TOWARDS A FRAMEWORK FOR DEVELOPING A CRITICAL STATISTICAL LITERACY

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In this theoretical paper we hypothesize a developmental framework for a critical statistical literacy based on our anecdotal observations from working with teachers and empirical evidence from a pilot study. We create the framework by taking Watson and Callingham's hierarchical framework for statistical literacy and reframing it as a developmental framework. We then combine it with Weiland's Critical Statistical Literacy framework, which serves as an end goal of the developmental framework. We provide excerpts from a pilot study as a proof of concept and make suggestions for future research.

INTRODUCTION

Statistical literacy has long been a construct of importance to study in statistics education (Ben-Zvi & Garfield, 2004; Gal, 2002). Statistics permeate every aspect of society, ranging from healthcare to education to the economy, which makes statistical literacy both an academic pursuit and a knowledge base necessary for citizenship (Nicholson et al., 2018; Weiland, 2017). Despite its centrality in informal and formal contexts, statistical literacy, and its role in interrogating the status quo, needs to be studied more in-depth in grades K–12 schooling. Additionally, little is known about how people codevelop statistical literacy and critical literacy (e.g., reading and writing the world; Freire, 1970; Gutstein, 2005). This work offers a theoretical framework at the developmental intersection between statistical knowledge and critical practice. We do so by rethinking Watson and Callingham's (2003) hierarchical framework for statistical literacy with a critical statistical literacy (CSL) framework (Weiland, 2017) as the goal and through a situated cognition theory of learning (Greeno, 1998; Lave & Wenger, 1991).

We argue that the combination of these frameworks is important for several reasons. CSL presents important practices but provides little insight into how to develop those practices. Although Watson and Callingham's (2003) hierarchical construct of statistical literacy provides levels of development that CSL is missing, it is rooted in a cognitive framework of learning, whereas our work is situated in a sociocultural perspective that comes with different ontological and epistemological assumptions. Therefore, we do not see levels hierarchically for development but consider attunement to practices through authentic participation in activities of the community, which aligns with the framing of CSL (Weiland, 2017). Furthermore, the hierarchical construct is created and validated through the process of measurement and is framed through the characterization of tasks rather than practices. We therefore reframe those levels in terms of practices in our Critical Statistical Literacy Developmental Framework (CSL-DK) and present a view of levels as a continuous process of attunement. In the sections that follow we briefly summarize the frameworks we are combining before presenting our theoretical framework for developing a critical statistical literacy and then presenting some examples.

CRITICAL STATISTICAL LITERACY FRAMEWORK

There are many definitions of statistical literacy that have permeated the field over the years, the most influential being Gal (2002). The CSL framework (Weiland, 2017) expands on the consideration of writing/statistical enquiry and views it at its intersection with critical literacy (Frankenstein, 2009; Freire, 1970), emphasizing the interrogation of the status quo and transformative action for more equitable and just societies. By developing their statistical literacy, individuals learn to understand language and statistical symbol systems and to communicate their understanding of these systems. By developing their critical statistical lens, individuals learn to "identify and interrogate social structures and discourses that shape and are reinforced by data-based arguments" (Weiland, 2017, p. 41). Critical statistical literacy also places an emphasis on an individual's subjectivity, so that the individual can identify both personal and societal biases and work to balance those tensions. The framework consists of 10 main practices (see Table 1) for reading and writing the world (Freire, 1970) with statistics. This framework has also been used by others in statistics education (e.g., Zapata-Cardona & Martínez-Castro, 2021).

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Table 1. Modified practices of a Critical Statistical Literacy (Weiland, 2017)

Reading	Writing
R1. Making sense of language and statistical symbols systems and critiquing statistical information and data-based arguments encountered in diverse contexts to gain an awareness of the systemic structures at play in society.	W1. Using statistical investigations to communicate statistical information and arguments in an effort to destabilize and reshape structures of injustice for a more just society.
R2. Identifying and interrogating social structures that shape and are reinforced by data-based arguments.	W2. Using statistical investigations to alleviate and resolve sociopolitical issues of injustice.
R3. Understanding one's social location, subjectivity, and political context and having a sociohistorical and political knowledge of self and understanding how it influences one's interpretation of information.	W3. Negotiating societal dialectical tensions when formulating statistical questions, data collection, and analysis methods and highlighting such tensions in the results of a statistical investigation.
R4. Evaluating the source, operationalization, collection, and reporting of statistical information and how they are influenced by the author's social position and sociopolitical and historical lens.	W4. Communicating one's social location, subjectivity, and political context to others and how it shapes one's meaning making of the world when reporting results of a statistical investigation.
R5. Interrogating the epistemological and historical underpinnings of statistical practice and how it has shaped data-based discourses and beyond.	W5. Using sociopolitical oriented epistemologies to create new statistical practices and ways of measuring the world.

