INTERNATIONAL PERSPECTIVES ON EARLY STATISTICAL THINKING: COMPARISON OF PRIMARY SCHOOL CURRICULA IN DIFFERENT COUNTRIES

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Due to the omnipresence of data, over recent decades, statistical thinking has fast become the new literacy that should be developed early in primary school. What are the issues and aspects that are realized and mentioned in the different curricula of various European countries? In this paper we will compare the status quo of early statistical teaching and learning across different countries (Ireland, Cyprus, Turkey, and Germany), explore similarities and differences, and garner insights into international practices.

INTRODUCTION

In today's data-driven society, vibrant democracies need well-informed citizens who can reason about data and probability when discussing relevant social issues and engaging in public decision-making processes (Engel, 2017). Indeed, over a century ago, H. G. Wells (1903) stated that "as an efficient citizen of one of the new great complex world-wide states that are now developing, it is as necessary to be able to compute, to think in averages and maxima and minima, as it is now to be able to read and write" (p. 189). At the end of the last century, Watson (1997) drew attention to the importance of statistical literacy outside the classroom and its important role in social decision making, the media, and in ascertaining risk. More recently, and with specific reference to early years and primary education, Ben-Zvi (2018) stated the importance of encouraging early statistical (and probabilistic) reasoning and thinking: "Today's students need to learn to work and think with data and chance from an early age, so they begin to prepare for the data-driven society in which they live" (p. viii). A recent book edited by Leavy et al. (2018) outlines international perspectives on enhancing statistical and probabilistic thinking in kindergarten and primary school. This book also draws attention to the importance of formative experiences with data and probability. Enhancing early statistical and probabilistic thinking is crucial—but it is interpreted differently across countries. In this paper, we present an exemplary analysis of the curricular treatment of early statistical thinking across selected primary school (up to grade 4) curricula internationally. We engage in a content analysis and curriculum mapping exercise to gain insights into various curricular perspectives on teaching statistical and probabilistic thinking in primary school. The purpose of the analysis presented in this paper is to show a quick exemplary and comparative snapshot of the implementation of early statistical thinking in different mathematics curricula of selected countries with the aim to document (a) which facets of early statistical thinking are realized in the curriculum and (b) which facets of early statistical thinking are neglected in the curriculum. Beyond that, this paper intends to encourage more detailed comparative curricula analyses in the future.

METHODOLOGY

Statistics is generally taught as part of the mathematics curriculum at the school level. For the purposes of this evaluation, it was decided to analyse the official academic standards that are recommended by the Ministry of Education in the country of study and to focus up to grade 4. Please note that the term curriculum can have multiple meanings: in some countries the Ministry of Education publishes the curriculum document, which includes an overview of different domains of mathematics such as number and operations or geometry and the learning objectives in each domain to guide instruction. In the case of Germany, the curriculum document of the federal state of North Rhine-Westphalia was selected because it is the state with the highest population (Ministry of Education (MSW) NRW, 2021). For the other three countries, the national curricula were selected: Department of Education (National Council for Curriculum and Assessment, 2022); Ministry of National Education

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(MEB, 2018); and Ministry of Education and Culture (MOEC, 2020). The curriculum documents were accessed through the relevant official websites of the ministries of education or curriculum development authorities. To ensure a fair and equitable comparison between countries, the comparison was limited to the analysis of these official curricula and did not incorporate attention to additional resources or textbook series. Most primary teachers in the four countries under study are generalist teachers that teach up to 12 subjects. Many are not experts in mathematics or statistics education and depend on the learning objectives stated in these documents to guide their practice. Consequently, when evaluating whether a curriculum met the criteria, we searched for explicit mention of the criterion within the curriculum documentation. We did not presume that a teacher may engage in additional practices that complement the stated standards, although we recognise that practice in classrooms may be broader and richer than that suggested by the national standards.

