STUDY AND RESEARCH PATHS FOR STATISTICS TEACHER EDUCATION AT SECONDARY SCHOOL LEVEL: AN EXPLORATORY STUDY

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We present the design and implementation of an inquiry-based activity based on a general proposal of the Anthropological Theory of the Didactic called "study and research path for teacher education" (SRP-TE) as a tool to reflect, develop, analyze, and experience proposals for teaching statistics at lower secondary school. Our proposal of SRP-TE starts from a textbook activity based on a statistical graph about water distribution in Brazil and turns it into an open activity that fosters different dimensions of statistical work (searching, collecting, cleaning, organizing, and exploring data), including interdisciplinary interaction. After its implementation, when asked about the content inherent in the inquiry, pre-service teachers seem to disregard many dimensions of the statistical work, especially those related to data handling.

INTRODUCTION

In Brazil, as in many other countries, there is a gap between the type of statistics that pre-service teachers learn at the university and the school statistics they will teach in compulsory education. The university subject *Probability and Statistics* does not seem to provide future mathematics teachers with the necessary epistemological and pedagogical tools for teaching. This means they need to look for other sources to overcome the difficulties found when teaching statistics. We can interpret it as an illustrative example of what Felix Klein called the "double discontinuity" between school and university mathematics (Winsløw & Grønbæk, 2014). With this in mind, our Ph.D. in progress seeks to answer the following research question: *What educational proposals for pre-service mathematics teachers can provide them with tools to design, analyze, and implement new instructional processes in the teaching of statistics in lower secondary school?*

Our research is framed within the Anthropological Theory of the Didactic (ATD) and the hypothesis that *study and research paths for teacher education* (SRP-TE) can lead teachers to question the content to be taught, as well as related scholarly knowledge for teaching the content (Barquero et al., 2022) is assumed. A second hypothesis, supported by our earlier works (Verbisck et al., 2022) and that of other writers (Batanero et al., 2011; Gould et al., 2018; Zapata-Cardona & Escobar, 2019), is related to the dominant conception of school statistics in secondary education. School statistics is reduced to numerical calculations of statistical measures (frequencies, means, medians, modes, deviations, ranges, quartiles, etc.) and to the interpretation of standardized graphical representations of distributions (pie charts, bar charts, histograms, scatter plots). This is especially evident when looking at Brazilian lower secondary school textbooks, particularly those mostly used in the country (Prestes, 2021; Verbisck, 2019). Even if the curriculum includes the item "plan and execute a sample survey," related activities are rarely included in textbooks. When they appear in the form of "inquiry activities," students are guided through all of the steps to be followed, and attention is turned to summarizing and graphically representing data rather than data collection and organization.

This paper presents an exploratory study of an SRP-TE for statistics that takes as a starting point a textbook exercise about water distribution in Brazil. The aim of the SRP-TE is twofold. It first seeks to get student teachers to investigate the question underlying the exercise to analyze the generative power of this question in terms of statistics and issues from other disciplines. Second, the inquiry and its analysis should provide student teachers with design tools to organize (and eventually implement and assess) a similar activity in a real educational setting. To sum up, the SRP-TE is intended to help teachers develop a "questioning-the-world" attitude for turning a textbook exercise into an inquiry activity.

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THEORETICAL FRAMEWORK

According to Chevallard (2015), teaching mathematics—and education in general—takes part in what he refers to as the paradigm of visiting works in contemporary society. Students' responsibility in this paradigm is to study content prescribed by rules and according to ready-made categories of different sizes: topics, areas, domains, and disciplines. It is the students' duty to "see and adore" these works without necessarily challenging their validity or worth. Hence, in this paradigm, themes and subjects are like monuments: students cannot change them; they do not need to understand why they exist; all they have to do is study them. The instructor, who is referred to as "the one who knows," is the one who asks the most questions regarding the value and validity of knowledge.

In contrast to this first paradigm, Chevallard (2015) describes the *paradigm of questioning the world* in which content is organized by *questions* and the knowledge needed to answer the questions. Teaching and learning processes become inquiry processes aiming at answering questions. In other words, students are the inquirers of the generating questions Q proposed by the teachers (or by the students themselves). There may be times when visiting works is required in the search for answers to a question, but with a specific *raison d'être* (answering Q). To study the conditions needed to transition to this second paradigm, Chevallard (2015) proposes a general inquiry format called *study and research paths* (SRP). The interplay between questions and answers plays a crucial role in the dynamics of SRPs. Students, helped by teachers, address an initial question Q, turn Q into derived questions Q_i, for which they learn or elaborate on their knowledge for answers A_i, finding new questions during the process which, in turn, call for new answers, etc.

