MaDiN – Teaching School Mathematics using the web

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Abstract: This report focuses on MaDiN, a project which is supported by the German Federal Ministry of Education and Research (bmb+f) in the programme "*Neue Medien in der Bildung*" (New Media in Education). The aim of this programme is to conceptualize new ways of learning and teaching with modern information and communication technologies. This line of research is realized in MaDiN (for: teaching school-mathematics using the web). The development of this multi-media and internet based learning and teaching environment, which includes most areas of school-mathematics, will be discussed here.

1. Introduction

Individually designed, self-determined and time-independent learning is essential to the conceptualization of new ways of learning and teaching with modern information and communication technologies. This line of research is realized in the programme "*Neue Medien in der Bildung*" *(New Media in Education)* supported by the German Federal Ministry of Education and Research (see http://www.gmd.de/PT-NMB/). One of these projects is *MADIN* (for: teaching school-mathematics using the web), which will be developed in collaboration with the universities Münster, Würzburg, Nürnberg/Erlangen and Braunschweig.[1] The main goal of this project is to provide a virtual environment for learning and teaching mathematics. This environment aims at students training to be teachers as well as at young teachers. It contains a modular-designed network of knowledge about all areas of school-mathematics.

In this report the goals and the conception of MaDiN will be discussed (see also Ernst/Niehaus/Stein 2001 and Eichler 2001 and 2002). The conception is based on a synthesis of theories of web-learning, mathematics education and general theories of learning. An evaluation study of using MaDiN in university lectures also influenced the concept. To illustrate the concept some examples of the module *didactics of stochastics* are presented.

2. Learning of mathematics – multi-media and web based

2.1. Main goals and principles of an environment of learning and teaching

[1] This report relates to the project „*Entwicklung einer dezentralen internetunterstützten Lehr-Lernumgebung für das Lehramtsstudium Mathematik*" supported by the bmb+f, working group Braunschweig (project manager: Prof. Dr. Uwe-Peter Tietze). This project is termed from 1.1.2001 to 31.12.2003.

Summarizing Dörr and Strittmatter (2002, 31) these are the following standards of multi-media based learning-environments. They should:

1. offer authentic exercises close to daily experiences which will motivate users and start the process of learning;
2. have an adequate presentation of the learning material.
3. include problem-solving and the possibility to construct knowledge in contrast to passive learning.
4. enable users to develop their own ways of learning in terms of time and information selection.

In appreciating these standards and specific features of the didactics of stochastics in contrast to other disciplines of school-mathematics there are the following main goals and principles for the development of MaDiN and especially the module *didactics of stochastics*:

- Problem-solving has evolved as the most essential demand on the German didactics for successful learning in school and university. This is especially the case after TIMMS and PISA.[2] Thus, problem-solving according to the learning theory of Bruner (see Stracka/Macke 1979) will be the most important approach (termed *problem-oriented approach* by the author) to the module *didactics of stochastics*. This approach should offer users access to the central ideas of a subject and open them a view to their own process of problem-solving.
- As problem-solving needs a foundation of knowledge (see Polya 1967, 146) and as even most students and teachers have only limited experience with stochastics (see Eichler 2002) there must be an adequate integrated mathematical and didactical foundation of knowledge, too. This foundation of knowledge is termed *theory-oriented approach* and accords to Ausubels theory of learning (see Stracka/Macke 1979).
- Due to the inhomogeneous experience of different students all approaches should offer different ways and depths to deal with a subject. For the development of a multi-media based learning environment Strzebkowski/Kleeberg (2002) define the interactive control as a main principle. This control will enable users to learn in their individual way. Beside this, the development pays attention to usability. Thus, Schnotz (2002) emphasizes the splitting of information into *topic* and *comment* so that information of each full-site can be reduced and can be viewed without scrolling (see also Nielsen 2001).
- Strzebkowski/Kleeberg (2002) define *didactical interactivity* as the core of multi-media based learning environments, which allows users to actively discover concepts or subjects. For this reason, it is necessary for the didactical interactivities to be summed up in a third *example-oriented approach*. Finally, this offers university teachers a source of material that can be integrated into lectures.