HEIRARCHICAL FRAMEWORK OF STATISTICAL LITERACY

Watson and Callingham's (2003) hierarchical construct of statistical literacy presents statistical literacy as a unidimensional construct with six levels of understanding: idiosyncratic, informal, inconsistent, consistent non-critical, critical, and critical mathematical (see Table 2). These levels, which have been extensively studied empirically using Rasch modeling, are based on a cognitive development model. As a note, Watson and Callingham do mention critical literacy and claim that their framework considers critical literacy at the higher levels. However, their perspective is more aligned with the critical thinking perspective of 'critical' than that of critical literacy (see Weiland, 2017 for further discussion). We argue some of the foundational practices of critical literacy are missing, such as questioning the status quo. For example, the status quo for statistical literacy is situated in the logic of discipline of statistics, where modern social statistics has troubling epistemological roots in the eugenics movement (Clayton, 2020), which is rarely interrogated in considerations of statistical literacy.

DEVELOPMENTAL FRAMEWORK OF CRITICAL STATISTICAL LITERACY

To help frame development of CSL, we draw upon situated cognition, which is a theory of learning that views an individual's development as mediated by cultural tools (Lave & Wenger, 1991). The process of learning occurs while situated in a community of practice (CoP), where individuals become attuned to the practices and norms of a CoP through engagement in those practices with a legitimate member of the CoP (Greeno, 1998; Lave & Wenger, 1991). People move from being periphery legitimate members of a CoP to legitimate members through the process described; this process is observable through their legitimate participation in the practices of the community (Lave & Wenger, 1991). Learning is viewed as an ongoing process of 'becoming' through 'doing' in meaningful and authentic ways. We liken this to the idea that to teach students statistics, we should engage them in authentic statistical enquiry (Wild & Pfannkuch, 1999). This can be seen through the focus on statistical enquiry and the statistical investigative process (Bargagliotti et al., 2020) in statistics education research. Through this learning perspective, we combine Watson and Callingham's (2003) levels with Weiland's (2017) CSL to propose our hypothetical developmental framework, described in Table 3.

Table 2. Hierarchical construct of statistical literacy (Watson & Callingham, 2003)

Level	Brief characterization of step levels of tasks
Critical Mathematical (Highest)	Task-steps at this level demand critical, questioning engagement with context, using proportional reasoning particularly in media or chance contexts, showing appreciation of the need for uncertainty in making predictions, and interpreting subtle aspects of language.
Critical	Tasks-steps require critical, questioning engagement in familiar and unfamiliar contexts that do not involve proportional reasoning, but which do involve appropriate use of terminology, qualitative interpretation of chance, and appreciation of variation.
Consistent Non-critical	Task-steps require appropriate but non-critical engagement with context, multiple aspects of terminology usage, appreciation of variation in chance settings only, and statistical skills associated with the mean, simple probabilities, and graph characteristics.
Inconsistent	Task-steps at this level, often in supportive formats, expect selective engagement with context, appropriate recognition of conclusions but without justification, and qualitative rather than quantitative use of statistical ideas.
Informal	Task-steps require only colloquial or informal engagement with context often reflecting intuitive non-statistical beliefs, single elements of complex terminology and settings, and basic one-step straightforward table, graph, and chance calculations.
Idiosyncratic (Lowest)	Task-steps at this level suggest idiosyncratic engagement with context, tautological use of terminology, and basic mathematical skills associated with one-to-one counting and reading cell values in tables.

