# INSTRUCTING FUTURE TEACHERS OF STATISTICS: THE BENEFITS OF EN-ACTIVE, ANALOGUE MODELLING

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*I report on special features of a course on statistics and probabilities at my university, where future teachers of mathematics in primary school are instructed on en-active representations of statistical situations and on their analogue modelling. I also report on empirical work with future mathematics teachers of primary school in Baden Württemberg who have been instructed to introduce simple, en-active representations of statistical concepts in the classroom.*

## INTRODUCTION

Since recent reforms in 2005 that follow the dispositions of the "Bildungsstandards", i.e., the adaptations of the National Council of Teachers of Mathematics (NCTM) Standards (2000) to the German Educational context, School Curricula in Baden-Württemberg now recommend introducing statistical concepts and statistical methods in primary school. The emphasis in Baden Württemberg is on "data analysis" rather than on probabilistic reasoning, in contrast to other States in Germany, like Sachsen Anhalt, where probabilistic reasoning plays a more dominant role. Thus it is only recently that stochastics has become a standard subject of the New Examination Regulation (Neue Prüfungdordnung) for primary school teachers in the six Universities of Education (Pädagogische Hochschulen) where most teachers for primary schools of the state of Baden Württemberg obtain their degrees. Up to now practicing teachers have seldom had any statistical training at all. One of the forms in which teachers are recently acquiring both content knowledge and pedagogical content knowledge in elementary statistics and in basic probability theory is by direct contact with staff at the universities. This contact materializes in different modalities: on the one hand students of these universities perform their practical training (Praktikum) in their schools, and, on the other hand, teachers regularly participate in so called "Lehrer Fortbildungen", i.e., additional "crash courses" in specific educational subjects.

I report on the particular features of my course on elementary statistics and probabilities for future teachers of primary schools and on future teachers' practical training on statistics and basic probability in primary schools of Stuttgart (Altenburg School), Kornwestheim (Eugen Bolz Schule), Marbach (Marbacher Grundschule) and Ludwigsburg. This work has been made possible by the openness of teachers in these primary schools: All teachers of the schools we visited have been extremely helpful giving interviews on their reactions to the new statistical and probabilistic issues in the mathematics curricula and on their personal progress in implementing these new topics.

It is important to note that in spite of the novelties in the curricula and the recommendations for early inception of statistical (and, to an even lesser degree, probabilistic) practices, the number of hours devoted to mathematics in primary schools has remained unaltered. One main concern of teachers has therefore been that they have to "steal" hours from the usual mathematical subject matters without impoverishing children's knowledge and training in those subject matters. This issue is crucial because the biennial evaluation tests, centrally organized by the Ministry of Education, are still mainly focussed on arithmetic and geometry. The first evaluation tests at the end of last year placed no emphasis at all on statistical ideas. This will change, as has recently been announced by the referents of the Ministry who periodically visit schools, although it is difficult to imagine statistics becoming preponderant. The number of hours that teachers are ready to devote to statistical and probabilistic practices goes from six 2-hour blocks in first and second grade to nine 2-hour blocks in third and fourth grade per school year. Our interventions were carried out during those blocks, and the discussions with teachers and future teachers on the interventions took place at the schools during extra hours provided by the teachers. The materials described in this paper form a tool box for en-active, analogue modelling

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of statistical scenarios that children learn in through playful yet ritualized activities and which, as suggested by robust results in applied cognitive psychology, provide good anchoring for successful mental simulations at later stages of their statistical development.

