TEACHERS' CLASSROOM PRACTICE AND STUDENTS' LEARNING

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This report focuses on a research project that combines three aspects of a curriculum concerning teachers' planning, teachers' classroom practice, and their students' statistical knowledge. The theoretical framework and methodology will be sketched. Next, the planning and classroom practice of four statistics teachers will be outlined. Finally, the report documents the knowledge and beliefs concerning statistics of five of each of the teacher's students.

INTRODUCTION

It is now 15 years since Shaughnessy (1992) first inquired into teachers' conceptions of probability and statistics. About ten years later, Batanero, Garfield, Ottaviani, & Truran (2000, p.5) declared this issue to be one of the "priority questions" that had not yet been sufficiently researched. And now, again, Shaughnessy (2007, p. 1001) claims, "there has been very little research into students' and teachers' beliefs and attitudes toward statistics." A compelling reason for further research into teachers' beliefs concerning mathematics is the assumption that the nature of mathematics teachers' thinking is the key factor in any movement towards changing mathematics teaching (Chapman, 1999). Furthermore, there is evidence that mathematics teachers' thinking determines both the students' knowledge and the students' beliefs concerning mathematics. Although many researchers stress differences between mathematical and statistical thinking, or mathematical and statistical beliefs (e.g., Gattuso, 2006), in this report, the theses of Chapman (1999) concerning the teachers' mathematical thinking will also serve as a central assumption concerning the teachers' statistical thinking. For this reason, the focus of the research project discussed in this report is on understanding:

1. The teachers' beliefs concerning the teaching of statistics (teachers' individual curricula);
2. The relation of the teachers' beliefs to their classroom practice (teachers' factual curricula); and
3. The relation of the teachers' beliefs to the knowledge and beliefs students attain after statistics courses (students' implemented curricula).

This report stresses the individual curricula of four German secondary schools statistics teachers (Grades 12 and 13), the factual curricula of these teachers, and the implemented curricula of five students for each of the four teachers. An additional focus is describing one crucial aspect of statistical thinking, i.e., the role of context (Shaughnessy, 2007).

THEORETICAL FRAMEWORK

In order to describe and structure the three levels of the curriculum, i.e., teachers' individual curricula, teachers' factual curricula, and students' implemented curricula, and in order to specify the term belief, the psychological construct of subjective theories (Groeben, Scheele, Schlee, & Wahl, 1988) is used. Subjective theories are defined as complex systems of cognitions (system of beliefs) containing a rationale that is, at least, implicit. Subjective theories contain subjective concepts, subjective definitions of these concepts, and also relationships among these concepts that constitute the argumentative structure of the system of cognitions (or beliefs).

The teachers' individual curricula are understood as subjective theories comprising instructional contents and instructional goals linked with these contents (Eichler, 2007a). A teacher's factual curriculum is the observable part of his curriculum. It provides evidence for the appropriateness of the reconstruction of the teacher's individual curriculum. In other words, the teachers' factual curriculum provides evidence about whether the teachers actually do what they say they intend to do. The students' implemented curricula comprise the statistical knowledge students attain as a result of classroom practice. Broers (2006) describes the construct of

© ICMI/IASE 2008. In C. Batanero, G. Burrill, C. Reading & A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference.*

statistical knowledge as the core of statistical literacy, statistical reasoning and statistical thinking (Shaughnessy, 2007). To structure the students' statistical knowledge, Broers' distinction between declarative knowledge, procedural knowledge, and conceptual knowledge is used in conjunction with the description of these three aspects of knowledge proposed by Hiebert and Carpenter (1992). In addition to actual knowledge, the students' implemented curricula also comprise beliefs concerning statistics or mathematics (Broers, 2006).

METHOD

In respect to the teachers' individual curricula and the students' implemented curricula, data were collected with semi-structured interviews. The four teachers were interviewed (for about two hours) before the beginning of the statistics course. The students were interviewed (for about 30 minutes) one week after the teachers finished their statistics course. From the several clusters of interview questions, those shown in table 1 will be discussed in this report. The interviews were transcribed verbatim. The interpretation of the transcripts adheres to the principles of classical hermeneutics (Schwandt, 2000). The objective of this first phase of reconstruction is to identify subjective concepts and to see how they are defined. The second phase concerns the construction of argumentative systems of knowledge and beliefs, i.e., teachers' individual curricula and also students' implemented curricula. The four teachers teach in German secondary schools (Grade 12 or Grade 13), where courses in statistics last about five months with three to five hours a week. The classroom practice of the teachers was observed and recorded (in writing) over this period. The process of interpretation and reconstruction is described elsewhere (e.g., Eichler, 2007a; Eichler, 2007b). Hence, the following discussion is restricted to the results of this process.