Anecdotal practices are characterized as a sole focus on anecdotal experiences for making sense of and creating arguments. Unlike the hierarchical construct, there is not necessarily any consideration of quantities, statistical ideas, or representations in these practices. Furthermore, because all personal experiences are situated in context, we do not claim that there is no consideration of context here, just that there is not a broad consideration of context. Instead, the context considered is the specific context of the person's experiences or beliefs they are drawing upon. In moving into other categories of practices in the framework, individuals become increasingly more attuned to the practices of a critical statistical literacy. This includes moving from:

- Individual view → aggregate view → nuanced balance between individual and aggregate views.
- Not explicitly considering context → considering the specific context of the issue occasionally to consistently → reading beyond the data and considering the broader socio-historical situatedness.
- Colloquial language → informal statistical language → formal statistical language → balancing between the two to best communicate to the intended audience

Another significant change we made is our consideration of Watson and Callingham's (2003) *Critical* and *Critical Mathematical* levels, which are characterized as the development of mathematical skills, namely the sophisticated use of proportional reasoning. It is also important to note that the researchers' work studied students in grades 3–9, with just over 60% of their sample consisting of students from grades 3–6 where a sophisticated understanding of proportional reasoning would not be expected based on most countries' standard curriculum. However, our interests are in the literacy required for critical citizenship at the end of high school. We hypothesize if Watson and Callingham used a sample of grades 9–12 students or even post-secondary adults, they would see different results in their assessment questions and models. Because of our focus, we changed consistent non-critical to consistent critical thinking to highlight the focus of critical thinking at that level. We also changed critical to emerging critical to capture the initial emergence of critical literacy practices that we have observed in our work. Finally, we changed critical-mathematical to critical statistical to acknowledge the end goal of our work, which is a critical statistical literacy.

To highlight how we view this process of development through a situated cognition lens see Figure 1, where the center is legitimate participation in the community with the outermost layer representing peripheral participation. The pathway drawn with arrows represents the back-and-forth journey of attunement to show that it is not a continuous straight path but rather, a freeform movement

back and forth between types of practice. We have also intentionally blurred the edges of the concentric circles to indicate that these are not clearly bounded categories but blurry borderlands.

Table 3. Critical Statistical Literacy Developmental Framework (CSL-DK)

Types of Practices	Brief Characterization of Practices
Critical Statistical	Using formal statistical language interchangeably with informal or even colloquial language to communicate important ideas to diverse audiences that maintains the meaning but is most appropriate for the intended audience. Statistical practices from the emerging critical practices are still common, but some practices are transformed based on questioning the status quo and are heavily criticized in the reading of statistical arguments. Critical statistical literacy practices are common. Transformative actions are commonly taken from reading and writing statistical arguments.
Emerging Critical	Using formal statistical language, exploratory data analysis, and informal inferential techniques consistently, interpreting and using basic probabilities and the logic of formal inferential statistical techniques to make sense of and communicate statistical information. Questioning of how common variables such as sex, gender, or race are measured based on lived experiences begins to emerge. Furthermore, the limitations of statistical practices for making sense of the world come into question but are not challenged in significant ways. The broader contexts of the issues being explored are considered and communicated but often in unspecific or informal ways.
Consistent Critical Thinking	Using formal statistical language, exploratory data analysis, and informal inference in the reading and writing of statistics is common and consistent. This includes the explicit consideration of variability. The specific context of the issue being explored is considered in conjunction with reading and writing statistics. Different possible interpretations of statistical information are considered or presented, including those that differ from pre-existing ideas. Informal inference takes into account frequentist perspectives of chance to make sense of and communicate statistical information.
Inconsistent	Using multiple pieces of statistical information and using statistical terminology inconsistently in conjunction with colloquial language. The use of chance to make sense of situations and create arguments is still based on intuitive notions, but with some calculated probabilities created or considered. Statistical sense making and communication are focused on signal, not noise, predominantly ignoring variability. The context of the issue explored is acknowledged and considered but inconsistently in the reading and writing of statistics. Statistical information that does not match pre-existing ideas is considered and acknowledged inconsistently and often dismissed or downplayed.
Informal	Using predominantly anecdotal practices in approaches to reading and writing but beginning to use or engage with basic tabular or graphical representations of data or intuitive notions of chance to make sense of information or create arguments. Multiple pieces of statistical information are considered in sense making but information that does not fit personal experience is questioned or disregarded. Interpretations of information still only favor pre-existing ideas. The possibility of the role of context is mentioned but not considered seriously or included in arguments. Terminology is colloquial in nature.
Anecdotal	Using only word of mouth, personal experiences, anecdotal stories, etc. to make sense of information or to present evidence of claims without considering other possibilities or perspectives or the possibility of other perspectives. The use of more than one piece of statistical information or consideration of broader contexts of issues is absent from the sense making or argumentation process. Possibly dismissive of personal experiences of others or statistical information or arguments that do not align with their own experiences.

