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# THE TEACHING OF PROBABILITY AND STATISTICS IN SCHOOLS: ISSUES FOR INTERDISCIPLINARY REFLECTION

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## 1. Introduction

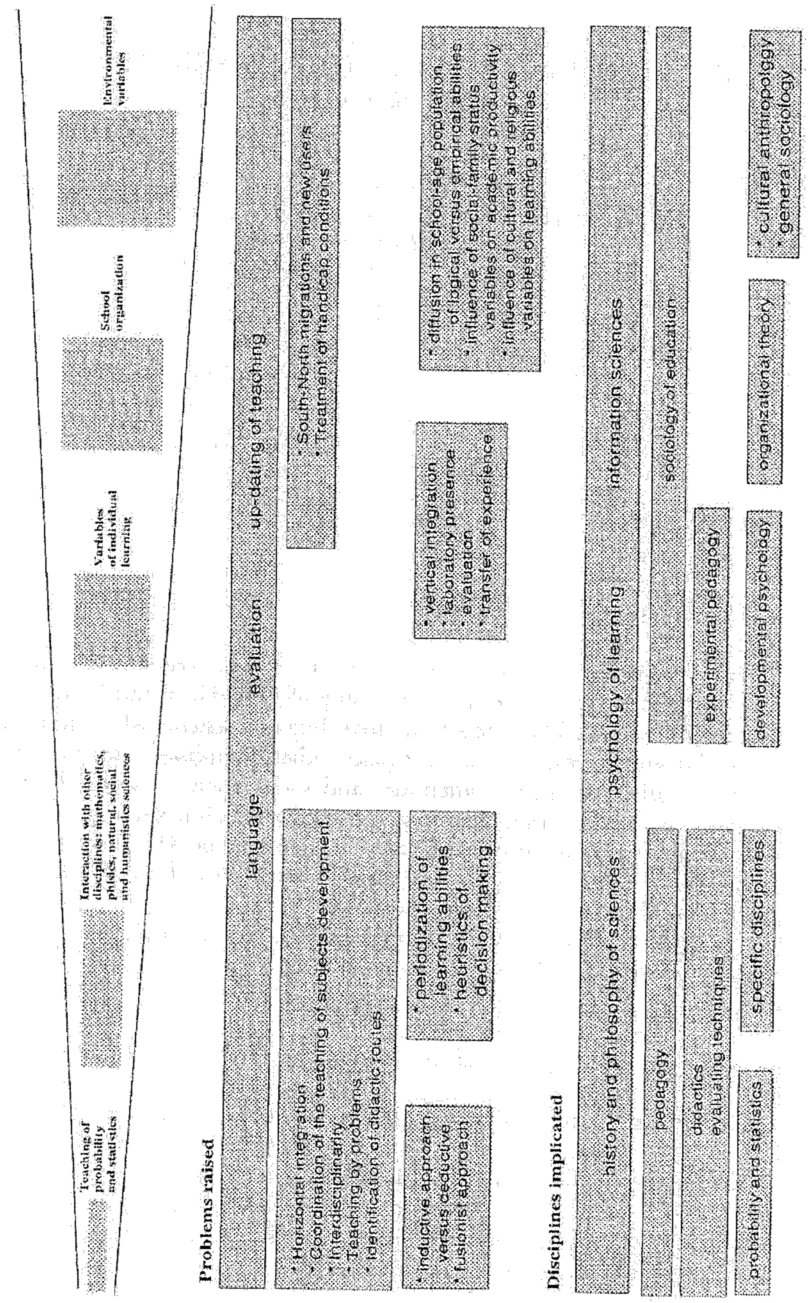
In the following discussion several issues are presented for interdisciplinary reflection on the teaching of Probability and Statistics in preparatory school. They are set out according to a scheme which links this particular subject, with, in the first place, other disciplines – mathematics, physical-natural sciences, humanities and social sciences; secondly, with the variables of individual learning and modes of scholastic organization; and finally, with environmental variables (see scheme 1). The scheme is organized around problems and disciplines involved in the didactic complex and incorporates many features of the learning-teaching process.

