From **Brunelli, Lina & Cicchitelli, Giuseppe (editors). Proceedings of the First Scientific Meeting (of the IASE).** Università di Perugia (Italy), 1994. Pages 377-383. Copyright holder: University of Perugia. Permission granted by Dipartmento di Scienze Statistiche to the IASE to make this book freely available on the Internet. This pdf file is from the IASE website at http://www.stat.auckland.nz/~iase/publications/proc1993. Copies of the complete Proceedings are available for 10 Euros from the ISI (International Statistical Institute). See http://isi.cbs.nl/sale-iase.htm for details.

PROPORTIONS, PROBABILITY AND OTHER MATTERS

Flavia Jolliffe
School of Mathematics, Statistics and Computing
University of Greenwich, Woolwich, London SE18 6PF, UK

1. Introduction

A study to investigate students' facility with proportions has been undertaken by the author and Fay Sharples of the University of Waikato in New Zealand over the period 1989 to 1992. The initial study was done during 1989 and 1990 and 64 students at the University of Waikato and 57 at Brunel University in the UK took part. Some results of this study have been reported elsewhere (Jolliffe and Sharples, 1991; Jolliffe and Sharples, 1993).

Students were given a short set of self-completion questions to test their understanding of proportion. They were instructed to "Give the answers which you *think* and *feel* are correct" and to write DK if they felt they did not know the answer. In all questions they were also asked to give a brief reason why they had chosen the answer they gave. The test took students about 25 minutes to complete.

All students in the initial study were at the start of their university courses at the time they were tested. Apart from 26 Brunel students who were registered for degrees in a mathematical subject, the students' main subject of study was in another discipline, but they were required to study some statistics. Many of these students were weak in mathematics whereas others had a good foundation in school mathematics and some had had instruction in probability. In spite of the variety of mathematical backgrounds and abilities the spread of responses was broadly similar in the two institutions and responses of the more mathematical students were not noticeably different from those of the others.

There was a tendency for students to think that events were equally likely, a phenomenon noticed also by others (e.g. Lecoutre, 1993), but more surprising and worrying was the relatively large number of incorrect responses due to arithmetical errors, to careless reading of the questions, and to an apparent lack of understanding of terms such as *estimate* and *product.*

2. A follow-up

We made some changes to our questionnaire after studying the results of the initial study. We made it slightly shorter, cutting out some questions which we felt were unsatisfactory in some way, such as two new versions of the "two children" problem where the chance that the other child is the opposite sex from the one observed is invariably said to be 1/2 regardless of the situation under consideration (Bar-Hillel and Falk, 1982).We amended the wording of some questions in the hope that they would then be clearer and we added a question asking the students the highest level (in terms of public examinations) at which they had studied mathematics previously. We kept the same instructions as before, and, as in the first version of the test, all questions were followed with "Why do you say this?" and nearly all were open-ended.

In the Spring of 1992, 127 students in New Zealand and 29 students in the UK, all of whom were taking statistics as a service course, completed the revised version of the questionnaire. As previously the New Zealand students were at the University of Waikato. Just over 2/3 of them had studied for a mathematics Bursary examination (taken in the final year at school) but would not have achieved a high mark in it, and about 70% were first year students. A random sample of 30 questionnaires completed by students at Waikato was taken for the purpose of this paper.

There were several differences between the UK students in the 1992 study and those in the earlier study. They were at a different institution – Thames Polytechnic – which has since become the University of Greenwich; reading for a different degree – Environmental Health – which involved them in acquiring expertise across a range of subjects, for example, Physics, Statistics, Microbiology, Technology, Law, Occupational Health and Safety; they were halfway through their first year although at the start of the Statistics course, and several of them were mature students with some work experience. Very few of them had studied mathematics beyond the level usually examined at age 16. The UK polytechnics traditionally taught more vocational courses than the universities and gave places to students having a wide range of abilities and entrance qualifications. In practice this meant that the more able students academically tended to go to universities as a polytechnic degree was thought in many circles to be second best. It is too early yet to say whether granting polytechnics university status in the Summer of 1992 has had any marked effect on the entrance standard.