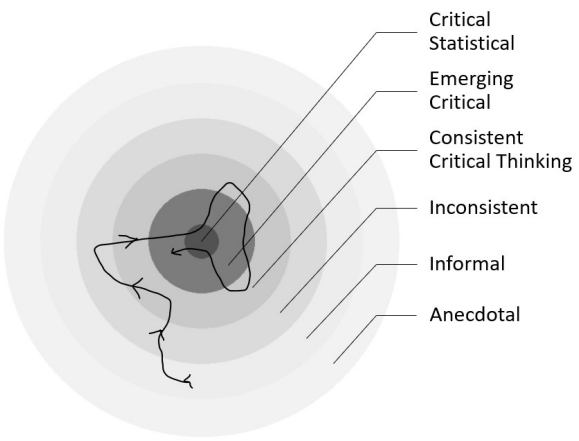

Figure 1. Representation of the process of development of a critical statistical literacy

PROOF OF CONCEPT

This theoretical work is part of a larger design research project (Ben-Zvi et al., 2018; Cobb et al., 2003) with the aim of developing teachers' CSL. We use data from our larger study to show proof of concept. The excerpts we present come from a female, elementary-grades teacher-participant from a graduate class focused on teaching statistics and probability, who consented to participate in the study. The data collected includes her assignments completed during the course. In the original study, we analyzed the participant's coursework to understand the co-development of her knowledge, practice, and identity (Weiland & Sundrani, 2022). Due to space constraints, the examples we provide here are of how we have used the framework to classify practices. The goal of our future work is to provide a more rigorous proof of concept of the development of practices through this framework.

One instance that serves as an example of how we have used the integrated developmental framework is the participant's response to exploring academic opportunity and discipline data at a school of her choosing. She states that "the statistics presented tells me Black students are more likely to have discipline problems than any other race ... non-Black students have a better chance to participate in gifted and talented programs." Further, "the statistics do not show the heart of the school. It's also possible that there could be some teacher bias presented. I also wonder if there are [more] non-Black students in ISS [in-school suspension] than presented. I've seen a lot of students sent to ISS just so they won't have a referral on their record. I also wonder if the students who are suspended or sent to ISS have been there more than once." Through this set of responses, the participant is considering the statistics presented, as well as the context surrounding those statistics. However, the use of colloquial language without formal statistical evidence places the participant at the Inconsistent level within the CSL-DK.

Later in the semester, the participant's assignment responses became more aligned with formal statistical language. In one investigation of polling data associated with parents' views on education amidst the pandemic, the participant was able to name the sampling method used and make conclusions couched in data. In addition, she worked to contextualize the statistics presented from the poll through her own experiences as a parent. She states, "the sample may not be representative of the area it represents. It doesn't take into consideration if the schools are public or private. Those who are in public school will have a different experience than those in private school. I also wonder if parents are able to fill out a survey for each child. I have two children in elementary and their experience, and needs are different for each. It also doesn't take into consideration if there's a stay at home parent." Through this response, the teacher participant recognized the limitations of the data because the results in the article aggregated all cases. This leaves little room for the reader to problematize issues important to parents of students in different types of school environments, grade bands, and other lived experiences. Because of the blending of statistical knowledge and contextual knowledge without a call for transformative actions, this would place the participant at the Emerging Critical level within the CSL-DK.

CONCLUSION

This theoretical work is initial. Our goal here was to hypothesize a new framework for the development of a critical statistical literacy by combining two preexisting frameworks. Though we

provided a proof of concept from a pilot study, an important next step is to consider this framework with a much larger set of data to see if it helps us describe people's development of a critical statistical literacy and to refine it appropriately. Currently, our argument for this new framework is theoretical and focused on differences in the theories of learning employed and end goals of the development. More empirical work needs to be done to show that such differences are observable and necessary. An additional area of research that our new framing of CSL-DK points to and that we see as fruitful is in tracing people's development of statistical literacy over time. We have shown a hypothetical way of considering such development here. We hope other researchers will join us in taking up these areas of work to better understand how individuals codevelop their statistical literacies and critical literacies.

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