® USING CONCEPT MAPS TO ASSESS PRE-SERVICE TEACHERS' UNDERSTANDING OF CONNECTIONS BETWEEN STATISTICAL CONCEPTS

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Concept maps are powerful tools for representing understanding of a concept. After designing a teaching sequence for the statistics content of a senior secondary mathematics syllabus, pre-service teachers were asked to prepare a concept map to demonstrate their understanding of the connection between the different concepts that had been included in the sequence. The concept maps prepared by the pre-service teachers were analysed in relation to what connections were made and the quality of the connecting statements. Results showed that these pre-service teachers had very different perceptions of the connections between the basic statistical concepts. Drawing the concept maps assisted the pre-service teachers to consider the concepts at a meta-level. How the concepts maps might be used as a tool for aiding the planning of learning sequences is worthy of investigation.

INTRODUCTION

The Australian Association of Mathematics Teacher's *Standards for Excellence in Teaching Mathematics in Australian Schools* (AAMT, 2006) identified the need for knowledge of conceptual understanding, and the ability to plan learning sequences to develop students' understanding, as essential to achieve excellence in teaching mathematics. To measure this understanding, assessment tasks for pre-service teachers should allow representation of understanding in an interconnected, meaningful organization. The two most essential indicators of understanding are *explain* and *apply* (Tomlinson & Tighe, 2006). The first of these, *explain*, measured visually as *concept maps*, is the focus of this paper. An assessment task is reported that used an individually constructed hierarchical concept map (cmap) to assess the extent of pre-service teachers' understanding of pedagogical content knowledge. Before the analysis is discussed relevant background in concept mapping and assessing understanding of statistical concepts is provided.

THEORETICAL FRAMEWORK

Ausubel's (2000) theory of meaningful learning, particularly its principle of linking new concepts to existing concepts in cognitive structures, underpins the process of constructing a hierarchical cmap. This linking may occur via *progressive differentiation* (a more general idea subsumes a number of less general ideas) and/or *integrative reconciliation* (merging of many ideas into one). The latter is a more cognitively demanding activity. By this theory, the cognitive structure of a learner is hierarchically organized and facilitates the assimilation and retention of new knowledge (Ausubel, 2000). How students perceive inter-connections between concepts of a topic may be inferred from individually constructed hierarchical cmaps. In contrast to traditional assessment, cmaps are unique because of the visual and hierarchical organization of meaningful interconnections between concepts (Ruiz-Primo, 2004). A cmap is a graph consisting of *nodes* (corresponding to concepts in a domain arranged hierarchically), *connecting lines* (indicating a relationship between the concepts, i.e., nodes), and *linking words* (describing the nature of the interconnections). A *proposition* is the statement formed by reading the triad(s) "$\text{node} \xrightarrow{\text{linking words}} \text{node}$." For pre-service teachers, deep understanding of the pedagogical content knowledge of a topic can be demonstrated through the purposeful organization of concepts into a meaningful hierarchy and explicit linking of interrelated concepts with rich descriptions (linking words) of the conceptual connections (i.e., drawing a cmap).

Numerous studies investigated the usefulness of cmaps as tools to illustrate students' understanding of mathematics topics. Findings from professional practice and research showed that cmaps have potential as teaching and assessment tools (Afamasaga-Fuata'i, 2005).

Investigations of the usefulness of cmaps to illustrate university mathematics students' evolving understanding, found that students' mapped knowledge structure became increasingly complex and integrated as a consequence of multiple iterations of presentation → critique → revisions → presentation (Afamasaga-Fuata'i, 2004). Two pre-service teachers receiving the same instruction on a concept each constructed vastly different cmaps, "one internalised the concept in its systemic interconnections, while the other continued to see it through a formalistic lens" (Schmittau, 2004, p. 576). More recently, research has demonstrated the value of cmaps for pre-service teachers as a pedagogical planning tool to provide an overview of a topic (Afamasaga-Fuata'i, 2006).

ASSESSING UNDERSTANDING OF A STATISTICAL CONCEPT

In statistics, models of cognitive development for various statistical concepts have commonly identified the need for an understanding of a concept before it can be applied. These include distribution (Reading & Reid, 2006), analysing and interpreting data (Jones, Langrall, Mooney & Thornton, 2004) and data handling (Watson, Collis, Callingham & Moritz, 1995). Understanding requires a relational (interconnected) set of links between the elements before increased cognitive activity can occur (e.g. Reading & Reid, 2006, p. 58-62). With increased interest in assessing understanding, recent trends in assessment (Garfield & Chance, 2000) include individual and group projects, case studies, authentic tasks, portfolios of work, critiques of statistical ideas or issues in the news, minute papers, and cmaps.