Some of the issues originate in the Italian reality; they may also be valid in other national contexts.

## 2. The conditioning of tradition (the Italian case)

Italian culture has strong humanistic traditions. This involves:

- development of linguistic abilities in an instrumental sense for the reading of "classics", to the detriment of the development of technical-scientific languages, contrary to what generally occurs in other countries (for example in Anglo-Saxon ones);




Scheme 1. *Probability and statistics in schools from an interdisciplinary viewpoint*

- greater depth, in the study of the development of thought, of those parts "close" to logical-deductive schemes (in the study of the History of philosophy one spends more time on Descartes, Kant, Hegel and up to Croce, than on Galileo, English empirism, pragmatism).

On the whole, the didactic courses peculiar to each discipline develop abilities and aptitudes for deductive processes, rather than inductive processes. Even the teaching of physics (and other experimental sciences) is closer to this general scheme than to the Galileian "method". It is done through the study of handbooks rather than by analyzing experimental results, even if simple ones. Often, teachers lament the scarcity of adequate structures. Actually, such scarcity represents a convenient *alibi* for the perpetuation of a bookish approach in experimental sciences also. It follows that it is almost never emphasized that the "rules" used to predict physical "facts" are the result of elaborations and calculations made on the basis of mathematical "models", suggested by "empirical" observations; that observations essential to interpreting the phenomenon examined are made in different contexts, from which features are detected which are considered relevant to the solution of the problem being studied. The planetary model of the atom is no more inadequate than others to explain certain phenomena, but it is not fit to explain all phenomena.

The usual courses in academic teaching are aimed at achieving "definitive truths" rather than at a "method" which, step by step, permits one to arrive at the interpretation of different phenomena with a "sufficient degree of approximation", as Guido Castelnuovo already reminded us at the begining of this century. The approximation concept itself is completely extraneous to the traditional culture and not even the introduction of computers, which necessarily produce results with a limited precision has led to greater awareness of its role, either in a numerical sense or in the sense of a model. The inductive method necessarily involves "approximation" and a "provisional" approach to the results achieved, in addition to a certain subjectivity in interpretation. All these concepts and abilities are not only neglected in traditional Italian teaching, but, in some ways, consciously rejected. Any "innovative" proposal must deal with this generally widespread situation, which also produces teachers. These teachers should be the creators of possible modifications in modalities, not only in relation to content, but also in facilitating the development of critical abilities, which is one of the declared goals of the school course.

## 3. Teaching by problems, teaching by models

The situation described has led to the affirmation of a methodology of teaching scientific disciplines, in particular of mathematics, by paradigms rather than by problems. One studies Euclidean geometry and calculates all the possible areas of plane figures, however complicated, but one does not talk about the fact that really it is a question of applying a model, that this is just an approximation useful for the study of property (for example, of allotments), because actually the earth's surface is spherical and within spherical geometry the axioms and assumptions of Euclidean geometry are not applicable. Once again the search for absolute truths overwhelms the awareness of approximation. This recognition, which sometimes occurs in relation to non-Euclidean geometries, generally appears as something strange rather than as something necessary for the interpretation of certain "facts".

The different mathematical models of space are never analyzed in a comparative way, yet non-Euclidean patterns are generally introduced in a way that subordinates them hierarchically to the Euclidean pattern.

It is considered preferable to reduce the real world to a "flat" world (without even mentioning it), whose historic necessity and utility are bound to the technically limiting instruments which are traditionally used in the study of geometry: (flat) blackboards and (flat) book and notebook pages. Here also, teaching has not taken advantage of the introduction of computers which would not only permit the use of realistic spherical images, but could also explain, for example, why the air-routes from Europe to America do not follow the parallels, as would seem natural from looking at an atlas (flat image), but follow geodesic lines. Analogously, one prefers to reduce the world to a deterministic world rather than to speak of approximations related to forecasts under conditions of uncertainty, and so necessarily based on probabilistic reasoning. Thus, one always prefers to remain anchored to ideal schemes, very far from the reality which one would like to represent.