As with the previous study, the results in this follow-up were interesting and not always what we expected. This paper discusses the results on three questions.

3. Numbered balls

The second question was about a blindfolded person taking a ball from a bag containing 10 balls which are identical except for their labels which are 9, 3, 6, 2, 7, 5, 0, 4, 8, and 1 (listed in the same order as in the question - see Appendix question 1) that is, each digit appears on exactly one ball.

Some students apparently did not understand the significance of *or* in this question and 10 thought there were two separate answers to part (a), 8 that there were two answers to part (b). The use of "or" in probability theory with its interpretation of "at least one of" is not a natural usage. When we drafted this question we were concerned with students' understanding of "at least" and "less than" and to some extent with how they would treat 5 which is included in the event "less than 7". We had expected them to count up how many numbers satisfied the criteria and divide this by 10, but many of those who gave a single answer first found two separate probabilities. They did not always then combine them correctly, for example two students multiplied them instead of adding.

Sixteen of the Waikato students in the sample but only 9 of the Thames students obtained the correct answer to part (a). The corresponding figures for (b) were 16 and 10. As expected there were some misunderstandings of "at least 8". Four of the Waikato students in the sample and two of the Thames students appeared to interpret it as "8 or less", that is, "at most 8", giving an answer of 9/10 to part (a). There were also other interpretations of the question such as "exactly 8" and "greater than 8". One Thames student said there is a probability of 2 in 9 of obtaining a number which is at least 8, and another guessed that the probability was 4 in 10. Three of the 59 students were unable to give an answer.

In part (b) four students clearly thought that "less than 7" meant "7 or less" as they stated that there were 8 out of 10 chances of getting a number less than 7. If they correctly realised that 5 was included in the event their final answer was 8/10. Another four students added the probabilities of 1/10 for a 5 and 7/10 for a number less than 7, that is, double counted the 5, and thus they too obtained a final answer of 8/10. The "Why do you say this?" for part (b) enabled us to sort out the two kinds of errors here.

Eight Waikato and four Thames students appeared to think there were six numbers less than 7, presumably forgetting to count the 0. "Less than 7" was interpreted as "more than 7" by a few and a student at Thames justified the answer 2/10 with the statement (quoted as written) "Between 5 and 7. There is no balls labelled except the ball of No. 6. The ball has to be less than 7 therefore there are only 2 balls i.e. 5 and 6".

4. A question about dice

This question (see Appendix question 2) was a revised version of a question asked in the previous study where the second sentence had started "The *product* of the two numbers". It was clear from responses in the earlier study (Jolliffe and Sharples, 1993) that many students thought that a product was a sum or did not know what was meant by a product. The changed wording appeared to have removed this confusion as only 2 students in the 59 gave any indication of considering sums and only two students felt unable to answer. However, as in the previous study, the modal response was "Equally likely" selected by 32 of the 59 students (54%) and 20 of these said that even and odd numbers on a single die are equally likely in explaining their responses. The response "is more likely to be odd than even" was selected by 5 students. The students at Thames were more inclined to choose "Equally likely" and less likely to choose the correct result of "Even" than the Waikato students. A summary of the results is given in Table 1 with combined figures for the previous study for comparison. Had students been aware that any number multiplied by an even number is even this would have been a very easy question. Would it have made any difference if we had lead them into this?

5. Sums and discs

This question (see Appendix question 3) was also asked in the previous study, but for the follow-up we put the player with two winning totals first as we had wondered whether some students in the previous study had selected the player with three winning totals as the one more likely to win because this was the first listed. As with the dice question, Waikato students did better than Thames students on this question. Thirteen of the sample of 30 Waikato students correctly thought that A had more chance of winning whereas only 5 of the 29 Thames students thought that A had more chance - 31% correct overall compared with 36% correct in the previous study. Eleven of the 13 and 2 of the 5 gave a correct argument for choosing A. Six of the Waikato students and 9 of the Thames students chose equally likely, and 9 of the Waikato students and 13 of the Thames chose B. In explaining their (wrong) choice of B as the more likely winner 12 students said that B has more winning totals, and 6 thought there were only six ways of combining the numbers on the discs that is, they did not realise there are two ways of obtaining the same total when the numbers on the two discs are different.

