PREPARING TO TEACH DATA ANALYSIS AND PROBABILITY WITH TECHNOLOGY

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Developing the pedagogical expertise needed to effectively engage students in learning data analysis and probability can be facilitated by engaging teachers in statistical thinking with technology tools. In this paper we present a framework and examples from teacher education materials designed to develop a specialized knowledge we call technological pedagogical statistical knowledge (TPSK).

Many national and international organizations and curricula promote the use of technology in teaching and learning statistics. Whether technology will enhance or hinder students' learning depends on teachers' decisions when using technology tools that are often based on knowledge gained during a teacher preparation program. Teacher education and research on teachers has been greatly influenced by Shulman's (1986) idea of teachers' pedagogical content knowledge (PCK). More recently, Koehler and Mishra (2005) and Niess (2005) have described technological pedagogical content knowledge (TPCK) as the integration of teachers' knowledge of content, pedagogy and technology, and TPCK is needed to effectively use technology to teach specific subject matter. Niess (2005) describes four different aspects that comprise teachers' TPCK: 1) an overarching conception of what it means to teach a particular subject integrating technology in the learning process; 2) knowledge of instructional strategies and representations for teaching particular topics with technology; 3) knowledge of students' understandings, thinking, and learning with technology; and 4) knowledge of curriculum and curriculum materials that integrate technology with learning. Considering the components of TPCK, we believe such a model should integrate mathematical/statistical content, technology, and pedagogy, with a focus on student thinking. Thus, a key feature in preparing teachers to teach mathematics or statistics with technology is to integrally develop teachers' TPCK.

The GAISE project (Franklin & Garfield, 2006) and the 2008 joint ICMI/IASE study are current examples of international awareness of the need for teachers to have a deeper understanding of data analysis and probability concepts and the ability to use simulation and data analysis tools, as learners and teachers (e.g., Konold & Higgins, 2003; Batanero, Godino & Roa, 2004). Although simulation and data analysis tools (e.g., graphing calculators, spreadsheets, *Fathom*, *TinkerPlots*, *Probability Explorer*) may be available in classrooms, there is a need for high quality teacher education materials. Such materials can help teacher educators become comfortable with and incorporate tools for teaching probability and data analysis in teacher education contexts. Materials should also support teachers to develop a specific type of TPCK related to statistics that includes a deeper understanding of: data analysis and probability concepts, technology tools that can be used to study those concepts, and pedagogical issues that arise when teaching students these concepts using technology.

OVERVIEW OF PROJECT GOALS

Through our project, *Preparing to Teach Mathematics with Technology: An Integrated Approach*[1] we have created a module focused on data analysis and probability that could be used in college mathematics education methods courses, mathematics or statistics courses, or in professional development workshops to prepare teachers to teach statistics with technology. For example, a mathematics or statistics department may want to use *Module 1* within a course on "Statistics and Probability for Teachers." The development and evaluation of *Module 1: Data Analysis and Probability* was completed in Fall 2007. This module is aimed to support a broad audience of teachers of age 11-18 students, and thus we made a purposeful decision to provide foundational ideas that would support understanding inferential statistics but to not include formal inferential statistics in the materials. *Module 1* is currently under review for possible

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commercial publication and has been field tested at several universities. The focus of this paper is to explicate the framework used in developing the module on data analysis and probability and to provide examples of how elements of that framework exist in sample materials.

A FRAMEWORK FOR DESIGN

In considering a framework for our design and development of the first module on teaching data analysis and probability with technology, we started with the notion of TPCK as described by Koehler and Mishra (2005) and Niess (2005). Their notions of TPCK are often displayed through a Venn diagram with a focus on the intersections of knowledge about content, pedagogy, and technology. We prefer to think about the development of teachers' technological pedagogical statistical knowledge (TPSK) as layered circles with a foundation focused on teachers' statistical thinking (Figure 1). Thus the innermost layer consisting of elements of TPSK is founded on and developed with teachers' knowledge in the outer two circles. Developing knowledge in the outer two layers of statistical thinking and technological statistical knowledge is essential to but not sufficient for teachers having the specialized TPSK. The elements in each layer in Figure 1 are descriptors of the major foci of the knowledge, thinking, skills and dispositions we aim to develop as teachers' TPSK in our materials.