An extreme feature of this kind of teaching by hypothetical-deductive schemes appears to be the same in the teaching of Probability. This is often inserted in Analyses and is considered as a particular application of analytic calculations (combinatorial calculation or others) rather than as an autonomous sector of mathematics - one which principally, but not exclusively, uses analysis as a tool, but finds its basis in problems which cannot be confronted within a simple logical-deductive model.

Instead, the most adequate scheme for the teaching of Probability and Statistics is of an inductive type:

SIGNIFICANT PROBLEM --> RESOLUTIVE MODEL --> RESULTS (FORECASTS) --> COMPARISON WITH DATA --> POSSIBLE MODIFICATION OF THE PREVIOUS MODEL etc.

It is important both to resolve the same (interesting) problem with many theoretical models, and to apply the same model to many significant problems. Above all, one should avoid using "recipe-books", and transmitting the (wrong) idea of a one-to-one correspondence between problems and models. That is, one must aim at developing the ability to "disentangle" new problems, not inserted within pre-constituted schemes, by using competences acquired through the solution of other problems.

## 4. Fusionism and interdisciplinarity

New teaching methods, better suited to developing independence in confronting new problems and oriented towards producing solutions based on experience with analogous cases, should be based on hypotheses of fusionism and work on interdisciplinary projects. From interdisciplinary work it is possible to map out interesting and significant problems to be confronted, and also to use the new capabilities given by computers (analysis of complex data, simulation).

The choice, in certain fields of application, of significant problems which are interesting because of the methods used in their solution, facilitates the clarification of both methodological notions and those related to application. Probability and Statistics represent the principle means for dealing with significant applications, and they permit the complete realization of the Galileian scheme:

OBSERVATION --> CONSTRUCTION OF A MODEL --> CALCULATION OF FORECASTS --> DETECTION OF NEW ADEQUATE DATA --> COMPARISON WITH THE FORECASTS.

This scheme also suggests a temporal order in introducing arguments: methods for data description, probabilistic models, inference. One can propose several prototypes of this kind of course, which are both "historical" and current and can help overcome the lack of consolidated experiences. Analyses of data from the field of (classical) physics is particularly instructive because such data correspond to essentially deterministic models in which the only fluctuations are due to errors of measurement. By contrast, analyses of data from biological and demographic spheres need intrinsic probabilistic models.

The difficulty of the statistical analyses of non-physical data, in fact, comes from what is the essential diversity of the relationships which must generally be highlighted. In practice, in contrast to physics, they always (or almost always) have to deal with relationships of functional dependence, because, even when the theoretical interpretation must rely on probabilistic schemes, the real (macroscopic) measures come from innumerable masses of homogeneous elements and lead to functional relationships. If, in order to produce homogeneous elements, the biologist or the demographer wants to divide the reference population according to all the characteristics which differentiate it, he would have to divide it into more and more restricted classes until it was reduced to single individuals. The probabilistic nature of the possible conclusions is therefore an aspect which relates not only to the expression of the results but also to the collection and elaboration of the data.

The physicist, whatever theory he creates, always knows it will have to be shown that future experiments confirm what his experiment has demonstrated. In other sciences which use statistics, observed facts which can be identically repeated do not exist, nor do they have, even when they are identical, an identical development.

Such considerations give rise naturally to the need to refer to more general mathematical models (of a probabilistic type) of which the schemes and models related to physical phenomena can be considered extreme cases. Fusionism is also opportune in the sphere of mathematics where the synergetic use of various (geometric, analytic, graphic) methods can be useful for the step by step solution of a certain problem.

Moreover, with such an approach it is possible to obviate another consequence of the present system: the development, sometimes even extreme, of analytical abilities to the detriment of synthetic abilities.

By proposing adequate courses, it is possible to teach something to individuals of any age; moreover, it is deleterious not to take the right moment to propose the best, most interesting *stimuli*. Children are good at using induction; it corresponds to the same mechanisms as linguistic (experimental) learning, which, as is well known, is much easier for younger people.