THE STRUCTURE OF SELECTED INTERNATIONAL PRIMARY CURRICULA

In most federal states of Germany, primary school is from grade 1 to 4 (ages 6–10), and the curriculum is specific for each federal state. The mathematics curriculum for primary school distinguishes several content domains such as Numbers and Operations, Geometry, Magnitudes and Measuring, and Data and Chance. The curriculum of North Rhine-Westphalia (the federal state with the highest population) for 2022–2023 offers vague and minimal components for "data." Upon completion of grade 4, students are expected to present and extract information from data in diagrams such as bar graphs and pie charts (MSW NSR, 2021). The national mathematics curriculum in Turkey (MEB, 2018) comprises four learning domains in primary school (grades 1–4, ages 6–10): Numbers and Operations, Geometry, Measurement, and Data Handling. Learning objectives in the data handling domain are structured by a four-phase data investigation process (formulating questions, collecting data, handling and analyzing data, and interpreting results) with an emphasis on making and interpreting tables and graphs. Irish schools are preparing for implementation of a revised primary mathematics curriculum in the 2022–2023 academic year. The curriculum for grades K–6 (ages 5–12) has five strands: Algebra, Data and Chance, Measures, Number, and Shape and Space. Each strand is accompanied by a research paper describing that strand's core mathematical concepts, skills, and processes (cf., Leavy, 2020, for the data paper). Except for the two kindergarten years, the overarching learning outcomes for each grade emphasise a four-stage process of statistical investigation. In Cyprus, the national mathematics curriculum follows an integrated structure: pre-primary (ages 5–6), primary (grades 1–6; ages 6–12), lower secondary (gymnasium) (grades 7–9, ages 12–15), and upper-secondary (lyceum) (grades 10–12, ages 15–18). The curriculum includes five strands: Numbers and Operations, Measurement, Geometry, Algebra, and Statistics and Probability. Benchmarks labelled as "success indicators" are provided for each grade. The Statistics and Probability strand at the primary school level focuses on the construction and interpretation of tabular and graphical representations and introduces basic concepts of probability. For example, the "success indicators" on statistics for grade 4 are (MOEC, 2020): "Constructing and interpreting bar graphs, pictograms, pie-charts, and line graphs" and "Describing and comparing datasets, based on their range and modal value."

A COMPARISON OF CURRICULAR TREATMENT OF EARLY STATISTICAL THINKING

With the advancements in computing and changes in data forms encountered in today's world, the Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II: A Framework for Statistics and Data Science Education (GAISE II) (Bargagliotti et al., 2020) advocates for developing data/statistical literacy starting from early school grades and provides recommendations for developing the new skills with an emphasis on data science topics. Based on this updated vision, the comparison is structured using the four-step statistical problem-solving process, organised into three developmental levels (A, B, and C) with the enhanced recommendations found in the GAISE II report. See also Batur et al. (2021) for an evaluation of the content of the learning outcomes related to statistics in the Turkish mathematics curriculum (Grades 1–12) in comparison with other countries, including Singapore, South Korea, the United States, and New Zealand, using the GAISE I framework (Franklin et al., 2007). Although the learning goals in each level stated in the report are not intended for specific grade levels, they are aligned with students' development in statistical literacy and thus we focus on level A content to compare curricular documents on early statistical thinking from pre-school to grade 4. Within each step we list the concepts, activities, and experiences expected of a student at level A: (a) Formulate statistical investigative questions; (b) Collect data/Consider data; (c) Analyse the data; and (d) Interpret results. These concepts, activities, and experiences in the above-mentioned process components of statistical problem solving served as deductive categories for our content analysis (Mayring, 2015).