6. Conclusions

Our first impression of the results of this study is that there were more wrong and strange answers than in the previous study, and possibly a higher proportion of no responses and "Don't knows" in some questions. There are a number of possible explanations for this - changes in the questions asked, known differences between the groups in the two studies, and a possible cohort effect reflecting changes in syllabuses and standards between 1989 and 1992. Although changes in the questions are in the control of the experimenter, it is difficult to collect information about and make adequate allowance for confounding factors in a study of this nature where one is relying on the willingness of the students to answer the questions. As is well known, it is difficult to experiment in education and much of the research in statistical education has taken place with a small number of volunteers in an artificial setting (see Hawkins *et al.*, 1992).

This does not mean that research is worthless. A study such as ours is more in the nature of an exploratory case study. It attempts to investigate the extent of common misunderstandings and to probe into reasons for them. Students tend to dislike probability questions and become disheartened when their answers are wrong. By finding out more about why those answers are wrong helps us to improve the situation by, for example, making sure that students understand both the language of probability theory questions and the mathematical terms used in them.

Bibliography

Bar-Hillel M. and Falk R. (1982), Some teasers concerning conditional probability, *Cognition*, 11, pp. 109-122.

Hawkins A., Jolliffe F. and Glickman L. (1992), *Teaching statistical concepts*, Longman, London.

Jolliffe F. and Sharples F. (1991), An investigation of the knowledge of proportion and probability that students bring to university with them, abstract in D. Vere-Jones (ed.), *Proceedings of the Third International Conference on Teaching Statistics*, International Statistical Institute, Voorburg, Vol.1, p. 370.

Jolliffe F. and Sharples F. (1993), Proportions and products - what do students know about these? *Teaching Statistics*, 15(3), pp. 78-80.

Lecoutre M-P. (1993), Cognitive models and problem spaces in "purely random" situations, *Educational Studies in Mathematics*, 24, pp. 559-568.

Appendix - Questions discussed in this paper

Note: Each question was followed by "Why do you say this?"

1. A ball is to be drawn by a blind-folded person from a bag containing ten balls. The balls are identical except for their labels which are:

9, 3, 6, 2, 7, 5, 0, 4, 8 and 1.

(a) What is the chance that the ball chosen is labelled 5 *or* with a number which is *at least* 8?

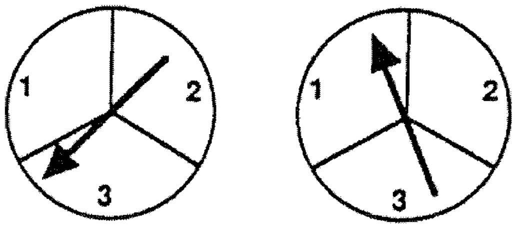
(b) What is the chance that the ball chosen is labelled 5 *or* with a number *less* than 7?

2. Two ordinary 6-sided dice are rolled. If the two numbers are *multiplied* together the answer is:

more likely to be odd than even ☐
more likely to be even that odd ☐
as likely to be even as odd ☐

Tick the box by the answer you select

3. Two discs are marked with numbers.



Each disc has a pointer which spins round. A game is played by spinning both pointers and then the numbers where they stop are *added*.

Player A wins if the *total* is 4 or 5.
Player B wins if the *total* is 2 or 3 or 6.

Which player has the greater chance of winning, or do they both have the same chance of winning?

Table 1 - *Results of the dice question*

	1992				1989/90	
	Waikato		Thames			
	N.	%	N.	%	N.	%
Odd	3	10	2	7	6	5
Even*	13	43	7	24	40	33
Equally likely	14	47	18	62	64	53
DK or no response	0	-	2	7	11	9
Total	30	100	29	100	121	100

* Correct answer