[2] See the statements of the main authorities of German didactics organized as *Gesellschaft für Didaktik der Mathematik* (GDM), *Verein zur Förderung mathematischen und naturwissenschaftlichen Unterrichts* (MNU) and *Deutsche Mathematiker-Vereinigung* (DMV), DMV/GDM/MNU 1998, DMV/GDM 2001 and GDM 2001.

- All approaches have to be close to reality. To convey statistics real data sets should be employed (see Pfannkuch/Wild 1999), since only real data can show advantages or limitations of statistical methods.
- The module should encourage activity. This means to discover concepts by interactive elements, search for data in the internet or acquire experience with useful software like Excel or Mathcad.[3] Furthermore, the module should raise students' acceptance of computers as an important medium for school-mathematics and as a learning-tool.

2.2. MaDiN: Conception of the surface

Basic structural element is a virtual desk (see fig.1), which covers a specific subject within stochastics education.

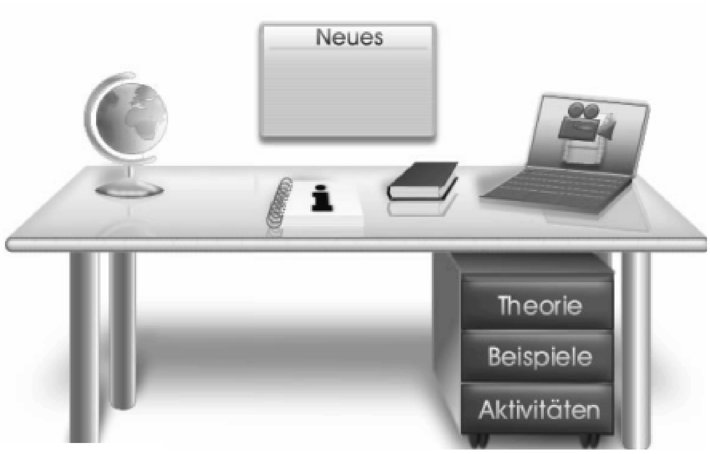

Figure 1: The MaDiN-desk

A network of desks constitutes the module *didactics of stochastics* in a tree-like structure that includes many links (on the base of html) between his branches (see fig.2). Finally, this module is part of MaDiN with its several modules representing most areas of school-mathematics.

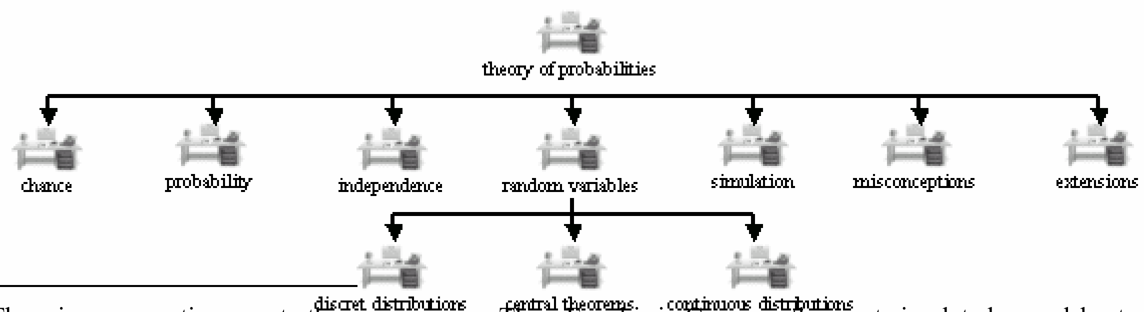

Figure 2: Tree-structure of the module theory of probabilities

[3] There is a pragmatic cause to this programmes. Thus Excel is in Germany the most circulated spreadsheet while for Mathcad it is possible to link Excel with a powerful and analytic software. A software like Fathom may be more useful but is currently not circulated (see Biehler/Kombrink 2002).