Process Component: Formulate Statistical Investigative Questions

As we see in Table 1, great similarities are evident across countries in that they all solely address posing statistical investigative questions and neglect consideration of all other learning goals. In Germany, the students are supposed to identify data from the immediate reality of life by the time they have finished primary school. In this respect, the generation of statistical investigative questions is not mentioned directly, but it is included indirectly. In Turkey, students in grades 2 and 4 are expected to pose questions that can be answered by collecting data from the classroom. Different types of statistical questions (descriptive, comparison, association) are not addressed in the curriculum. Similarly in Ireland, different types of questions are not addressed. In grades 1–4, students pose statistical questions of interest to them, which subsequently motivate cyclical statistical investigations. In Cyprus also, pre-primary and early primary school students are expected to pose questions of personal interest that can be answered by collecting data from their immediate environments (classroom or home).

Table 1. Realization of level A concepts, activities, and experiences to "formulate statistical investigative questions"

Level A concepts, activities, & experiences	G	T	I	C
Understand when a statistical investigation is appropriate				
Pose statistical investigative questions of interest to students	x	x	x	x
Pose statistical investigative questions about one variable … and extend to include comparison and association questions				
Experience different types of questions in statistics				

Key: Germany (G), Turkey (T), Ireland (I), and Cyprus (C)

Process Component: Collect Data / Consider Data

There are broadly similar expectations for how primary students collect data (see Table 2). In Germany, emphasis is placed on understanding that data are information, knowing how to collect data, and understanding that a variable measures a characteristic of the data representing the same information across observational units. In Turkey, with the emphasis on collecting data based on the problem posed in grades 2 and 4, students are expected to understand data as information and how to collect data. However, the curriculum does not specify concepts such as variables, data values, and variable types (categorical/quantitative) nor experience with interrogation of the given data sets to understand the context of the variables and consider possible missing values. In Ireland, the emphasis in the early kindergarten years is on explaining data as collections (of objects) that possess specific attributes or hold information. Data are described as being all around us, as being collected and represented in different ways, and that they function as a tool to help us interpret the world. In grades 1 and 2, children are introduced to data being qualitative or quantitative, with each type requiring different representational forms (graphs) and summary measures (statistics). In Cyprus, the situation is very similar to Turkey. Although the national curriculum does expect students in the early grades (pre-school to grade 4) to collect and record data of personal interest, it does not include any learning objectives (or "success indicators") on fundamental concepts related to data collection (e.g., variable, variable types, or data variation). The success indicators are presented in the curriculum as mathematical practices, where mathematical problem solving is presented as a main mathematical practice for statistical reasoning.

Process Component: Analyse the Data

The similarity across countries continues into expectations for how primary students should analyse data (see Table 3). In German primary schools, students are supposed to present data and frequencies in diagrams and tables; to extract data from calendars, diagrams, and tables; and to interpret the data representations to answer mathematics-related and consumer-relevant questions. In some specific cases, key features of distributions such as the mean as the equal share are taught (but not explicitly mentioned in the curriculum). In Turkey, there is an emphasis on analyzing data in all primary grades (1–4). Students are expected to read simple data tables for one or two groups (grade 1); organize data using tree diagrams, tally charts, and frequency tables and make picture graphs (grade 2); read information from picture graphs and make conversions from graph to tally chart and frequency table (grade 3); and use different representations (bar chart, picture graph, table, tree diagram) to display collected data (grade 4). However, understanding the concept of distribution and using distributions to analyze data are not addressed in the curriculum. In comparison with Turkey, there is no explicit mention of different types of representations in the Irish curriculum. However, except for association, the curriculum of Ireland has a comprehensive treatment of data analysis and a strong emphasis on distribution. Across grades 1–4, graphs are considered reasoning tools that can be used to provide answers to the initial questions posed. Graphs host or represent distributions of data and communicate measures of centre and variability. Measures of centre are described as one number summaries of distributions, and the range is described as a measure to capture variability. It is mentioned that distributions can be compared based on analysis of their shapes. In Cyprus, the curriculum puts emphasis on data organization and representation using appropriate displays. In preschool, students are expected to present data in picture graphs, whereas in grade 1, bar graphs are also introduced. In grades 2 and 3, tabular representations of data are introduced, and students are expected to represent the same dataset in multiple ways (bar chart, picture graph, table). In grade 4, students are expected to construct, both by hand and through use of technology, the tabular and graphical representations introduced in previous grades, as well as pie-charts and line graphs, which are first introduced in grade 4. Expectations concerning the description of key features of distributions are limited to the description of datasets based on their range and modal value in grade 4.