Moreover, by being accustomed to using inductive reasoning "early" they can avoid the hardening of the logical-deductive structures which are given priority in traditional teaching, even in scientific disciplines. Yet Italian schools show very strong trends toward conservation. The causes of this must be sought in the general situation in which the Italian school system operates, a situation which excludes competitivity, given that conservation is not assured by the quality of the product or by the efficiency of the system. Therefore any innovative proposal meets obstacles and involves high costs of various origins. It is impossible to leave these costs and the relative benefits of proposed innovations out of the analysis; hence the importance of a system of evaluation which can be used to monitor productivity and to suggest corrections useful for "adjusting the aim".

## 5. Variables of individual learning

Psychological investigators – as Fischbein (1975) reminds us – have found interest in the late development of the concept of probability, particularly in the growth-related development of children. What are the motivations for this delay, and for the recent (relative to the moment in which he was writing) interest? He lists some reasons, such as "the fact that behavioral phenomena are in themselves of an aleatory character", and that only recently has psychology arrived at the idea of needing probabilistic models to interpret behavior. Another is the "halo effect" produced by other sciences, contiguous to psychology, which are based on the theory of probability: in particular decision theory, information theory and cybernetics[1]. Since then, work and discussion on the axiomatic bases of probability have reopened the discussion on the meaning of the concept of probability itself, imposing a serious reconsideration of subjective features, or of the degree of confidence of every subject in events occuring within its field of interest.

Without doubt, Piaget and Inhelder's research[1] (see Piaget and Inhelder, 1951)) constitutes the first reference for this kind of study which subsequent authors have had to deal with. From our point of view, its didactic possibilities have not been completely taken advantage of in the current environment. Moreover they have not been directly elaborated by other authors, although they are always in the background, ready to guide those who want to consider them. For example, the result of the close relationship in the formation of the concept of probability as a function of age constitutes an undeniable guide for the organization of school work. It is also worth remembering the three stages of development identified by Piaget and Inhelder, because within age limits (which possibly can be shifted by usefully focused teaching) they represent a recognized background for the psychological genesis of the idea of chance.

In the first stage (before 7-8 years) a lack of operative abilities is observed, in the sense that the reasoning formulated by children at that age might be regulated only by a largely intuitive system, without set inclusion being managed, or even used. At this stage, the boy or girl might not distinguish between the possible and the necessary.

The second stage (from 7-8 to 11-12 years), characterized by the beginning of logical arithmetical operations, shows early embryonic

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[1] It could be interesting to remember the introduction of Piaget and Inhelder's book, in which we read: "Un mathématicien connu par ses travaux sur la théorie des probabilités" (he is probably referring to E. Borel) "nous souggérait un jour l'étude de ce problème psycologique: n'existerait-il en l'homme normal, une intuition de probabilité aussi fondamentale et d'usage aussi fréquent que, par exemple, l'intuition du nombre ?".

developments of the idea of chance. Boys and girls at this age cannot manage to complete an exhaustive analysis of the combinatorial possibilities related to a certain situation. This is due to the fact that formal hypothetical-deductive thinking, which represents the substratum for formulating such a list of possibilities, is not yet completely developed. The children cannot support the techniques of reasoning which – according to Piaget – are the basis for the recognition (needed in the judgement of probability) that every possible outcome may occur.

In the third stage (over 11-12 years), when combinatorial operations are understood and stabilized, a synthesis between combinatorial possibilities and probability can be given.

That is, children should be able to imagine the entire arc of sampled events and to represent to themselves that only some of those events, in subsequent experiments, may be realized. They are also able to distribute probabilities in this arc of events, by stating that some events will happen more easily than others.