Table 1. Clusters of the semi-structured interviews

Interview with the teachers	Interview with the students
Contents of statistics instruction	Statistical concepts
Goals of statistics instruction	Uses of statistics instruction
Teaching and learning statistics	The nature of statistics

TEACHERS' INDIVIDUAL CURRICULA AND TEACHERS' FACTUAL CURRICULA

Institutional boundaries shape the instructional planning of the four statistics teachers. First, the written curriculum for Grades 12 and 13 and institutional demands concerning final examinations prescribe two main content areas, i.e., probability (discrete and continuous distributions) and inferential statistics (hypothesis testing or confidence intervals). Neither the content-based written curriculum nor the demands concerning the final examinations comprise central ideas of statistical literacy, statistical reasoning or statistical thinking, e.g., the role of the context. A second boundary involves the fact that, in Germany, teaching statistics before Grade 12 is not common. Hence, the four teachers plan their statistics curricula with the assumption that their students have at most little statistical knowledge.

According to these two institutional boundaries, the four teachers' individual curricula (and observed factual curricula) concerning the instructional contents are similar and comprise two main aspects. The first focus is on probability theory including the concepts of chance, random experiments, probability, combinatorics, and binomial distribution. The second focus is on (inferential) statistics, and, in particular, on hypothesis testing and confidence intervals. Data analysis in terms of descriptive statistics has no relevance for the four teachers.

While the teachers' individual curricula are similar concerning the instructional content, they differ considerably concerning the main objectives of statistics instruction. For example, one of the teachers, Mr. D, mentions, "And that's what I am trying to illustrate here as well, that you get models of approach this way, but of course become better afterwards. That you will, of course, somehow get quite far with relative frequency, but that if you have such problems afterwards, elections for instance, you will [...] advance towards reality in confidence intervals. This means showing them, as well, that mathematics, if it takes place in the applications [...] that there are quite often problems which you can solve with maths. [...] That students are enabled to better categorize mathematical models, which determine our economic condition."

The brief quotation illustrates the main objective of Mr. D, i.e., to develop statistical methods as a process, the result of which will be both the ability to cope with real stochastic problems and the ability to criticise. In contrast to the individual curriculum of Mr. D, both the main objectives of the other teachers and their methods that are (in the teachers' perspective) necessary to attain the main objectives differ with respect to two dimensions. The first dimension of the teachers' principle orientation can be described with the dichotomous pair of a static versus a dynamic view of statistics. The second dimension can be described with the orientation on formal statistics versus statistical applications (Table 2).

Table 2. Main objectives of the teachers' individual curricula

	Main objective	Methods
Mr. D	Coping with real statistical problems; attaining the ability of critical facility	Developing statistical methods in a process by examining identical realistic problems at several points in his statistics courses (e.g., elections).
Mrs. I	The ability to analyse and to structure (partly real) statistical problems	Posing various statistical problems; calling on students to create problems of their own making
Mr. J	Establishing a theoretical foundation for statistics	Teaching statistical concepts with regard to the stringent logical structure of statistics
Mrs. K	Establishing a theoretical foundation for statistics, and understanding scientific applications of statistics	Presenting statistical concepts and methods with repeated emphasis on their structural nature

The observation of the teachers' instructional practice provides strong evidence that the teachers pursue their main objectives (or rather their central beliefs; Thompson, 1992). For instance, Mr. D actually poses problems concerning elections at several times during his curriculum, using real data sets. At various times, he resumes the process of developing statistical methods. In these phases, he emphasises that a 'new' method (e.g., confidence intervals) improves the potential to describe the world, in contrast to 'old' methods (e.g., frequencies). Hence, Mr. D emphasises both a process-oriented or dynamic view of statistics and real statistical problems. In contrast, the classroom practice of the other three teachers shows different styles with respect to both the teaching orientation and the role of the context.