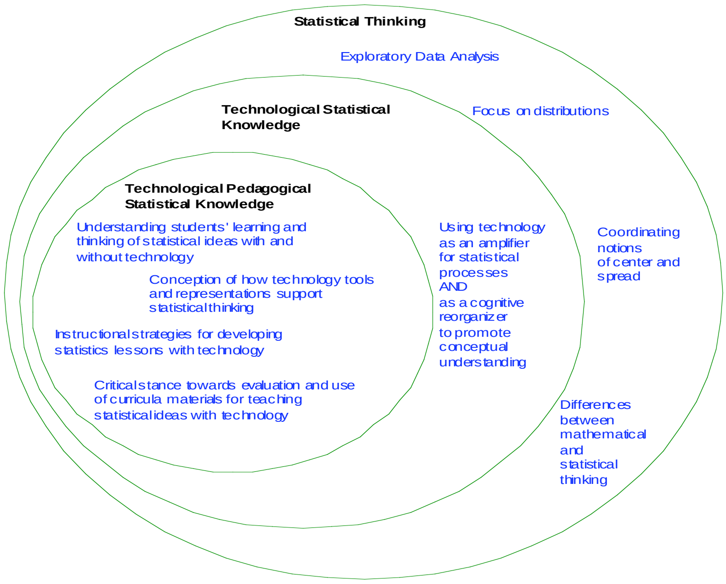

Figure 1. Framework for developing teachers' TPSK

Statistical Thinking as Foundational Content Knowledge

For many teachers, both prospective and practicing, engaging in statistical thinking is a different process from that in which they have been engaged when teaching and learning mathematics (e.g., delMas, 2004). Thus, it is important to engage teachers as active learners and doers of statistical practices. In our *Module 1* materials, we focus on four big ideas that can be supportive of learning to teach data analysis and probability: 1) engaging in exploratory data analysis, 2) attending to distributions, 3) conceptually coordinating center and spread in data and probability contexts, and 4) developing an understanding of and disposition towards statistical thinking as different from mathematical thinking.

In our materials, teachers are engaged in Exploratory Data Analysis (EDA). Teachers may think of analyzing data as a linear process that begins with formulating a question, passes through data collection and analysis, and ends with an answer to that question; however, the practice of data analysis is seldom direct (Konold & Higgins, 2003). Using an EDA approach, a search for patterns often leads one to ask new questions or collect additional data. Using different representations may highlight or cloak different attributes and cause new questions to form. Thus, EDA is often an iterative process that is further informed by the personal experiences one has with the context of the data. In our materials, we purposely use data that is likely of interest to teachers (e.g., national school data, college retention data, vehicle fuel economy, birth data) to motivate data analysis and probability tasks. Using such contexts promotes the practice of asking questions from data, and tools like *TinkerPlots* and *Fathom* facilitate using a variety of data displays to analyze data that can motivate other questions.

As many have noted, examining distributions is at the core of statistical thinking. The technology tools promote graphical representations as a primary display that draws attention to data as individual cases as well as an aggregate. Our materials engage teachers in examining distributions graphically and characterizing the data with such constructs as bins (Rubin & Hammerman, 2006) and a modal clump (Konold & Higgins, 2003) before computing statistical measures. Questions promote the comparison of distributions as means to transition to thinking about data as aggregate. We also pair a quantitative and qualitative variable in initial bivariate data analysis and use this to transition to examining bivariate quantitative data.

Another major focus on statistical thinking is to help teachers conceptually coordinate center and spread of a distribution in data and probability contexts. In data distribution contexts, this means being able to understand measures of center like mean, median, and midrange with respect to individual deviations from those measures, that is attending to variation. We use this notion in a univariate context to help students consider measures of variation (e.g., residuals, sum of squares) in a bivariate context when modeling with a least squares line. Just as measures of spread are useful in describing a distribution of data, in a probability context, it is useful to consider intervals that represent a reasonable range of values in describing distributions of data collected in a context with random variables, particularly variation from expected values within a sample and variation of results across samples. The inverse relationship between sample size and variation from expected values is often counterintuitive for teachers and students. Samples with smaller sample sizes are more likely to have results that vary considerably from expected, while larger sample sizes tend to decrease the observed variation. This relationship is important for teachers and students to develop and to use when considering the likelihood of a given result. Throughout the materials we aim to develop teachers' understanding that the way one thinks in statistical contexts is often different from thinking used in pure mathematical contexts. The materials provide experiences for teachers to consider how statistics is a tool for answering questions and that answers to questions are highly connected to the context of the data and rarely are strong enough to make definitive or conclusive statements. Data analysis and probability are much more about making informed decisions than computing values and using complicated formulas. Modeling linear relationships includes error, and we should expect variation from some "center" in probability contexts.

Statistical Technological Knowledge

Although statistical thinking is foundational in our framework, we use technology tools to engage teachers in tasks that simultaneously develop their understanding of statistical ideas with technology skills so they may experience first hand how technology tools can be useful in fostering statistical thinking. Technologies such as *TinkerPlots*, *Fathom,* and *Probability Explorer* were created to allow users to have dynamic control over data—meaning that as data changes in a document, representations of that data dynamically update. In *Probability Explorer* and *Fathom,* as data are randomly generated, graphs can be simultaneously "building" so that variability in a distribution can be seen as stabilizing as a sample size increases. Further, tools like *Excel*, *Fathom* and *TinkerPlots* allow users to drag data points within a graph and notice the effect on tabular representations and statistical measures, which can be a powerful tool for exploring data in meaningful ways.