The desk itself is built up of desktop-areas and drawers as second structural elements. These structural elements will offer three different ways to discover a subject. In detail:

- the drawer *Aktivitäten* (= *activities*) contains complex problems besides tests and simple exercises. Here, a problem is defined as a real situation or a situation close to reality. Problems should include the main ideas of a subject and should be sufficiently complex to provide the possibility of several mathematical models and prevent intuitive solutions. Thus this drawer is the center of the problem-oriented approach, which is based on the theory of learning by Bruner (see Stracka/Macke 1979);
- the drawer *Theorie* (= *theory*) is the center of the theory-oriented approach. It will offer users two pages, containing on the one hand didactics and on the other hand mathematical theory. In respect of Ausubels theory of learning, the *Überblick* (= *overview*, the "i" in the center of the desktop) represents an *advanced organizer* whereas in the drawer named *theory* the *progressive differentiation* takes place. The *consolidation* is established in the drawer *Beispiele* (= *examples*) including examples of newspapers and magazines and furthermore mathematical deepenings or exercises with extra software like Mathcad. In this approach the drawer named *activity* represents *exercises*;
- in context of the example-oriented approach, the drawer *examples* contains didactical material like pages of schoolbooks or excerpts of teacher-interviews. Furthermore, it includes *didactical interactivities* (see section 2.1). A list of these interactivities is linked to the "notebook" on the desktop. Further lists can be reached via the "book" and the "globe" on the desktop containing literature and links to the World Wide Web.

Beside these structural elements there are tools for integrating MaDiN into the organisation and practice of university lectures:

- It is possible to administer a lecture online and provide several lectures for several user-groups.
- To accompany each lecture or user-group, a forum exists for the communication between students or university teacher and students.
- With a recorder-tool the university teacher is able to prepare series of bookmarked pages, which represent ongoing lectures or which are important for a preparation or subsequent study of these lectures.
- An author-tool makes MaDiN dynamic in that way users can develop and integrate their own sites and desks into MaDiN (into a separate module).

3. The module didactics of stochastics

The present state of MaDiN and especially the module *didactics of stochastics* considers both the theoretical issues and the results of the *formative* evaluation. While the results of the evaluations are implicit included in the following discussion, their design and their results will be pointed out after the discussion of the different approaches of the module *didactics of stochastics*.

3.1. The problem-oriented approach

For two sections of the module *didactics of stochastics* the structure of problems will be specified. For problems related to data analysis there were two appropriate classes of problems identified:

1. A set of data is given. An exploration by flexible use of data analysis methods results in descriptive and - if carefully applied - in prescriptive statistical statements about a real situation. Examples of this class of problems (P1) are the study of the weather or the eruption of Old Faithful (see Nordmeier 1993 and Shaughnessy/Pfannkuch 2002).
2. Statistical statements are omnipresent in newspapers or other media. Most of the statements are published without the underlying data. Here, a class of problems (P2) can be the reconstruction of missing data (see for an example Eichler 2001).

For the two classes of problems[4] the structure of problem-solving is shown below:

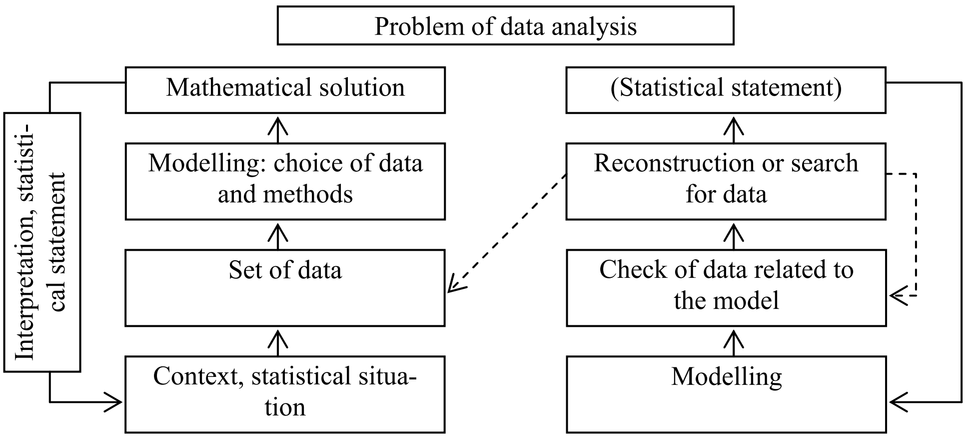

Figure 3: Structure of the process of problem-solving in data analysis

In his theory of stochastic thinking Bea (1995) pointed out three classes of problems, *recognizing stochastic situations*, estimating *subjective probabilities* and *computing objective probabilities*. The classes of problems of MaDiN include operating with subjective and objective probabilities. A simple model of the possible steps of problem-solving to each problem is shown below[5]:

[4] See to this aspect also the reports of Moore 1997 and Pfannkuch/Wild 1999 related to the term of statistical thinking.
[5] It is not always necessary to use each step of problem-solving to each stochastically problem as not all steps are always applicable.

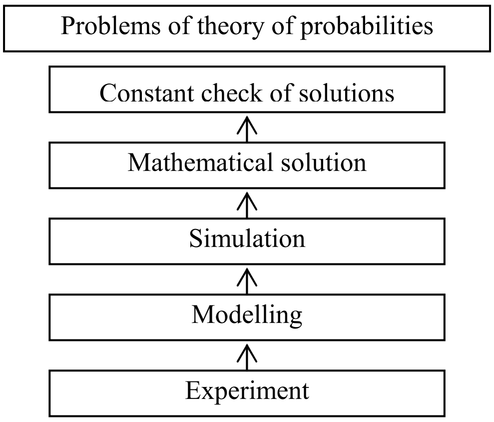

Figure 4: Structure of the process of problem-solving in theory of probabilities

A set of heuristics for any mathematical problem and its classifications is pointed out by Polya (1967) or Tietze (2000). There are a few heuristics that are especially related to statistical or stochastical problems. For example visualization or clustering are essential heuristics for problems of data analysis (see Tukey et al. 1982) as experimentation or simulation can be used for problems of probability theory. Any given heuristic must be able to help the problem-solver cope with barriers in the process of problem-solving. And also every given heuristic will be classified in a mode of meta-cognition into the several steps of problem-solving. The problem-oriented approach has the following structure:

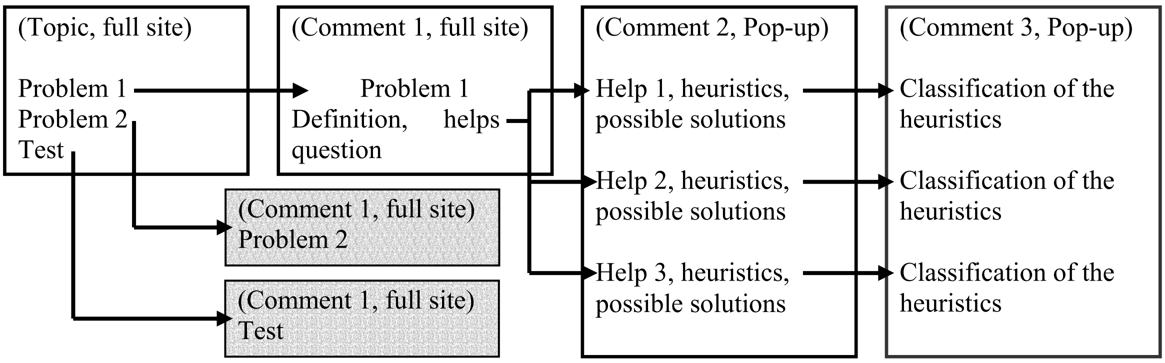

Figure 5: Structure of the problem-oriented approach

While the focus is on the heuristics, suggestions of solutions or mathematical definitions and methods (as link into the theory drawer) were also included.