Table 2. Realization of level A concepts, activities, and experiences to “collect data/consider data”

Level A concepts, activities, & experiences	G	T	I	C
Understand that data are information	x	x	x	x
Understand how to collect and record information from the group of interest	x	x	x	x
Understand that a variable measures the same characteristic and results in data values that may fluctuate	x			
Understand that within a data set there can be different variable types			x	
Interrogate the data set to understand the context of the variables …				
Understand that data may contain errors, have missing values …				

Table 3. Realization of level A concepts, activities, and experiences to “analyse the data”

Level A concepts, activities, & experiences	G	T	I	C
Understand that distribution describes the number of times an outcome occurs			x	
Represent the variability of variables using appropriate displays (e.g., tables, picture graphs, dotplots, bar graphs)	x	x	x	x
Describe key features of distributions (center, variability, shape)	x	x	x	x
Recognize distributions can be used to compare two groups			x	x
Observe whether there appears to be an association between two variables				

Process Component: Interpret Results

As evident in Table 4, all countries, including Germany, emphasise the use of statistical evidence to answer the statistical questions and communicate the results. In Turkey, the learning objectives at the upper primary grades focus on interpreting results. In grade 3, students are expected to read and interpret simple data tables, and in grade 4, students are expected to make interpretations and predictions from bar graphs and solve problems using the information from data displays. The curriculum does not address the generalizability of conclusions to other groups or describing differences between two groups in experimental conditions. In Ireland, across grades K–4, data interpretation is framed within the statistical cycle where data are used to address statistical questions, and graphs are used as reasoning tools from which to interpret findings and make conclusions. In Cyprus, students across the early grades are expected to engage in interpretation of data presented in graphs in order to respond to statistical questions. The statistical questions concern only single datasets, except for grade 4, where students are expected to compare two datasets based on their range and modal value(s). Students in grades 2 and 3 are expected to pose questions about a dataset when provided with a graphical display of the data (e.g., "Pose questions that could be answered based on the information provided in the following bar graph"). Grade 4 students are expected to be able to compare two datasets (e.g., via range).

Table 4. Realization of level A concepts, activities, and experiences to "interpret results"

Level A concepts, activities, & experiences	G	T	I	C
Use statistical evidence to answer the questions and communicate results.	x	x	x	x
Make statements about the group or population recognizing that conclusions are limited to these groups and cannot be generalized to other groups			x	
Describe the difference between two groups with different conditions				

CONCLUSION

It is evident that current curricula represent many desirable practices, such as viewing statistics through the lens of a cycle of investigation. This is a welcome perspective compared to historically disjoint renderings and perspectives of what constitutes the activity of 'doing statistics.' The importance of relevant questions and the role of graphs as communicative tools, evident across all national contexts, will serve to embed primary-level statistical activity as meaningful practices that contribute to the skill set required of 21st century citizens. There remain opportunities for development and growth, in particular with regard to understanding different types of questions, understanding of distributions and using them to compare data sets, consideration of association, and engaging in a simple comparative experiment to compare the results of different conditions that are recommended by the GAISE II report. When speaking of statistical thinking we have also to think about probabilistic thinking. The GAISE II report addresses probability in relation to understanding uncertainty in the context of sampling and making generalisations beyond data (stated in levels B and C). In this paper, we concentrated on level A and statistical thinking only, and we have shared some insights into how early statistical thinking is implemented in the curricula of four different countries. Of course, this is only a small glimpse into the situation of early statistical thinking in Cyprus, Germany, Ireland, and Turkey, but it can serve as a starting point for further explorations and broader analyses.

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