Barquero et al. (2018) argue that teacher education proposals also need to be conceived within the new paradigm of q*uestioning the world*. Thus, they consider implementing *study and research paths for teacher education* (SRP-TE) as a way to provide teachers with tools for their professional development. An SRP-TE consists of five modules. It starts with an open question $Q_{0\text{-TE}}$ related to a teaching issue, such as *How should we teach statistics in the final grades of lower secondary school?* (*Module 0*). Student teachers are then asked to carry out an SRP that could be implemented in a real classroom (*Module 1*). The SRP experienced is then analyzed using epistemological and didactic tools spontaneously accessed by the students or provided by the educators (*Module 2*). In *Module 3*, teachers design and implement an SRP under specific conditions and, finally, they analyze it and share their experiences (*Module 4*). In pre-service teacher education, student teachers do not always have the opportunity to implement their designs in real classroom conditions and the SRP-TE ends with the design proposal and a priori assessment. This is the case we consider here with a group of pre-service teachers of the Pedagogical Residency Program of the Federal University of Sergipe, Brazil.

METHODOLOGY AND DESIGN OF THE ACTIVITY

The Pedagogical Residency Program is one of the initiatives that make up Brazil's National Policy for Teacher Education and aims to improve the engagement of pre-service teachers in compulsory education. This engagement must contemplate, among other activities, classroom management and pedagogical interventions, guided by an experienced schoolteacher and supervised by a university lecturer implementing the program (the last author of this contribution).

The strategy we employ begins with a mathematics textbook exercise about water distribution in Brazil, which focuses on a topic that is both timely and paradoxical: Brazil has ample water, but at the same time water is scarce. The textbook introduced this issue as follows:

> Brazil has approximately 13.7% of the total fresh water supply in the world and is a territory rich in terms of water. However, the country is experiencing serious problems related to both the degradation of water quality, especially in the vicinity of urban areas, and the lack of control over excess water and water shortages, which affects several Brazilian locations. Floods affect Brazilian cities, but water scarcity also imposes serious restrictions and high costs on the economic and social development of large cities (Giovanni Júnior & Castrucci, 2018, p. 26, our translation).

The textbook exercise only focuses on minor components of the graph that summarizes the issue. (See Figure 1.) In a first analysis (Verbisck et al., 2022), we show how the exercise can be turned into an inquiry-based activity by considering the "water paradox in Brazil" as a generating question. Addressing it derives into questions about water distribution and use in Brazil, availability of related data, accessibility, and treatment. To test the productivity of the generating question, we decided to take

this scenario as the starting point for an SRP-TE (Barquero et al., 2018). We first present this methodological tool based on the ATD, and then describe the experimentation of the SRP-TE with a group of 16 pre-service secondary school teachers of the Pedagogical Residency Program in Brazil.

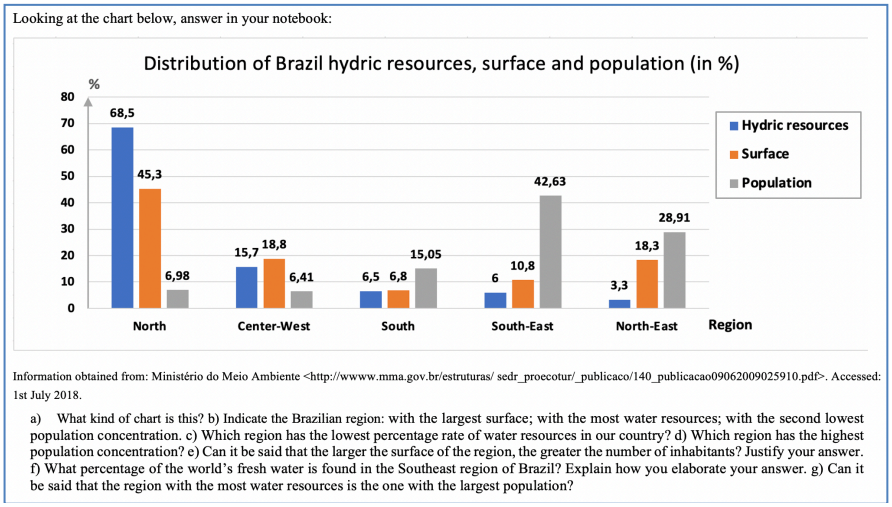

Figure 1. Brazilian exercise (Giovanni Júnior & Castrucci, 2018, p. 26, our translation)

The SRP-TE activity was proposed to this group of 16 pre-service teachers in four 90-minute sessions, which consisted of tutorials offered by the university teacher. The sessions were held virtually (through Google Meet) due to the COVID-19 pandemic. We used the last four sessions of the academic year in November and December 2021. The sessions were led by the first author and the regular lecturer, while the other authors acted as observers. The students were already familiar with the ATD approach, particularly the SRP format because the lecturer had already implemented it with this group in other situations. The students were also used to working in autonomous teams outside the classroom between sessions. The sessions were based in the modules of the SRP-TE structure.