3.2. The theory-oriented approach

The conceptions of the theory-oriented approach are twofold, using the principles of Ausubels theory of learning and recognizing the division of the pages into topic and comment (see section 2.1.). As pointed out in section 2.2. the theory-oriented approach is realized in most of the drawers and desk areas. But the focus of attention here lies on the theory drawer, where the progressive differentiation (sensu Ausubel) takes place. Taking into account the completeness of didactical and mathematical theory (see section 4) the division of information into topics and several comments is particularly important. Firstly, there are two full pages of topics, one of didactical and one of mathematical theory in the theory drawer. The page containing the topics is divided into two parts, one part text and one part thumbnails. Here, one comment containing explanations such as definitions or formulas is linked to the text. Another comment containing an example out of or close to reality is linked to a variable designed thumbnail and furthermore a last comment including an excursus is linked to a thumbnail of a uniformly designed excursus-button (see fig.6).

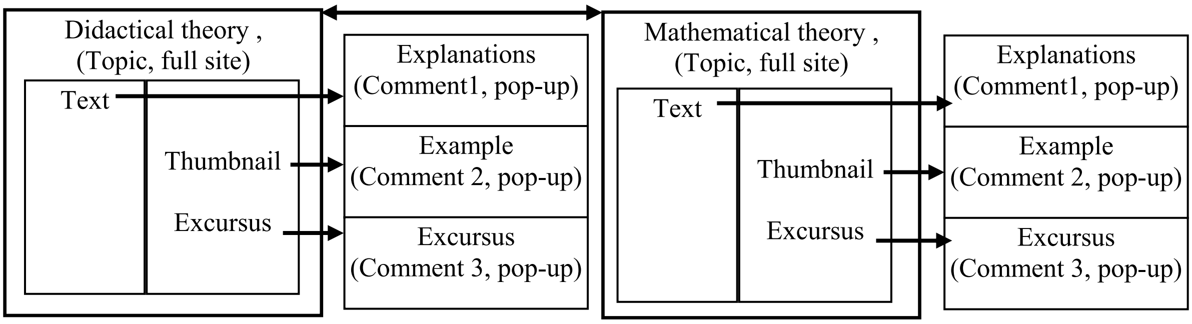

Figure 6: Structure of the theory

3.3. The example-oriented approach

The example-oriented approach is not separable from the other approaches. It includes on the one hand elements, which will be used by a university teacher in lectures. On the other hand part of its elements - especially the didactic interactivities - are integrated into the theory discussion and also into problem-solving. As pointed out above they will visualize the central ideas of a desk subject. For all desks central ideas are connected with one interactive element, which visualizes this idea in several situations and matters under flexible modification of the element.

For example the desk named *discrete distributions* includes the central idea of the importance of parameters, which describe the shape of several distributions. The model-style of these theoretical distributions is another central idea. Thus, the interactive elements visualize a distribution like the binomial distribution once as theoretical distribution, where parameters are changeable, and once as comparison between the theoretical distribution and simulations based on the settings of parameters (see fig.6).