Cmaps have been recognised as important for visually organising course content to aid student understanding (e.g., Icaza, Bravo, Guinez & Munoz, 2006; Bulmer, 2002). However, the use of cmaps to assess understanding of statistical concepts has been limited. Verbalising while completing cmaps demonstrated tertiary students' poorly expressed understanding of relationships between significance level and other concepts (Williams, 1998), but the use of cmaps was not evaluated. Cmap representations provided Schau and Matten (1997) with useful information about what students did not understand and to assist scoring they used *select-and-fill-in type* cmaps (pre-drawn with some nodes and linking descriptions removed) rather than originally-drawn cmaps. The former were useful for assessing pre-specified understanding of the concept but not for investigating unique qualities of the understanding. This paper investigates the question: What is the nature of interconnectedness of pre-service teachers' understanding of statistical concepts as evident from their individually constructed concept map? This information was needed to improve assessment rubric criteria and to identify good and poor cmapping practice to provide exemplars for future cohorts.

CONTEXT

At a regional Australian university, the pre-service teachers destined to deliver the content of the *Stage 6 Syllabus General Mathematics* (NSWBOS, 2002), were taught to design classroom teaching sequences introducing concepts developmentally. They were introduced to cmaps and provided with cmapping activities and professional reading on cmaps. Assessment for the course involved five assignments. The second included the design of a teaching sequence for the topic "Statistics" and construction of a comprehensive concept map to illustrate a hierarchical network of interconnections between the listed concepts for the whole topic (i.e., *cmap task*). The topic "Statistics" (NSWBOS, 2002, pp. 24-31, 58-63) has seven sequentially-organised subtopics. Each includes a list of concepts. The cmap task (25% of the assignment grade) is the focus of this paper.

This task was innovative in its use of a cmap to allow pre-service teachers to illustrate their understanding of statistical concepts. Such use of cmaps must include (i) a *task* that allows a student to provide evidence of his/her understanding, (ii) a *format* for the response and (iii) a *scoring system* to evaluate the cmap (Ruiz-Primo, 2004). For the pre-service teachers, the *cmap task* required construction of "a comprehensive topic cmap to include all listed concepts hierarchically organised into an interconnected network, including meaningful descriptions of the relationship between connecting nodes and illustrative examples for the more specific concepts, preferably towards the bottom." The *format* (type) of cmap required was *construct-map (concepts provided)* (Ruiz-Primo, 2004). The *scoring system* was an *assessment rubric* (summarised in figure 1), adapted from Novak's scoring scheme (Novak & Gowin, 1984). Also in figure 1 are the weightings (contribution to the overall cmap score) and the ratings. The cmaps were graded by the lecturer for formal assessment in the course. These were the pre-service teachers' first assessed cmaps. The following analysis of the cmaps was designed to inform the development of a richer assessment rubric and identify exemplars for the pre-service teachers.

Course-criteria	*Description*	*Weighting (%)*	*Rating*
CC1	Inclusion of given concepts plus any additional relevant concepts	56.5	all, most, some, few or none
CC2	Ranking of concepts from most general to most specific	8	all, most, some, few or none
CC3	Inclusion of linking words on lines to describe meaningful relationships	8	all, most, some, few or none
CC4	Inclusion of appropriate illustrative examples	8	level of necessity and appropriateness
CC5	Structural complexity	6.5	continuum from complex to linear
CC6	Multiple branches - nodes with multiple outgoing/incoming links	6.5	number of occurrences
CC7	At least four hierarchical levels	6.5	continuum from less than four to more

Figure 1. Course-criteria (CC) assessment rubric for the cmap task

METHODOLOGY

The already-assessed cmaps were analysed quantitatively and qualitatively to develop a richer view of the pre-service teachers' understanding. The quantitative analysis included a regrading of the cmaps using revised criteria and a comparison with performance on the course-criteria. The qualitative analysis included a synthesis of crucial visual features to enrich the view of understanding.