Our design of the SRP-TE considered an *a priori* analysis of the exercise to check its generating power in terms of possible statistical activities (and of other disciplines) that could appear during the inquiry (Verbisck et al., 2022). For reasons of space, the next section presents the implementation of the SRP-TE together with the design activities that were taken as part of the *in vivo* analysis.

IMPLEMENTATION OF THE SRP-TE ACTIVITY

In the first session, the educators presented a school exercise that only contained the introductory statement and the graph displayed in Figure 1. In teams of four, the students were asked to discuss the following proposal: *Based on the situation presented in the school exercise, what questions can be raised about the issues considered, and about the data used?*

In teams, the students wrote down the issues and organized them to be presented; they designated a communicator to pool ideas. They asked questions that enlarged the initial situation: *How are water resources distributed in Brazil? Do geographical factors influence water distribution in each region of the country? What factors cause water scarcity? How much untreated sewage is returned to rivers? Is there water storage for dry periods? Can water scarcity cause water shortage in the future? What can be done to make better use of available resources?*

To prepare for the second session, the educators grouped the questions into six topics: water resources and distribution, sharing water resources among regions, quality of water resources, climate conditions (rainfall), water consumption, and policies (government and citizen actions). However, questions related to the data presented in the exercise (such as *What variables does the exercise present and how were they measured? Where were these data taken from? Is the source reliable? How can we validate the data?*) did not appear.

At the beginning of the second session, we presented the complete school exercise (including the questions). It was interesting to see that the pre-service teachers were surprised by how poor/ limited the textbook questions were in comparison to the richness of the questions they had prepared.

Afterwards, we presented their questions grouped into topics and added some general "methodological" questions they had not considered about data access and reliability that could fit in each topic.

After the discussion on the questions elaborated by the teams, we provided each team with a set of questions and asked them to analyze the different questions:

- Group 1. Resources: *How much water is there in the five regions of Brazil? How is it measured?*
- Group 2. Climate: *Where does the water come from in the five Brazilian regions?*
- Group 3. Consumption: *What is water consumption like in the five regions of Brazil? What factors cause water scarcity?*
- Group 4. Sharing: *What strategies exist to distribute water from one region to another?*

The students were asked to note all of the sources they consulted, both those sources that were used and those that were discarded; to appoint a secretary to write down everything being done; and to appoint a communicator to prepare a summary with the main results to be presented. Due to time constraints, the students were asked to focus on one question per team.

In the third session, one representative of each group presented their answers and possible derived questions to the question assigned to them in the last session. The students reported that they had trouble locating sources relevant to the question they were seeking to answer, a lack of information and data on water resources in the five individual Brazilian regions, and of current studies on the themes investigated. A lot of the information they found was redundant and outdated (from decades ago), but they did not have time to update or discard it.

After the presentations, we proposed one last activity. As we mentioned earlier, our goal for this SRP-TE activity was to help the pre-service teachers develop an attitude (change of paradigm) for turning a textbook exercise into an inquiry activity. Bearing in mind the themes discussed in the previous sessions, the last task for each team of student teachers was to design an instructional proposal (a project or a longer activity) adapted to the lower secondary school level. For the design, they were asked to follow a list of design-oriented questions: *What is the initial question? How is the question posed; what is proposed to be done; and what tools are available to students? How much class time is the educational proposal going to need (approximate length of the activity)? How should the activity be completed? What content themes, domains, or disciplines (beyond mathematics) could be addressed in the proposed activity?*

The groups started to outline their proposals in this third session but, due to the short time remaining, they had to continue working on them after the session. Sharing of results was scheduled for the last session. The four groups presented different teaching proposals that included the following generating questions:

- Q_{Group1}: *Why is water consumption limited in certain regions of Brazil?*
- Q_{Group2}: *Is it possible to collect rainwater to use at home?*
- Q_{Group3}: *Given the problems related to water quality degradation, what is the percentage of degraded water resources?*
- Q_{Group4}: *If water resources are abundant in Brazil, why is there water scarcity and/ or rationing in all regions of the country?*

We focus here on the teaching proposal from Group 2 that interested us the most. (See Table 1.)