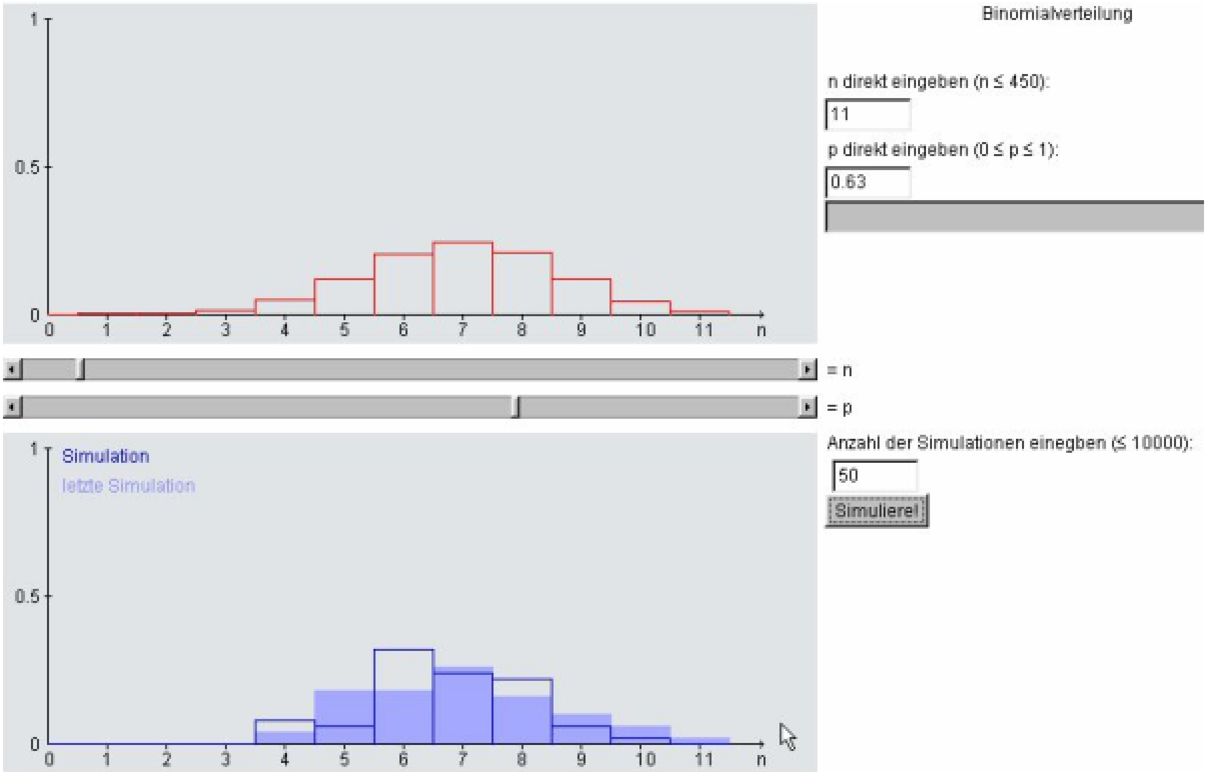

Figure 7: Didactical interactivity

Another goal of interactive elements is to support learning in its critical phases. For example misconceptions in employing conditional probabilities are often discussed (see Bea 1995). Two strategies to overcome these misconceptions seem to be the visualization of conditional probabilities with the standardized square and the transformation of probabilities to absolute frequencies (see Gigerenzer 2002). Both are integrated interactive elements of the desk.

The visualization of stochastic models limitations is a third goal using interactive elements. Therefore it may be possible to show that stochastical and especially statistical methods are only heuristics for solving stochastical problems and that results must be interpreted in context of the specific situation. Here, interactive elements are able to show the difference between several methods and where their advantages or limitations come from. One example is the integration of two methods of regression – minimizing the absolute and the square residues – into one interactive element that enables users to change the given set of data.

Finally, interactive elements should be self-explanatory and encourage and develop user interest.

4. Evaluation of MaDiN and its results

"*For the educational technology field, evaluation was now being viewed as an integral and ongoing part of the instructional development process.*" (Ross/Morrison, see Fricke 1995)

Evaluation is an integral part of the development of MaDiN, too. The definition used to limit the term evaluation is based on the reports of Götz (1999) and Tergan (2000). Here, evaluation is defined as ongoing investigation of the quality of educational courses – especially a multi-media and internet based learning and teaching environment – during the steps of planning, development and practice. As seen above, the definition of the potential users, the conception of the structure-elements and the surface of MaDiN is determined by theoretical considerations and is not subject of empirical evaluation. The development in detail is mostly shaped by the results of the *formative evaluations*.[6]