Research-criteria	*Description*	*Weighting (%)*	*Rating*
RC1	*Content* - inclusion of the given concepts	16.6	3 = majority, 2 = some, 1 = few
RC2	*Hierarchy* – concepts organised from more general towards the top to less general towards the bottom	16.6	3 = majority, 2 = some, 1 = few
RC3	*Links/Node* – computed average number of links per node	16.6	3 = more than 1.2, 2 = from 1.2 to 1.0, 1 = less than 1
RC4	*Branching* – count of nodes with greater than 2 outgoing links	16.6	3 = more than 7, 2 = from 7 to 5, 1 = less than 5
RC5	*Merging* – count of nodes with greater than 1 ingoing links	16.6	3 = more than 6, 2 = from 6 to 4, 1 = less than 4
RC6	*Linking Words* – quality of linking words	16.6	3 = mostly deep, 2 = mixture, 1 = mostly superficial

Figure 2. Research-criteria (RC) assessment rubric for the cmap task

All twenty-one pre-service teachers enrolled in the course were invited to make their cmaps available for analysis. One of the two researchers (the authors) was the course lecturer responsible for assessing the cmaps using the course-criteria. The six research-criteria (figure 2) were developed to achieve reduction of emphasis on content, more precise measurement of interconnectedness, and distinction between superficial and deep linking words. Thus more emphasis was placed on the indicators of rich and meaningful interconnections, which are more reflective of a deep and connected understanding. The weighting and rating for each criterion are included in figure 2. All rating scales were 'countable' except for RC6, which used two levels of linking word quality: 'superficial' (for *membership of*, *example* or *belonging to*) and 'deep' (for *nature of the connection at a working level*, e.g., explaining exactly how one concept is used for, or contributes towards, another concept). The two researchers applied a consistent procedure to individually rate each criterion, compared ratings, and negotiated disputed ratings. Qualitatively, each cmap was analysed to determine examples of good and poor representations.

RESULTS

Of the 21 pre-service teachers enrolled, two withdrew, two submitted late and two did not consent, hence 15 cmaps were analysed. First the results of the quantitative analysis are presented, then the qualitative. The number of pre-service teachers achieving each rating level for the six research-criteria (table 1) shows that at least 40% attained the preferred rating (3) on all criteria except *Branching* and *Merging*. However, *Merging* and *Linking words* had more than 30% in the least preferred rating (1). These concentrations of undesirable ratings were for the criteria that best measured the interconnectedness of the understanding.

Table 1
Number of pre-service teachers (n=15) at each rating level for the research-criteria

Criteria/Rating	*3*	*2*	*1*
RC1	8 (53%)	6 (40%)	1 (7%)
RC2	7 (47%)	5 (33%)	3 (20%)
RC3	6 (40%)	7 (47%)	2 (13%)
RC4	3 (20%)	9 (60%)	3 (20%)
RC5	4 (27%)	5 (33%)	6 (40%)
RC6	6 (40%)	4 (27%)	5 (33%)

The total scores (out of 18) across the six research-criteria for each pre-service teacher (PST) are shown in table 2, along with the course-criteria scores and rankings for each. The correlation coefficient (0.73, $p < 0.01$) between the course-criteria and research-criteria scores indicates that there is a strong association. Thus cmaps that scored well on the course-criteria still scored well on the research-criteria, despite the reduced weighting of the content criterion. Comparing rankings, cmaps PST03 and PST06 were scored as better (shifted into the top four) while PST04 and PST01 dropped rankings. The cmap PST07 was ranked as poorest whereas PST015 improved in ranking. Features that characterised the varying-quality cmaps follow. Note that some reproduced concept maps have been reduced and are intended mainly to show structure (e.g., figure 6), hence individual words may not be readable.

Good Concept Maps

Cmap PST05 (figure 3), consistently ranked best, demonstrates a good structural organization, an effective positioning of illustrative examples at the bottom and good branching and merging (examples identified in figure 3). One clear proposition is "*Data Analysis* is a *mathematical process*" (P1 in figure 4). Despite its high rating this cmap, along with only one other, did not follow the convention of enclosing concepts within geometric shapes to distinguish them from linking words. Despite this, most nodes were identifiable, but one was problematic (figure 4) because it contains both more than one concept (*measures of centre*, *dispersion*) and linking words (the relationship between, and). This produces the poorly constructed proposition: "*Analysis* of *the relationship between measures of centre and dispersion*" (P2). A node should only contain one concept and no linking words.

Table 2
Concept map scores and rankings based on the course-criteria and research-criteria

PST	*Course-criteria score (out of 25)*	*Research-criteria score (out of 18)*	*Course-criteria score (ranking)*	*Research-criteria score (ranking)*
05	24.0	16	1	=1
04	23.6	14	2	=5
01	20.4	14	3	=5
09	20.0	16	4	=1
02	18.8	12	5	=9
06	18.4	16	6	=1
03	17.6	16	7	=1
08	15.6	12	=8	=9
13	15.6	14	=8	=5
07	15.2	9	=10	15
10	15.2	14	=10	=5
16	12.8	10	12	=12
17	10.8	10	13	=12
11	10.0	10	14	=12
15	9.2	11	15	11
Key		better (top four)		poorer (bottom four)