Table 3. Classroom practice and the role of the context

Teacher	Social norm
Mr. D	His students predominately work on realistic problems comprising real data sets. New statistical concepts often evolve from previous problem solutions.
Mrs. I	New statistical concepts mostly evolve from teacher-directed explanations. The students of Mrs. I work partly on routine tasks and partly on more extensive problems, which the students sometimes create by themselves. The huge amount of tasks posed by both Mrs. I, or her students, involve realistic situations but unrealistic data sets ("In a German city, 50 percent of the citizens are infected by HIV …").
Mr. J	His lessons involve teacher-directed explanations of new statistical concepts followed by student work on routine tasks. He seldom uses real data sets or realistic problems but prefers various tasks concerning dice, cards or urns.
Mrs. K	Her lessons involve totally teacher-directed explanations of new subject matter followed by student work on routine tasks. To some extent, the tasks involve realistic data sets. Mrs. K finishes every topic by presenting a written summary in which the statistical concepts and linkages between different concepts are described.

THE STUDENTS' IMPLEMENTED CURRICULA

As an example, the structure of the statistical knowledge of one of Mr. D's students, namely Friederike is shown in Figure 1. The single statistical concepts are understood as the students' declarative knowledge (Hiebert & Carpenter, 1992). Friederike remembers the quoted concepts and is able to explain these concepts (except those in double brackets, e.g., conditional probability). In addition, each concept contains an assessment of whether the student's explanation of the concept is appropriate (+) or not (0).

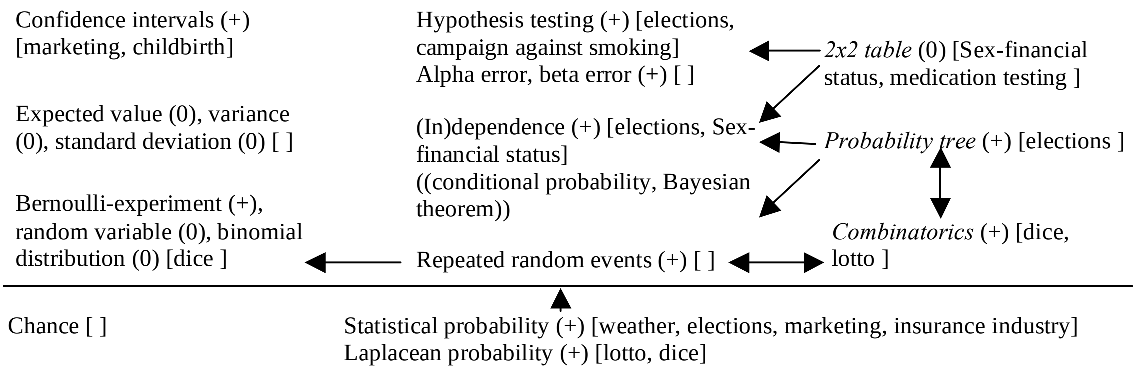

Figure 1. Statistical knowledge of Friederike

The three concepts on the right hand side, e.g., probability trees, are understood as the procedural knowledge. The conceptual knowledge is represented in three ways:

1. The clusters of concepts represent the students' knowledge concerning relationships between statistical concepts within a representation form (Hiebert, & Carpenter 1992).
2. Another aspect of the conceptual knowledge comprises the connections of different representation forms. Such relationships exist between statistical concepts and applications of the statistical concepts (in square brackets), e.g., the relationship between the concept of hypothesis testing and its application, e.g., elections.
3. The third aspect of the conceptual knowledge is the relationship between several clusters of statistical concepts, which is represented by arrows.

The students of the four teachers differ remarkably concerning:

- The nature of the explanations of statistical concepts. For example, some students explain statistical concepts, e.g., the expected value, solely formally, "Expected value, yes, that is μ is equal n times p" (Greta, student of Mr. D). Otherwise, some students explain statistical concepts meaningfully by trying to grasp the meaning of a statistical concept, "Expected value, yes. Given that the probability of being ill is 0.5, and given that there are 1000 persons. Then you will expect an average of 500 ill persons" (Hans, student of Mr. D).
- The nature of situations the students use to explain the statistical concepts. Partly, the students mention realistic situations to explain statistical concepts. In doing so, some students use realistic situations, which they have examined in school; some students use realistic situations, which they have not examined in school. Partly, the students use situations concerning urns, cards, or dices with little relation to reality.
- The quantity (and the quality) of relations among several clusters of statistical concepts. For instance, Friederike (Figure 1) mentions seven relations among clusters of statistical concepts. Most of these relations involve parts of Friederike's procedural knowledge like the relations between the probability tree and the concept of (in) dependence. As the students mention different relations, the students predominately 'define' similar clusters of statistical concepts like the cluster involving the concepts of expected value, variance, and standard deviation.