## SPECIAL FEATURES OF COURSES ON STOCHASTICS FOR FUTURE TEACHERS OF ELEMENTARY SCHOOLS

The main feature of my course Daten und Zufall, i.e., Data and Randomness, for the so called Module 4 (that corresponds to a fourth or fifth semester course in a standard curriculum for future teachers of primary schools) is my emphasis on work with concrete materials following the educational program for primary school based on “natural” frequencies put forward by Martignon and Wassner (2005) and based on the empirical results on the advantages of natural frequencies as information formats that foster stochastic reasoning (Hoffrage, Gigerenzer, Krauss & Martignon, 2002). Future teachers learn how to design their statistical units with children en-actively and with materials that allow an analogue modelling of discrete statistical scenarios. The materials proposed are flexible items like plastic cubes, cards and magnet plates. This hands-on approach to modelling follows on the one hand, the E-I-S (i.e., En-active→Iconic→Symbolic) principle of Jerome Bruner (1960) and, on the other hand, the programme of ecological rationality in statistical literacy for risk assessment (Kurz-Milcke, Gigerenzer & Martignon, 2008). The postulate of the course is that if such analogue representations are “constructed” by children using adequate materials, this activity will provide good anchoring for future knowledge based mental modelling of statistical scenarios fostering statistical and probabilistic reasoning. Results about the advantages of this analogue modelling of statistical scenarios have been obtained by Martignon and Kurz-Milcke (2006), Martignon, Laskey and Kurz-Milcke (2007), Kurz-Milcke and Martignon (2006), Martignon and Krauss (2007), Kurz-Milcke, Gigerenzer and Martignon (2008), Kurz (2008) and by Brase (in press). Brase, in particular, performed three beautiful and interesting experiments that reveal how powerful iconic, analogue representations can be as fomenters of Bayesian statistical reasoning thus surpassing by far other types of representation formats. During my course I also stress the advantages of analogue modelling as a preparation for work with a computer-software like *TinkerPlots* that can be implemented already at the end of fourth grade (Biehler, 2007). This software, designed for young children from fourth grade onwards, is based on the principle that young users visualize how icons representing individual items, like small coloured squares representing “members of our class”, become dynamically clustered together to form, say, columns of histograms. Colours and shades of colours are used to encode scaled values of features. Future teachers learn to implement a smooth transition from en-active analogue modelling with Tinker-Cubes to iconic modelling with *TinkerPlots*.

## THE TOPICS OF PRACTICAL TRAINING AND EXPERIMENTS

Future teachers of primary school perform their practical training in schools in Stuttgart and around Stuttgart. Elementary school teachers receive these students and let them carry out experiments and small projects with children, on topics corresponding to the class programs at the time of the trainings. Due to the restrictions on the number of hours that teachers can dedicate to statistics in elementary school (see Table 1) we concentrate on central topics of great utility for further development. All teachers are present during interventions and participate in discussions before and after the interventions. Special meetings with teachers are organized to further instruct them for their implementation of teaching practices on other statistical topics. They all express great interest in representational devices for data collection and data description, for sorting and classifying data and for instructing children on proportions as they are used for probabilistic comparisons. The topics we treat are strongly interlinked with other mathematical themes treated in elementary school and can, in some cases, even be seen as applications thereof. Table 1 presents the topics treated so far by our groups of future teachers and the materials used for en-active representation. We will not be able to describe our work concerning all concepts and all materials named in Table 1 because we will rather concentrate on some of them.

It is important to note that primary school in Baden Württemberg consists of four grades only: children complete primary school at the age of nine years (sometimes ten depending on their birthday and their school beginning).

Table 1. Statistical concepts introduced in the four grades of primary schools in Baden Württemberg

| Class | No. of hours per year | Contents | Materials | Links to other mathematical subject matters |
|---|---|---|---|---|
| 1 | 12 | Sorting and classifying<br>Betting<br>Trees | Plastic forms in different colours<br>Unfair and fair dice | Arithmetic: Counting |
| 2 | 12 | Histograms<br>Tallies<br>Two-by-two Tables<br>Quantified categorizations | Dice<br>Tinker Cubes<br>Magnet Plates,<br>Plastic Plates | Arithmetic: decomposing a number as a sum of two numbers, addition and subtraction |
| 3 | 18 | Arithmetic mean<br>Median<br>Data collection | Tinker Cubes<br>Magnet Plates<br>Urns | Arithmetic: Addition, subtraction, multiplication and simple divisions |
| 4 | 18 | Proportionality<br>Urn arithmetic<br>Trees | Urns<br>Tinker Cubes | Arithmetic: multiplication, division; proportions<br>Geometry: Similarity |