Therefore, reviewing what has been one of the principal branches of the historical development of probability from 1500 (Cardano, Galilei, Pascal, Fermat,...), Piaget and Inhelder recognize a direct parallel between the achievement of combinatorial abilities and the development of the idea of probability in youth. In fact, the latter represents a prius of this: "(...) le jugement de probabilité s'organise en sa géneralité *par un sorte de choc en retour* de l'opération sur le hasard" (see Piaget and Inhelder, 1951).

If Piaget's study only demonstrates combinatorial potential in teenagers, Fischbein (1975) explores the possibility that specific educational forms can accelerate the process of acquiring combinatorial abilities, and in the same way as those related to probability. "Probabilistic intuition" – he states – "is not spontaneously developed, except in a very restricted sphere. The understanding, interpretation, evaluation, and forecasting of probabilistic phenomena cannot be entrusted to simple uncultivated intuition, neglected and abandoned in a rudimentary stage of development, under the pressure of operative schemes which cannot articulate with them" (see Fishbein, 1975, p. 18).

Starting from Kahneman's and Tversky's analysis of the use of conceptual short-cuts (behavioral heuristics, according to the definition used in psychological literature), Agnoli (see Agnoli, 1991a,b; Agnoli and Krantz, 1989) also forcefully insists on the educational implications of these ideas. According to what she reports and has directly experimented with, it seems that the acquisition of logical concepts (substantially, as for Piaget, set inclusion and intersection, or implication and logical product in the language of propositions) appears stably in boys and girls from 11-13 years old and reduces the frequency of errors rather generally (see Agnoli and De Zuani, 1991a,b).

With regard to the origin of errors, this author also proposes to consider it, in adults as in children, as rooted in school teaching which too often does not recognize that certain decisions are taken in conditions of uncertainty[2]. If school teaching tends to emphasize the solution of problems of a deterministic character (that is, those problems for which just one correct answer is foreseen) and is not helpful in recognizing situations containing inherently uncertain results, then the children's intuition cannot open up to the presence of events to which a degree of personal confidence, a probability, can be assigned .

Another question, directly connected with what precedes, relates to eliminating the ambiguity of everyday language: if, for example, probable is confused with possible, then their respective negations, in the absence of semantic clarification, can lead to identifing improbable with impossible. But these unclarified linguistic features pervade large parts of mathematics teaching, as Baruk (1985) has shown with an abundance of particulars drawn from the 'homework' of her French students. She dedicates an entire section of her book to the existence of understanding and its relationships to the meaning which must be attached to a signifier.

## 6. Variables linked to school organization

School organization can be crucial to the possibility and incisiveness of the teaching of Probability and Statistics. In almost all educational systems such teaching, because of its recent introduction into academic programs, encounters all the problems related to every novelty (accentuated by the general ignorance of its basic concepts in almost the entire population) .

Often, one notes the lack of connection between different levels of study. For example, a daring reform regarding primary and middle school levels, which has promoted probability and statistics in the corresponding programs, has not been followed up in secondary schools, so that accumulated capital may be quickly lost, producing a rapid loss of acquired model-building abilities and a return to perceptual ones.

[2] A central problem of decision making in conditions of uncertainty, which are usually those which everyone must confront, is that individuals sometimes decide in ways very different from what economists and statisticians insist they should, that is, respecting prescriptions based on rational criteria. Notwithstanding this, one cannot affirm that individuals are irrational, indeed Descartes observed that intelligence is the good most widely spread among human beings.

Hungarian and also Italian experiences, for example, have shown and still show problems of the lack of vertical connection.

Moreover, a lack of horizontal connection is noted almost everywhere. A background reference analogous to that represented by historical synchronism, which is the supporting structure of teaching in the humanities complex, has not been widely used for the physical-natural disciplines.

In the last ten to twenty years some steps have been taken, above all for the linking of physical, chemical and natural sciences. Connections established between these disciplines and mathematics are less decisive, and practically non-existent with respect to Probability and Statistics, not withstanding that the latter could represent an important methodological adhesive.