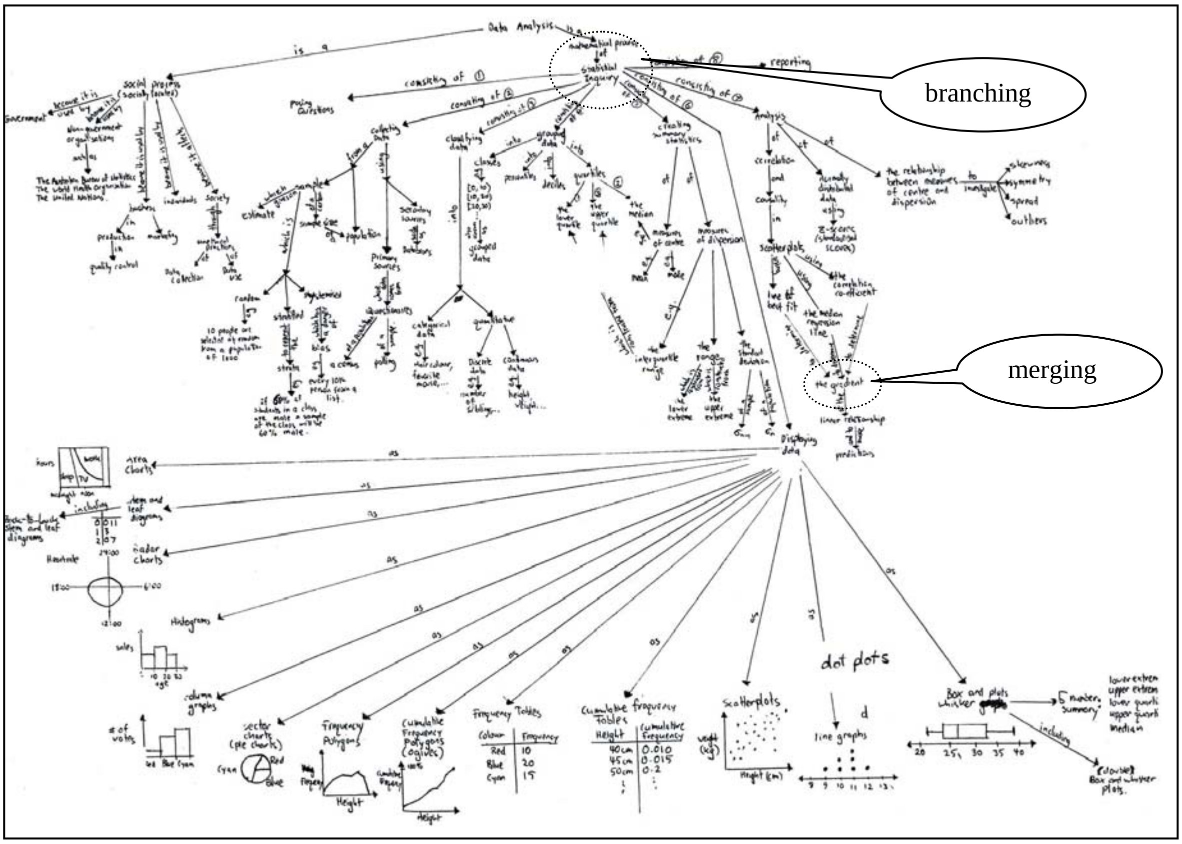

Figure 3. Best cmap PST05

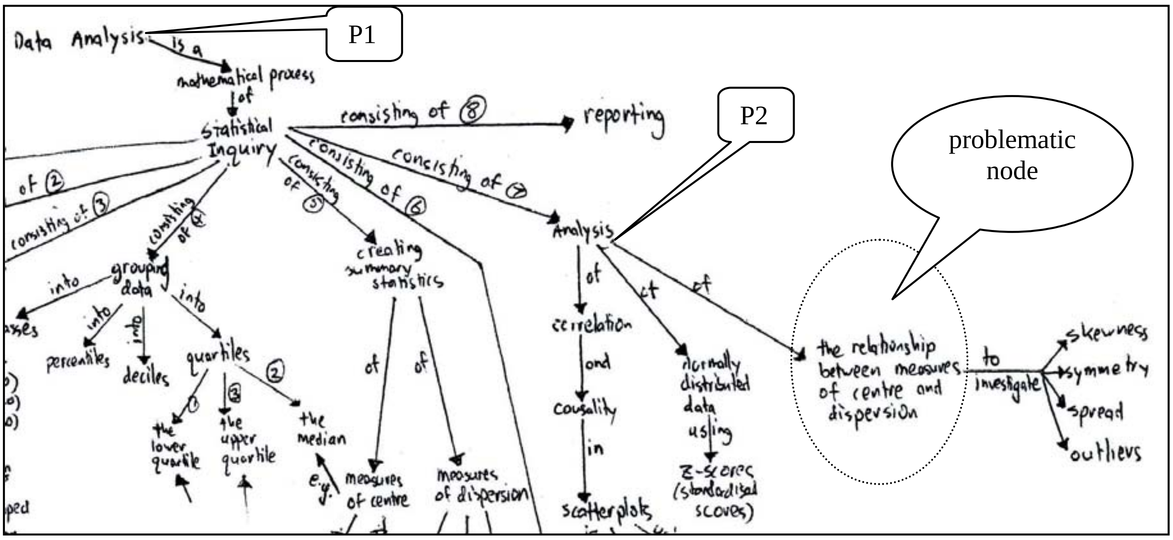

Figure 4. Part of cmap PST05 showing propositions and a problematic node

Another good cmap PST09 (partially shown in figure 5), consistently ranked in the top four, was rated highly for content, number of links per node, branching and merging. However, the hierarchical organization and quality of linking words was only rated as a two. One proposition that demonstrates deeper meaning is "*Two sets of Data* for which we calculate *correlation coefficient* which show *correlation* but not necessarily any *causality*" (P3). This cmap includes an example of a confusing combination of linking words and nodes (L1). The mapped propositions are "*Scatterplots* for which we can draw a *a line of best fit*" (L1a) and "*Scatterplots* for which we can draw a or *median regression line*" (L1b). The linking word "a" is redundant in L1a and "or" is unnecessary in L1b.