## MAGNET PLATES IN SECOND GRADE

Magnet plates are proposed to future teachers as materials of great flexibility. In general they are just magnetic plastic squares in different colours, on which children can stick dots or name tags. These magnet plates are to be placed on magnetic blackboards, present in all German class rooms. The following type of intervention unit is regularly discussed during my course on Data and Randomness for future teachers of primary school. Here I describe a typical intervention, which was performed as an experiment in Stuttgart and around Stuttgart (n=191), where second graders learned to use magnet plates for representing data on the outcomes of two dice (Weustenfeld, 2007), and was conducted with the support of eight future teachers. The experiment began with children rolling two large blue/red dice and discussing their results. The second phase was designed for children to work in the front part of the classroom and “construct” numbers from 2 to 12 by placing pairs of (blue and red) magnet plates on the blackboard, forming columns, such that the sum of the dots in each pair of plates corresponded to one fixed number from 2 to 12 written under the column (see Figure 1). The intention of this phase was twofold: Children en-actively constructed a histogram representing the “natural” frequency−in the terminology of Hoffrage, Gigerenzer, Krauss and Martignon (2002) and Zhu and Gigerenzer (2006)−of possible outcomes with two dice, and they were exercising the arithmetic of number decompositions. The issue here was to establish whether the construction of concrete histograms fomented children’s probabilistic thinking for later bets on which outcome was more likely. Similar bets in a control group *without* the previous construction with magnet plates have consistently illustrated how effective the previous construction can be.

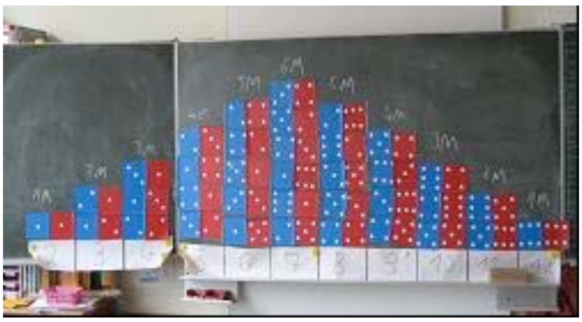

Figure 1. A histogram constructed by means of magnet plates

The term “Säulendiagramm” (histogram) was written on the blackboard, and children were asked to describe this structure in their own words. The experiment proceeded with bets on the result of rolling dice (Weustenfeld, 2007). At a later stage the histograms of the decompositions for the numbers 2 to 12 were reconstructed by children with Tinker Cubes on their tables, (for a description of Tinker Cubes see the following section and Figures 2 and 3) using one colour for each number (e.g., “red” for the number 7). The construction of these towers of assembled cubes, one colour for each number and as many cubes for a tower as possible decompositions of the number, brought children one step further in their understanding of how to encode statistical information. By performing such interventions and/or by analyzing the results, future teachers appreciate the effects of training children in representing individual items with magnet plates and/or Tinker Cubes and teaching them how to view histograms as the result of an organic transition from observations on individuals to “towers” composed of “representatives” of these individuals (Kurz, 2008).

TINKER CUBES AND TINKER TOWERS

Another type of en-active construction that future teachers learn during my course on Data and Randomness is based on the use of what I have called Tinker Cubes (Martignon & Kurz-Milcke, 2006). Tinker Cubes are extremely flexible elements for representing items, individuals and feature profiles. They are plastic cubes that can stand for themselves or can be assembled into Tinker Towers as in Figure 2.

Figure 2. A Histogram made of Tinker Cubes

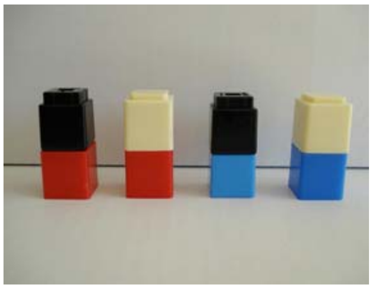

Figure 3. Tinker Towers encoding two features (red/blue for “girl/boy”; white/black for “preferring Math/preferring German”)

One important feature of Tinker Cubes is that they are “oriented”, that is they have a bottom and a top. Tinker Towers, obtained by assembling two or more Tinker Cubes together, also have a bottom and a top. Thus Tinker Towers are to be seen as ordered “vectors” of feature values, represented by colours encoding feature values and forming “profiles” of items/individuals. Tinker Towers can be arranged in different orders, following a predisposed rule. In Figure 2 for instance, Tinker Towers represent fruit preferences in a group of seven year old children, and have been arranged in decreasing order. In two second grade classes of the Altenburg School in Stuttgart future teachers observed the effects of analogue modelling with Tinker Cubes. This unit required two blocks of 2 hours each. The first task children had to implement was to examine their own attitudes towards mathematics and German. Three questions were written on the blackboard:

1. Do children in this class tend to prefer German to mathematics?
2. Do girls in this class tend to prefer German to mathematics?
3. Are children who tend to prefer mathematics necessarily boys?

