified*	Mathematical context *Not specified*	Non-mathematical context *Not specified*

Finally, in the class taught by the professor, the enacted curriculum, some fragments of the discourse are analyzed, in which intuitive probability intervenes when concepts such as experiment, event, and result are explained. The professor dedicates two sessions (both analyzed) to explain the concept of probability and goes back to games of chance (before starting with frequentist and classical probability), although the professor introduces examples in a non-mathematical context related to the business career that students chose. The class is taught on the Zoom platform and the images and speech analysis are obtained from recordings made from two complete sessions. Table 3 displays the record sheet for one of these sessions.

Table 3. Class observation record sheet

Code				
Image  		Speech ...So I throw a dice, that´s my experiment, if I throw it twice as I has already explained, I have two experiments, and suitable dice are the possible outcomes, if I throw a dice only once, they are one, two, three, four, five and six, can anything else happen? No, that list you see there is called a collectively exhaustive list, what are the possible events?		
Structure conceptual		**Representations systems**	**Phenomenology**	
Concept field:	**Procedural field:**	**Representation**	**Math contexts**	**Non mathematical context:**
Level 1 *Terms* Dice	Level 1 *Abilities* Roll one die at a time Throw a dice twice		Situation	Situation Dice roll
Level 2 *Concepts* Experiment List Outcome	Level 2 *Reasoning* *Get rolling a die results*		Make a list with the results obtained	
Level 3 *Conceptual structures*	Level 3 *Strategies*			

Content in the two observed classes (the enacted curriculum) and in the unit 3 official curriculum are compared (see Table 4). There is almost no relationship in representation systems between the official and enacted curriculum. No representation types are proposed in the official curriculum. The professor focuses on a verbal, pictorial, and tabular representations in the enacted curriculum, under the belief that different representations help students to better understand the concept of probability.

Table 4. Comparative table on representations systems between curriculum types

Systems/Representations	Verbal	Numeric	Pictorial	Tabular	Graphic
Business International curriculum					
Professor´s class observation	X		X	X	

Finally, the phenomenology in terms of situations and contexts was observed for the official curriculum. Table 5 shows a coincidence in an educational context, although the professor takes a second approach by using a professional context to share example of executives in a business class, as displayed in Table 3. The official curriculum does not specify the type of mathematical context, and the professor uses a non-mathematical context in the enacted curriculum.

Table 5. Comparative table about phenomenology between curriculum types

Systems/Representations	Personal	Educational	Professional context	Mathematical Context	No mathematical context
Business International curriculum		X			
Professor´s class observation		X	X		X

CONCLUSIONS

So far, it has been observed that there are some similarities and differences in the meanings of probability between the official curriculum and the curriculum taught by the professor. First, the official curriculum starts from a frequency meaning, whereas the phenomenology and the representation systems are not specified, at least in the first part of analysis.

The components of meaning share conceptual structure, and the field that is most enhanced is the conceptual. The phenomenology in the professor´s class is explained through a non-mathematical context, as when the notion of meaning is analyzed in the history of probability. The most important reference found in the historical development of the meaning of probability is games of chance, also present in the enacted curriculum. Although the official study program curriculum does not mention it, it would be important to determine how the probabilities will be approximated.

REFERENCES

Batanero, C. (2005). Significados de la probabilidad en la educación secundaria [Meanings of probability in secondary education]. *Revista Latinoamericana de Investigación en Matemática Educativa, 8* (3), 247–263.

Batanero, C., Chernoff, E., Engel, J., Lee, H., & Sánchez, E. (2016). *Research on teaching and learning probability.* Springer. https://doi.org/10.1007/978-3-319-31625-3

Cañadas, M., Gómez, P., & Pinzón, A. (2018). Análisis de contenido [Content analysis]. In P. Gómez (Ed.), *Formación de profesores de matemáticas y práctica de aula: Conceptos y técnicas curriculares* (pp. 53–112). University of the Andes.

Dolores Flores, C., & García González, M. (2012). *¿Hacia dónde reorientar el currículum de matemáticas del bachillerato?* [Where to reorient the high school mathematics curriculum?]. Plaza & Valdez Editores.

Eichler, A., & Erens, R. (2014). Teachers' beliefs towards teaching calculus. *ZDM Mathematics Education, 46*(4), 647–659. https://doi.org/10.1007/s11858-014-0606-y

Hacking, I. (1975). *The emergence of philosophical probability: A philosophical study of early ideas about probability, induction and statistical inference* (2nd ed.). Cambridge.

Hernández, J., Zamora, R., & Lupiáñez, J. L. (2020). Estudio comparativo de los significados y expectativas de aprendizaje del límite en tres libros y el currículo official [Comparative study of the meanings and learning expectations of the limit in three books and the official curriculum]. *PNA, 14*(4), 241–269. https://doi.org/10.30827/pna.v14i4.13736

Makón, A. (2004). Métodos comparativos en ciencias sociales: Algunas reflexiones en relación a sus ventajas y limitaciones [Comparative methods in social sciences: Some reflections in relation to their advantages and limitations]. *VI Jornadas de Sociología.* Facultad de Ciencias Sociales, Universidad de Buenos Aires.

Mateos, G. & Morales, A. (2002). Historia de la probabilidad (desde sus orígenes hasta laplace) y su relación con la historia de la teoría de la decisión [History of probability (from its origins to Laplace) and its relation to the history of decision theory]. In *Historia de la probabilidad y de la estadística* (pp. 1–18). A.H.E.P.E.

Mayer Celis, L. (2015). *Rutas de incertidumbre. Ideas alternativas sobre la génesis de la probabilidad, siglos XVI y XVII* [Uncertainty routes. Alternative ideas about the genesis of probability, 16th and 17th centuries]. Fondo de Cultura Económica.

Restrepo, L. F., & González, J. (2003). La historia de la probabilidad [The story of chance]. *Revista Colombiana De Ciencias Pecuarias*, *16*(1), 83–87. https://revistas.udea.edu.co/index.php/rccp/article/view/323856

Rico, L. (2012). Approach to research in didactics of mathematics. *Avances de Investigacion en Educacion Matematica, 1,* 39–63. https://doi.org/10.35763/aiem.v1i1

Rico, L. (2013). El método del análisis didáctico [The method of didactic analysis]. *Union: Ibero-American Journal of Mathematics Education,* *9*(33), 11–27. https://union.fespm.es/index.php/UNION/article/view/801/512

Rico, L., & Fernández-Cano, A. (2013). Análisis didáctico y metodología de investigación [Didactic analysis and research methodology]. En L. Rico, J. L. Lupiánez, & M. Molina (Eds.), *Análisis didáctico en educación matemática. Metodología de investigación, formación de profesores e innovación curricular* (pp. 1–22). Comares, S.L.

Ruíz, B., Albert, J., Pinto, J., Tauber, L., Alvarado, H., & Zapata-Cardona, L. (2021). Red Latinoamericana de Investigación en Educación Estadística. Necesidades, expectativas y realidades [Latin American Network for Research in Statistical Education. Needs, expectations and realities]. *Revista Mexicana de Investigación Educativa–RMIE, 26*(88), 69–94.