Ben Zvi (2000) has provided a useful way of organizing how a technology tool can support statistical thinking. Building from the work of Pea (1987), Ben-Zvi provides a useful lens on statistical technology to amplify or reorganize one's statistical work. Technology tools are typically used in two different ways in statistics. They can amplify our abilities to solve problems. The idea of an amplifier is that the tool expedites a process that could be completed without its use. For example, technology tools can be used to quickly compute and order numbers, generate large lists of pseudorandom numbers, and to generate graphical representations efficiently. Technology tools can also be seen as a reorganizer. Through dynamic features of dragging, the linking of multiple representations, and overlaying measures on graphs, technology tools can help students reorganize their statistical conceptions.

In our materials we use various features in technology tools as both amplifiers and reorganizers. For example, teachers use *TinkerPlots* and *Fathom* as an amplifier to create conventional graphical representations like box plots, histograms, and dot plots as well as ordering data in a table and computing measures like mean and median. However, the most common use of the technology tools is as a reorganizer of how we typically analyze data and conduct probability simulations. For example, linking the data icons in a plot to the data cards in the collection in *TinkerPlots* and *Fathom* could be seen from the reorganization perspective, as the linking helps students juxtapose the individual case with the aggregate. In addition, overlaying statistical measures such as mean and median on a graphical representation can help reorganize the way teachers and students conceptualize these measures in relation to a distribution.

Technological Pedagogical Statistical Knowledge

Our ultimate goal is to develop the inner circle of knowledge representing TPSK. This knowledge encompasses the knowledge in the outer circles in Figure 1: statistical technological knowledge and statistical thinking. We use findings from research on students' understandings of statistical ideas to make points, raise issues, and pose questions for teachers throughout the materials. After teachers have engaged in examining a statistical question with a technology tool, we often ask them pedagogical questions aimed at developing their understanding of how technology and various representations can support students' statistical thinking. In addition, we continually encourage teachers to consider the pedagogical implications of the differences between mathematical and statistical thinking. An entire chapter is dedicated to teachers examining a video and written work of students' engaging in a comparison task with *TinkerPlots*. The videocase provides an in-depth opportunity to analyze how technology can support or hinder students and to push teachers to consider how their students will typically engage in data analysis tasks differently than the ways the teachers did as learners.

We also capitalize on the ability of some software to be used as a *tool-builder* to make interactive exploratory diagrams that can be used by learners to gain conceptual understanding of a statistical idea. Such diagrams allow learners to engage in dynamic manipulations, observe effects of their activities, and reflect on those effects to develop a more meaningful conception of a statistical idea. Teachers use several pre-made interactive exploratory diagrams created in *TinkerPlots* and *Fathom* in our materials. Teachers then discuss benefits and drawbacks and instructional strategies for using interactive dynamic diagrams with students in settings including: 1) individuals working alone at a computer, 2) small groups of students working together with one computer, 3) small groups of students working on individual computers but allowed to discuss their results as a group, and, 4) whole group discussion with the interactive diagram displayed using a projector and teacher and students working together.

EXAMPLE FROM MATERIALS

It is difficult in the page limitations to provide many examples directly from our materials. Figure 2 contains an excerpt from Chapter 3 in *Module 1*. As teachers are learning to use the technology (*Fathom*) to analyze a sample of vehicle data and answer questions, they are introduced to how certain approaches can help students. In this example, teachers are explicitly asked a pedagogy question focused on how a graphical representation could influence students' data analysis. Although teachers may struggle in responding to this type of question, the presence of such questions throughout our text create opportunities for pedagogical perturbations that can prompt reflection and critical thinking. Such perturbations and reflections may help teachers develop technological statistical pedagogical knowledge.

In exploring this data set, the teachers observe that some types of vehicles have better City miles per gallon (mpg) than others. In particular, they may notice that the four cases considered as outliers were all Hybrid engines. The data set contains vehicles of three different Engine types: Standard, Diesel, and Hybrid. Upon making such observations, teachers (and their students) are often prompted to explore a new question. This is an important feature of

Exploratory Data Analysis—analysis of data leads to more questions, which leads to further analysis. At the beginning of this Section, teachers consider the following question:

Question: *Which type of engine gives vehicles the best fuel economy in the city?*

Chapter 3-Section 3: *Comparing Distributions Using Center and Spread*

To examine this question, we need to use two attributes in the data set (bivariate data) with one quantitative attribute (City mpg) and one qualitative attribute (Engine). Having students examine one quantitative and one qualitative attribute together can provide a transition into working with bivariate data with two quantitative attributes to answer a question.