The instructor and six future teachers distributed Tinker Cubes to all children, who were sitting in groups of four at tables in the classroom. The first phase was called “Collecting data”. During the first phase children were instructed to en-actively construct their class as represented by Tinker Towers. Two children were selected to collect the data on their classmates’ preferences.

They had to go from table to table and ask each child "Do you prefer Mathematics or German as subject matter in your class?" It had been previously established that each child had to make up her or his mind for one subject only. The two "Forscher" (data collectors) used sheets of paper where all the names of children had previously been written. They dictated the data to the instructor who wrote the data on the blackboard. During the next phase each child had to "construct" the class on his table. Each boy in the class was to be represented by a blue cube and each girl by a red cube. During the second phase children assembled towers of size two, using white for encoding "mathematics as favourite subject" and black for "German as favourite subject". At the end of phase two each child had his own set of towers on his or her side of the table. The next step was to perform quantified categorizations, by organizing the towers in en-actively constructed two-by-two tables. The instructor introduced the concept of proportion: children answered questions on the proportion of, say, those who like mathematics being girls. Expressions of the type "***k*** out of ***n***" had to be used for answering these questions.

From the analysis of this type of experiment future teachers perceive how proportions can be used by second graders for quantifying categorizations and for answering questions of the type "How many girls in this class prefer mathematics?." One crucial element of our intervention is precisely the introduction of the expression proportion and its consistent use. Various studies (Koerber, 2003) have namely illustrated that proportional reasoning is not well mastered at primary schools in general and have proposed consistent use of external representations for fostering it. First, empirical results on controlled experiments that measure how sustainable the introduction of proportions represented by means of Tinker Cubes at an early stage can be have recently been obtained by Kurz (2008). All that second graders have to learn is to assess proportions of the type "*k* out of *n*", without having to compare proportions, which is far more difficult and in our studies was introduced through instruction in third and fourth grades (Martignon & Krauss, 2007). What second graders can already learn is both to assess conditional proportions and to invert them, in order to answer questions like "How many of the girls prefer maths to German?" or "How many of those children who prefer mathematics are girls?" These inversions are crucial for an organic development of good Bayesian reasoning.

## INSTRUCTING FUTURE TEACHERS ON THE ADVANTAGES OF ANALOGUE MODELLING IN FOURTH GRADE

Future teachers of primary school at the University of Ludwigsburg learn how populations and samples become central themes in third and fourth class. The focus is on analogue modelling of populations and their samples and on encoding features by means of Tinker Cubes and Tinker Towers made of cubes in different colours. I adopt the German translation of the children's book by Smith and Armstrong (2003) titled "If the world were a village" for interventions in third and fourth grade that are later extensively analyzed by future teachers in my course. This book is a thoughtful quantitative description of different populations and subpopulations of the world, conceived and designed for young children: It suggests that the reader imagines the world as a village of 100 people. The book consists of small chapters on subdivisions of the world-village of 100 people into subpopulations, according to special features like nationalities, languages and religions. For instance the first chapter treats the continents of the world: In our world world-village of 100 people 61 of them are Asians, 12 Europeans, 5 North Americans, 8 South Americans, 13 Africans, etc. Here again future teachers perform repeated experiments with fourth graders in which children construct towers of Tinker Cubes for encoding specific features and are then able to answer questions of the type "How many Christians in the village are Europeans?" and "How many Europeans are Christians?" Kurz (2008) has now concluded subtle experiments that establish how this training in analogue modelling also fosters the understanding of percentages later on in sixth grade.

## DISCUSSION

The materials presented here are part of a tool box for en-active representations and analogue modelling, which enable children to grasp statistical situations and concepts of proportionality in playful yet ritualized environments. Future teachers at Ludwigsburg are learning to develop their own techniques with these materials and can test them in their practice hours and internships in schools. The work presented here is guided by the belief that early analogue modelling, as implemented by future teachers in primary school, will in the long run, lead to sustainable learning effects of children in the statistical domain. This belief is firmly supported by work in applied cognitive psychology (Kurz-Milcke, Gigerenzer & Martignon, 2008; Brase, in press) that demonstrates how analogue representations foster and strengthen both statistical and stochastic reasoning.

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