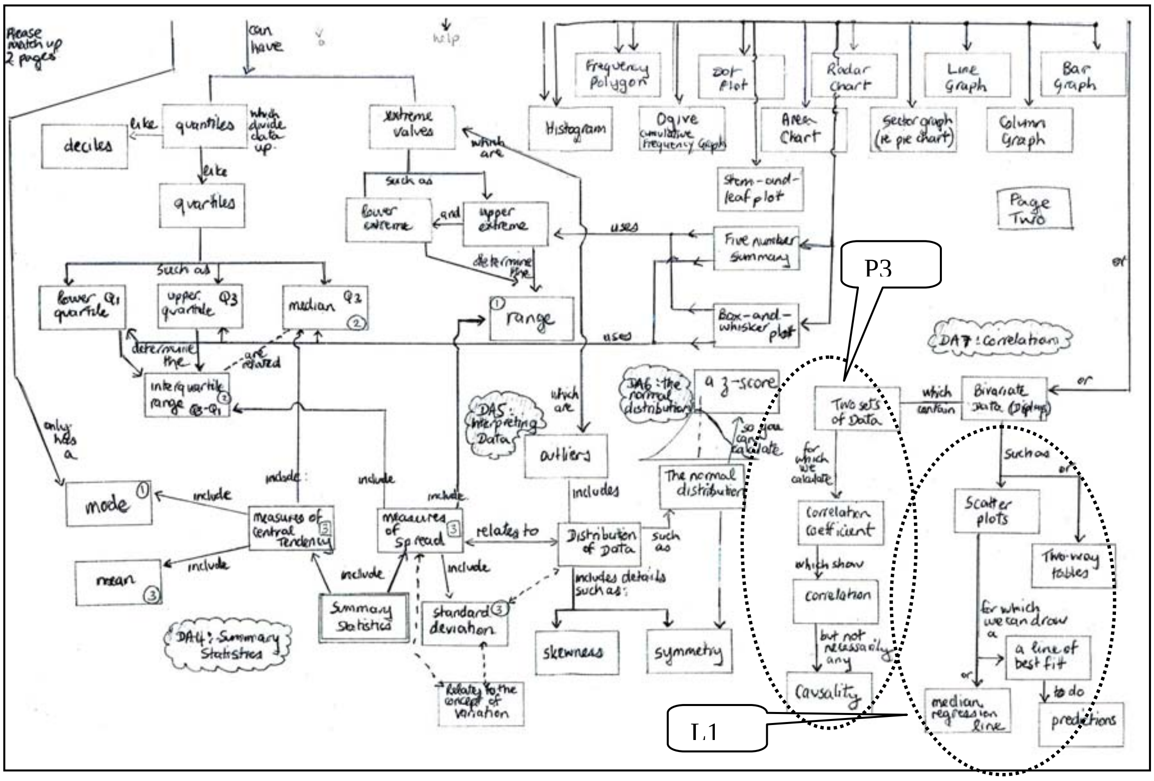

Figure 5. Lower half of good cmap PST09

Poor Concept Maps

Cmap PST11 (figure 6) had low ratings due to many of the given-concepts missing, no linking words, and no merging. This cmap however does show good branching. Cmap PST17 (figure 7) has a good overall hierarchical structure and some branching and merging. The poor quality of linking words and more than half the given-concepts missing, consistently placed the cmaps in the lowest rankings. One superficial proposition is "*Data Collection* can be *Census*" (P4). Proposition "*Quantitative* is either *Discrete*" (P5) does not form a correct mathematical statement. Despite many poor propositions, there was a deep proposition: "*Interpreting* through *Summary*" (P6).