The *formative evaluation* was carried out in regular university lectures, thus students evaluated MaDiN as experts (see Mandl 2000, 89). The main questions of inquiry considering the quality or functionality of the three approaches described above, the student acceptance of the modules and in the broadest sense the effectiveness of learning with MaDiN. Table 1 provides an overview of the location of field testing and the main approaches used:

Table 1: Settings of evaluation

Module	Place	Design of practice	Main approach
Didactics of data analysis	Didactical seminar for advanced students	Self-learning	Theory- und problem-oriented approach
Didactics of analytic geometry	Didactical seminar for elder students	Preparation of lessons, subsequent study of the lecture	All approaches
First lecture of analysis to all students in mathematics	Attendant lecture for students of educational sciences	Preparation of a mathematical talk and subsequent study of the lecture	Theory- and example-oriented approach
Didactics of theory of probabilities	Didactical seminar	Employment into the lecture for demonstration	Example-oriented approach

All results discussed here are based on qualitative studies as essential part of the evaluation. The methods[7] of these studies are:

- questionnaires before using MaDiN on students knowledge and experience of employing computers and students attitude to this employment;
- observations while students used MaDiN;
- written statements about MaDiN as learning environment;
- semi-structured interviews while and especially after using MaDiN.

[6] *Formative evaluation* is defined as proof of quality during the development-process (see Götz 1999). So-called *summative evaluation* would be done in a wide practice situation of MaDiN.
[7] For the methods of evaluation see Friedrich 1997 or Fricke 2000 and for the settings of qualitative studies see Lamnek 1993.

Summarizing the large body of results there are three main results considering content, structure of content and interactivity.

Students need both didactical and mathematical contents. As much as mathematical theory without its integration into didactics is not useful, students reject attempts to deal with didactics without specific mathematical underpinnings. Students did not criticize pages or desks, which include didactical and mathematical contents representing each other. Since the students' knowledge of stochastics is extremely inhomogeneous several ways of receiving information in several stages of depth is mandatory.

The content structure is essential to a learning environment. Especially for complex systems of information like MaDiN users should be able to identify their location within the module, desk or page at every stage. Students criticize developments that make them feel *lost in hyperspace*. This impression becomes stronger as the series of links grows or as the programme offers users various *horizontal* links to full-paged sites of topics and comments and *vertical* links from topics to comments parallel to each other.

As postulated by Strzebkowski and Kleeberg (2002) students judge interactivity to be the core of an internet based learning environment. Here, didactical interactivity is more important than interactivity of control making the pages and the whole module more useful.

5. Summary

The aim of the programme "*Neue Medien in der Bildung*" to develop new ways of teaching and learning employing modern information and communication technologies has led to the following theses:

- The learning of didactics means to learn subjects, concepts, methods etc. as well as to grasp the process of learning. This necessitates on the one hand the use of an elaborate theory of learning adapted for a multi-media based environment like MaDiN. On the other hand the process of learning needs to be a subject of meta-consideration as well.
- Considering the low stochastical experience there should be a completeness of information related to didactical and mathematical information. That means the programme must not offer isolated didactical or mathematical ideas, subjects or concepts.
- Both the results of the evaluation and theoretical consideration require that didactical interactivities need to be implemented in computer-based learning environments.

It should be acknowledged that the conclusions above and the fact that the module on stochastics is an integral part of an extensive network of linked modules representing most of school mathematics. Thus MaDiN offers not only singular, isolated highlights of didactics, which may lead to the sigh, the 101st animated proof of the central limit theorem is enough once and for all (see Borovcnik 1996, 49 and also Baumgartner 1998, 61). Only the completeness of all linked modules of MaDiN, the integration of learning theories and especially the consideration of web-specific learning processes makes MaDiN a serious attempt to employ new ways of computer and internet based learning and teaching.

6. Literature

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