On one hand, interdisciplinarity has had scarce attention in pre-university schooling; on the other, statisticians have invested little in the didactics of their discipline. It follows that in many cases the teaching of Probability and Statistics ends up being residual, unconnected with other disciplines, not even with mathematics where it is generally located. It is not by chance that the contents of Probability and Statistics are almost always introduced at the end of the academic year, or even of the educational cycle.

The construction of the curriculum, the definition of course prototypes, the planning of moments of interaction with other disciplines, the accumulation of a wealth of cases, are all goals to be pursued with great determination if we want to achieve results which are not banal or fragile.

Much national evidence emphasizes the importance of evaluation. The problem has different aspects. Firstly, it is clear that a system of evaluating and monitoring innovative experiences, such as that of teaching our subjects, is necessary to nurture the process of improvement.

We also need to consider that, in preparatory school, evaluation of the results of study is common. However, attention is directed at the final results of problems or exercises, and at the completeness of definitions. Instead, for our disciplines, and also because of their recent entry into teaching programs, priority should be given to evaluations relative to the process of reaching a result, and to the learning and use of models for reduction of problems. In other words, the evaluation of the learning of superior intellectual processes should be promoted, according to Bloom's taxonomo-cognitive objectives, which include the capacity to make applications, analyses and syntheses.

The actual teaching of Probability and Statistics is highly conditioned by the presence of corresponding components in the examinations. This is the evidence, for example, in the United Kingdom.

For our country, at the level of middle school, even though Probability and Statistics amount to 1/5 of all programs, examination problems on these topics comprise only in 5-10 % of the total questions. This certainly does not help.

One notes a divergence between modalities of teaching at university and pre-university levels in the experiences of many countries. Although pre-university teachers are generally trained in universities, when they return to schools they often, more or less quickly, regress to more common but not preferable schemes of teaching: from an interdisciplinary to a monodisciplinary approach; from an approach through models to perceptual evaluation; from an experimental attitude to a dogmatic one; from a predisposition to continuous up-dating to the defensive use of acquired training.

Until now, little has been studied about the presence of logical-deductive abilities and inductive abilities respectively in the school-age population. The experience of any teacher could testify that they are present in different proportions from student to student and this proportion is markedly linked to his/her original environment. Deductive abilities appear more widespread within higher social categories than within those less advantaged, as is immediately evident if one refers to the two categories of full-time students and part-time working students.

Generally, it is possible to argue that logical-deductive approaches to teaching can appear less democratic than inductive ones, to the degree that advantaged social classes are a minority in the school age population.

With regard to the teaching of Statistics and Probability, we need to add that it is largely justified by the contribution which it can make to understanding real experiences and problems. It leads to a preferring an inductive approach.

The link between the culture of origin and learning ability has been even less investigated.

To introduce a stimulating element for reflection, it is contended that more oppressive or dogmatic religions stimulate the acquisition of deductive abilities and hence predispose students to learning through axioms-theorems; while, naturalistic religions contribute to the expression of inductive abilities and predispose to learning starting from experience. On the other hand, if a culture or a religion forbids gambling, several possibilities for the learning of probability are precluded.

In the case of cultural minorities in which a non-standard language is predominant, such as in the ghettos of the United States, examples of a propensity to mathematical ovelinguistically based learning have been noted. Immigrants of very distant language families, such as Asian immigrants to the United States, have demonstrated great success in mathematical fields at university level.

These kinds of topics have been little considered in the debates in West European countries which have been relatively homogeneous from a cultural viewpoint. They will become more and more important however, as the flux of immigration from less developed to more developed countries increases.

## 7. Some suggestions

It would be useful to create a network for the exchange of experiences and information amongst centers, teachers and administrators. It is possible that many of the problems pointed out in the preceeding sections have been confronted locally, and that some results are already available. Knowledge of these could result in important and widespread effects.

ISI and IASE should promote research on the teaching of Probability and Statistics, taking into consideration the involvement of these subjects with other disciplines. Some possibilities have been indicated here. They need to be probed further, preferably in an international workshop, after analogous preparatory national workshops, of a very selective nature, with representatives of numerous disciplines. It could be an occasion to return to reflection on a wide horizon.

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