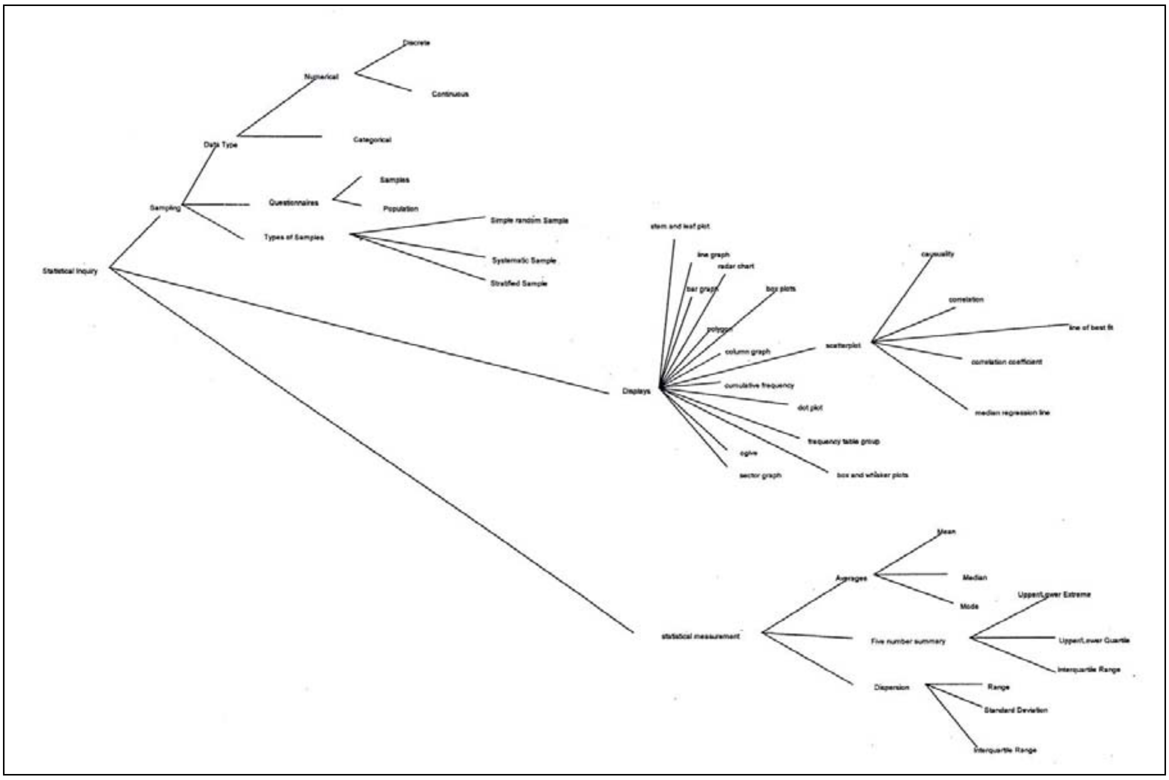

Figure 6. Poor cmap PST11

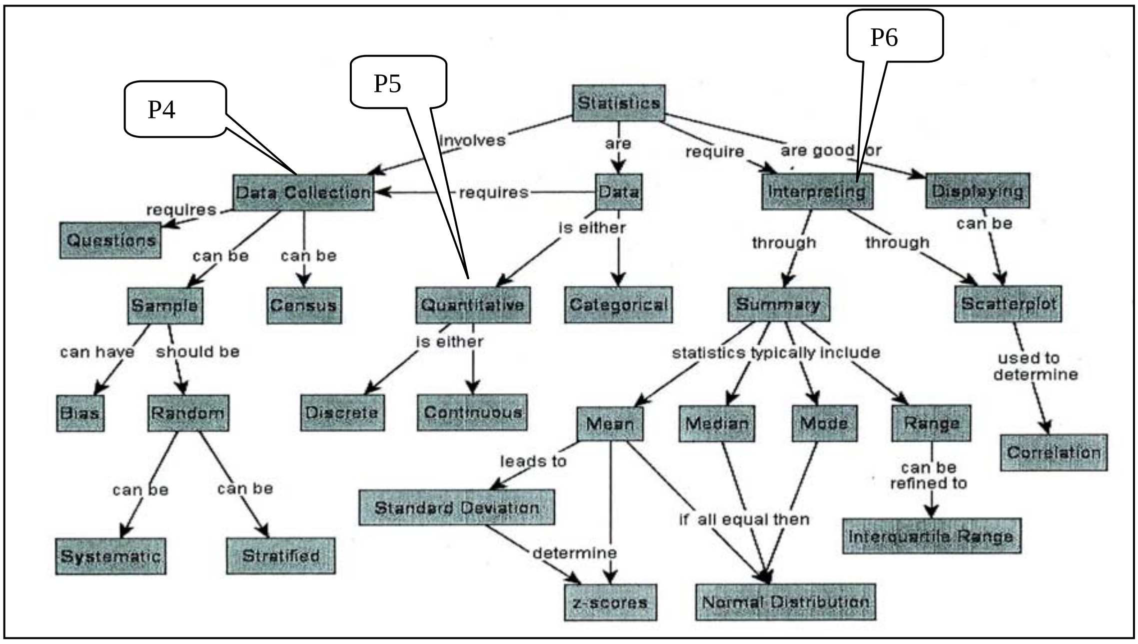

Figure 7. Poor cmap PST17

Visually confusing cmaps

Cmap PST10 (partially reproduced in figure 8) has indistinguishable hierarchical levels and free flowing links that complicate, not facilitate, the tracing of propositions through to the end nodes. This is further complicated by the inclusion of illustrative examples that are not effectively aligned. Cmap PST15 (figure 9) has no hierarchical structure, with concepts and connections radiating out from a central concept. This cmap had low ratings on content and merging but achieved a rating of two for branching and quality of linking words.