One way to begin examining the data with attention to the two attributes is to overlay the qualitative attribute on top of the dot plot of the distribution of the City mpg. This action will recolor the icons according to the categories of the qualitative attribute and display a legend explaining the coloring.

To overlay a legend attribute to a graph:

1. Click and drag the name of an attribute from the case table and point to the interior of the plot window. Directions will appear as shown in Figure 3.10. You only need use the Shift or Ctrl keys if it is not clear which type of attribute you are dragging, or if you want to purposely use an attribute a specific way (e.g., if the categories of a qualitative attribute have been entered using numeric codes such as 1, 2, 3, you may have to use the Shift key to force *Fathom* to recognize the data as categorical).
2. Release the mouse and notice the appearance of the legend and that different shapes and colors are represented (see Figure 3.11). If the legend attribute is qualitative, shapes and colors will be used, if the attribute is quantitative, a color gradient will appear.

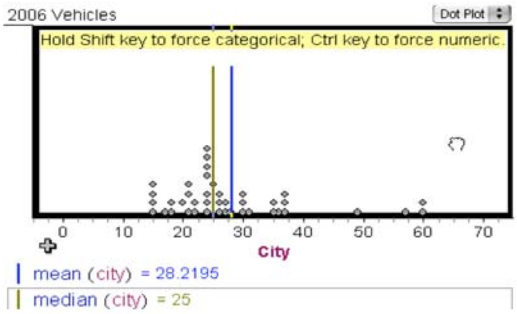

Figure 3.10

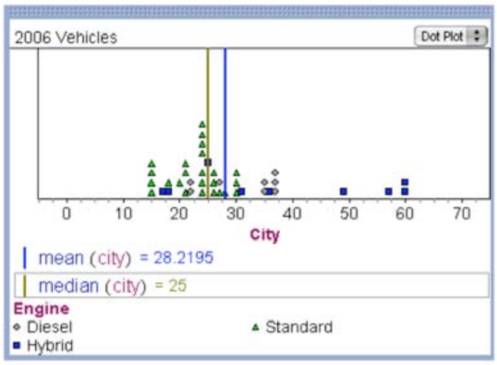

Figure 3.11

Q14 (*Focus on Mathematics)*. Viewing Figure 3.11, what can you say about the City mpg for each of the Engine types?

Q15 (*Focus on Pedagogy)*. How can overlaying a categorical (qualitative) attribute on a dot plot of a numerical (quantitative) attribute influence students' ability to examine data?

The graph in Figure 3.11 is a good way for students to begin to coordinate two attributes in a data set, and thus is a first step in learning to conduct bivariate data analysis where one variable is quantitative and one is qualitative. The graph can promote comparisons of three distributions.

Figure 2. Excerpt from *Module 1* that illustrates our integrated approach to developing TPSK

EVALUATION AND IMPLICATIONS

Evaluation instruments were developed and used with a control group and three experimental groups. Questions on the content section were selected from Garfield (2003) and others from the ARTIST database (https://app.gen.umn.edu/artist/index.html). Due to the small sample sizes, a Wilcoxon Signed Rank Sum test was used to compare the differences in gains from the pre to posttest with an alpha level of 0.10. The gains experienced by students in Experimental I (n=18) were significantly higher (p=0.10) than the Control group (n=15), specifically with items related to content knowledge (p=0.007) and technology (p=0.058). The materials were revised and re-tested in Fall 2006 (Experimental II, n=15) with a slight drop in overall, content, and technology gain scores and an increase in pedagogy gain scores (although not significantly different). The overall gain score in Experimental III (n=32) was not significant at the 0.1 significance level (p=0.134) in comparison to the Experimental II group. In addition, the gain scores in the content subsection for Experimental III students were significantly higher than those in Control (p=0.001), Experimental I (p=0.048) and Experimental II (p=0.074). Although there was a slight drop in the pedagogy gain scores from Experimental II, the Experimental III pedagogy scores were still higher than those in the Control or Experimental I groups, with slightly less variability.

The small sample sizes in all our groups and the difficulty in assessing pedagogical knowledge on a paper-and-pencil test contribute to the difficulty in making any claims based on this data. However, the qualitative data (video, written work--including lesson plans, and interviews) suggest that teachers' TPSK is further developed by their use of *Module 1*. The materials are now used as the standard curricula for 5-6 weeks of a 15 week course at North Carolina State University that focuses on the use of technology in teaching and learning mathematics. Outside field-testing occurred at different universities and the materials have been used several times at these sites and others in the United States of America. However, it is clear from feedback that instructor materials and training are needed to help teacher educators realize the full potential of the materials.

NOTE

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