In this proposal from Group 2, it is interesting to observe that they took the question about the problem of water shortage that exists in the region where they live (northeast Brazil) seriously. They propose a generating question—collecting rainwater for household use—demanding both theoretical and practical answers. The derived questions they listed follow the logic of the generating question, and not the knowledge they used to answer it. They thus include several areas of study, such as biology, geography, and chemistry, as they themselves pointed out. Interdisciplinarity seems to gain strength thanks to this generating question. However, at no time do they consider the search for data as work related to statistics. Mathematics content is present, but statistics does not seem to be part of this enquiry, even though data searching is certainly an action that would appear in different derived questions. It is even more surprising given the fact that the initial problem was taken from a statistics textbook, and they knew that. They also searched data during the second module of the SRP-TE, when addressing the initial generating question by themselves.

Table 1. SRP proposal from Group 2

Q_0: Is it possible to collect rainwater to use at home? *Context*: Using the context of the water distribution SRP, we start from how water is distributed in our city, making students realize that in our region there is relatively little water. The idea of trying to save water is hence put forward, since water distribution is a process that is hardly used in Brazil. To do this, we will talk about the climate and how rainfall can help alleviate this problem. We then introduce our Q_0 to them. Throughout the SRP, the students can use whichever tools are available, such as books, mobile phones, etc. NOTE: Considering most of the students are from rural areas, they usually use rainwater, as piped water supplies are not always available in the homes in those areas. *Questions that may arise when answering Q_0*: Is the amount of rainfall in my region enough for a person to live on rainwater alone? How much water does an average person use in their daily life? How can rainwater be stored or treated and distributed to be used in most household activities? How much rainwater can be harvested? How can we collect as much rainwater as possible? Would this save money on water bills? Can rainwater be used for drinking? Is it possible to convert it into drinking water? How many gallons of chlorine would have to be used if 300 gallons of water were collected? *Subjects and contents*: Mathematics: ratio, magnitudes and measures, functions, areas of figures, volume. Geography: Climate of the region. Chemistry: Stoichiometry. Biology: Micro-organisms and bacteria, water cycle. […]. *Estimated time and conclusion*: We believe it will take about a month of class time to complete this inquiry, since we anticipate two weeks of discussions, one to learn about the issues that arise in the process, and another to prepare and present the final answer. We hope that the study will conclude with an answer to the initial question, in which the students use the knowledge acquired throughout the lessons to say whether it is possible to carry out what is asked or not. If the answer is yes, it would be necessary to show a project that in practice fulfils what is asked in Q_0 and estimates how much water can be collected. If it is not possible, the students should argue why and under what conditions it could be feasible.

DISCUSSION AND CONCLUSION

As noted, due to the COVID-19 pandemic, the teacher education activity consisted of few sessions, using the very last classes of the academic year as well as online interactions. Although considered to be a pilot study of a potential SRP-TE, it provided us with some interesting ideas regarding the proposal and how it might be used in the future. The virtual sessions entailed the difficulty of facilitating teamwork, as well as the fact that the most significant work was completed outside of the classroom. However, we were able to discover certain favorable factors for the activity's further development. First and foremost, the group of future teachers was familiar with the ATD and had already taken part in other SRPs. The student teachers were used to crafting their own questions and questions not limited to mathematics. They outperformed our a priori analysis in this regard. They expanded the range of questioning beyond geography (resources and climate) to include social, political, economic, and environmental issues (sharing and quality), allowing statistics to be considered "an essential tool for the formation of a critical attitude about current social, political, cultural, and scientific issues in the study of interdisciplinary or cross-cutting themes" (Batanero et al., 2011, p. 7).

However, we want to stress the fact that, among the contents activated by the SRP, aspects related to statistics and data processing appeared only tangentially. The student teachers did not focus on the search and use of data, as we did in the a priori analysis. Having little time to undertake the education activity explains the students' choice to focus on topics for which they found studies that were easily accessible. It is also worth noting that the statistical work that underpins many studies is not always visible or accessible. This raises the problem of defining "what it is to study a question Q" more clearly and, more specifically, when an answer to Q is acceptable or not. For instance, it is important to establish an agreement with the pre-service teachers about why "copying" directly accessible information is not acceptable as an answer: the information found during an inquiry must be tested and, if possible, teachers should partially reproduce the evidence provided.

Finally, the invisibility of data processing reappears when students are asked to associate curricular knowledge with the development of their teaching proposal. This is a return to the invisibility

of some statistical activities that teams incorporated in the proposal but did not identify as part of the content to be taught (while they did identify other content). We can talk about a certain invisibility of the knowledge needed in relation to statistics and data analysis, which may be supported by the scarcity of the school terminology. It is difficult—not only for student teachers—to identify and even name activities related to data collection, organization, cleaning, structuring, and presenting that are not part of the curriculum description. We can relate this situation to the *transparency of knowledge objects* discussed by Margolinas (2014). This points to yet another challenge for the implementation of future SRP-TE about statistics in primary and secondary education. We are currently working on gaining greater knowledge of the status of some statistical notions and tools according to teachers' perceptions and professional practices.

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