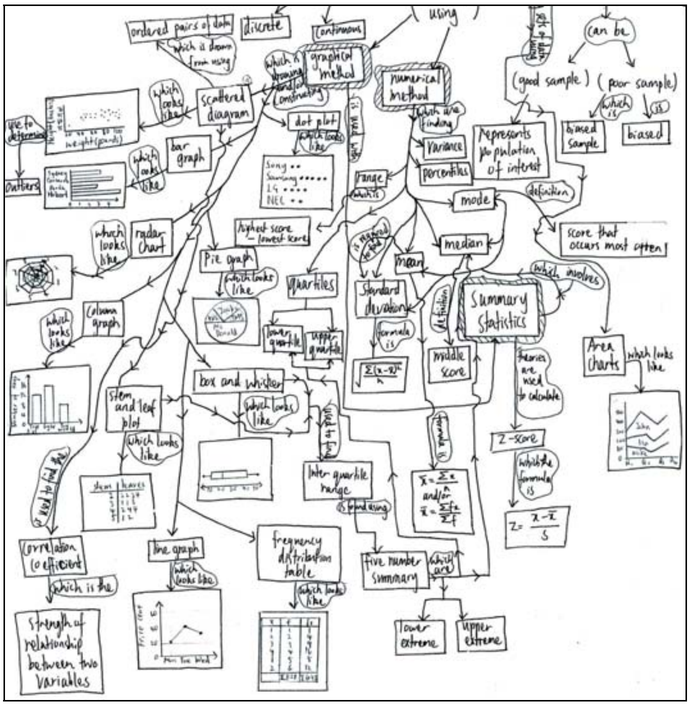

Figure 8. Part of visually confusing cmap PST10

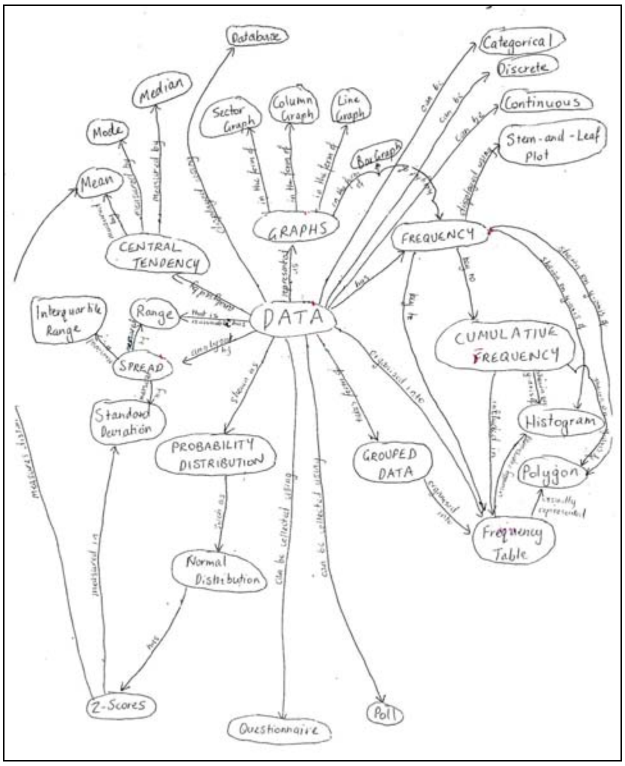

Figure 9. Visually confusing cmap PST15

DISCUSSION

This research investigated the use of cmaps to assess pre-service teachers relational (deep) understanding, which is necessary since "a teacher's instructional actions at any moment are … influenced ... by what the teacher understands about what (he/she) is teaching" (Thompson & Saldanha, 2003, p. 96). A deep understanding of statistical concepts, as proposed by the authors, should be indicated by rich and meaningful (a) concepts (*content*) appropriately incorporated, (b) hierarchical organization of interconnecting concepts (*hierarchy*, *links/node*, *branching*, *merging*) and (c) description of interconnectedness (*linking words*). The research-criteria increased the emphasis placed on (b) and (c) compared to the course-criteria.

The variety of cmaps indicates that the pre-service teachers' understanding of concepts is interconnected in very different ways. Although more than half of them included the majority of the concepts (*content*), the low ratings for some pre-service teachers indicate that they struggled to manage, and meaningfully include, all concepts as nodes. This reduced the amount of understanding that could be demonstrated. More important than content, for demonstrating understanding, was the positioning of a concept within a *hierarchy* to illustrate relative generality. While many cmaps demonstrated "more general" and "less general", there was little indication of the notion of "equal generality". The latter indicates lack of recognition of equal status concepts.

Deep understanding requires an interconnectedness of concepts, which is better indicated by *links/node* than an absolute count of the links. Those cmaps with more links per node suggested that the majority of pre-service teachers tended to look for interconnections. A deeper view of interconnectedness is provided by *branching (progressive differentiation)* and, the more cognitively demanding, *merging (integrative reconciliation)*. Very few cmaps demonstrated high ratings for both these cognitive processes. The shift of rankings with the good cmaps was mainly due to differences in the *links/node* and *branching*. Similarly with the poor cmaps, which suggests that these criteria are discriminating well between rich and poor knowledge of interconnections. Criteria so far discussed were focussed primarily on the structural complexity, but the