Besides statistical knowledge, the students differ in reference to their statistical beliefs. The students mention different beliefs about the relevance of statistics for both society and the students' life. For instance, Friederike (student of Mr. D) mentions, "So, I think, for example, that you can use this in the insurance business. That is, in all the areas where you need a certain amount of accuracy, but you can never be 100 percent certain. Yes, in insurance, where some things that very probably require a premium that is very high. And other things, which are not so probable, are not such a great risk. They would rarely have to pay a compensation." As Friederike believes that statistics are a tool to solve real problems emerging in the society, Sabine (student of Mrs. I) negates any relevance of statistics, "I wonder if you really need that stuff later. I like random. However, I think we better would leave things to chance instead of question them." Analysing both the similarities and the differences as discussed above, there are patterns concerning statistical knowledge and the statistical beliefs of the four teachers' students. A brief overview of these patterns is shown in Table 4.

Table 4. Statistical knowledge and statistical beliefs of the students

Students of	Statistical knowledge and statistical beliefs
Mr. D	The students predominately showed meaningful explanations (see above) of the statistical concepts. In contrast, they do not use formal explanations. The five students mention 7 (Friederike, see above), 6, 4, 3, and 2 relations among statistical concepts (overall 22 relations). The students of Mr. D believe that statistics is highly relevant for society. To explain this relevance, the students use various realistic situations that they predominately have examined in school. In contrast, the students of Mr. D believe that statistics would have little relevance for their own life.
Mrs. I	Predominately formal explanations of statistical concepts; 6, 5, 4, 4, and 3 relations (overall 22); two students assign statistics no relevance, two students assign statistics little relevance for society (using solely situations from school); all students assign statistics no relevance for their own life.
Mr. J	Predominately (vague) formal explanations; 4, 3, 2, 2, and 1 relations (overall 11); all students assign statistics little relevance for society (using solely situations from school); no students assign statistics relevance for his own life.
Mrs. K	Predominately meaningful and formal explanations; 9, 7, 6, 6, and 5 relations (overall 33); all students assign statistics high relevance for society (using predominately situations from school); four students assign statistics some relevance for their own life, one student assigns statistics no relevance for his own life.

DISCUSSION

This brief overview highlights some of the patterns concerning the relationships among the three levels of a curriculum referring to the small sample of four teachers and their students. One crucial aspect is that the four teachers do not differ concerning instructional content. However, they differ considerably concerning their objectives. Thus, the students' implemented curricula are independent of instructional content, but they are highly dependent on the teachers' objectives or mathematical beliefs. For example, the students of Mr. D and the students of Mrs. I have similar statistical knowledge concerning the explained statistical concepts, the 'defined' clusters of statistical concepts, or the quantity of relations between clusters of statistical concepts. However, they differ considerably concerning the nature of explanations (meaningful versus formal), and they also differ concerning their statistical beliefs. A possible rationale for these differences could be the two teachers' fundamental different definition of the role of the context. It seems, that the restriction towards pseudo-realistic statistical applications (Mrs. I) seems to hinder both students' ability to meaningfully explain statistical concepts and students' beliefs that statistics is useful for understanding decision-making in society. Otherwise, the impact of the teaching styles on the students' statistical knowledge and beliefs is ambiguous. For instance, just as Mr. D, Mrs. K uses realistic situations and real data sets for their teaching. As Mr. D has a cognitive constructivist orientation, Mrs. K has a direct transmission view (Staub & Stern, 2001). However, in contrast to the findings of Straub & Stern, the students of

Mrs. K reveal similar or higher statistical knowledge in comparison with the students of Mr. D. Further the students of Mrs. K reveal a profound understanding of the relevance of statistics for both society and the students' own lives. It seems the tendency of Mrs. K to discuss, once in a while, real statistical applications and to provide concise summaries about certain topics seems to have a great impact on both her students' statistical knowledge and her students' beliefs about the relevance of statistics.

To conclude: on the one hand, there is a definite need to produce guidelines for good practice in teaching statistics, particularly with regard to statistical reasoning, statistical literacy or statistical thinking. On the other hand, we must have a much greater understanding of teachers' beliefs and the impact of these beliefs on students' knowledge and students' own beliefs concerning statistics. Only then will we be able to make sound proposals that could result in changes affecting teachers' individual curricula and students' implemented curricula.

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