criterion most reflective of understanding, is the expression of interconnections (quality of their *linking words*). Low ratings on this criteria for one third of the cmaps indicated that pre-service teachers either did not understand the interconnections, or found it difficult to express their understanding. The variety of representations created by these pre-service teachers, from the same list of terms, is indicative of their very different understandings of the interconnectedness of statistical concepts. This is consistent with Schmittau's (2004) findings when two pre-service teachers produced very different concept maps despite similar contexts. To better prepare pre-service teachers for teaching concepts to their students, there is a need to support them in reflecting on how their mapped interconnections could be best described.

IMPLICATIONS

The research-criteria should be used as a revised assessment rubric. In addition, another criterion should be included to assess the validity of mapped propositions (i.e., " $\text{node} \xrightarrow{\text{linking words}} \text{node}$) in terms of their grammatical and mathematical correctness. This would encourage the pre-service teachers to describe the mapped interconnections more accurately using rich and meaningful complete statements. To assist the pre-service teachers in interpreting the rubric, two important points should be stressed. First, the ranking procedure should be clarified, clearly explaining that concepts should be aligned to indicate equivalent generality levels (horizontal alignment) as well ordered in terms of more or less general (vertical arrangement). Second, the process of linking nodes should be encouraged beyond single connections to include *progressive differentiation* between, and *integrative reconciliation* across, concepts. The revised rubric provides a good basis for educators who wish to assess understanding by using cmaps. Now researchers need to investigate the reliability and validity of using this revised rubric. By constructing hierarchical cmaps, pre-service teachers have the opportunity to critically reflect on their own understanding thus developing a connected view of the syllabus content. The power of the cmap is not just in constructing a cmap for assessment purposes but in being able to use it as part of the learning process. Thus research is also necessary to determine whether constructing cmaps is more beneficial before or after the development of teaching sequences.

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