

# Statistical Reasoning Adequacy on Elementary School Teachers of Mathematics for Interdisciplinary Work, in Lavras, Minas Gerais, Brazil

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## Introduction

Every day people receive from the media innumerable pieces of social and economic information with statistical aspects, represented by graphs and tables. In this context, the teaching of statistics has basic importance to provide the citizens a foundation for the understanding of its reality, by means of the discussion and manipulation of this information, and, at the same time enabling them to make well-informed decisions and not to become hostages of the interpretation of data. The ability to master statistical reasoning configures a new type of inclusion and differentiation.

To strengthen such necessities, in accordance with the National Curricular Parameters elaborated and published by the SEF/MEC, subjects as Combinatorial Analysis, Probability and Statistics are recommended in the block of content "Treatment of the Information" of the programs of Mathematics for Elementary School, since these subjects make possible "the development of particular forms of thought and reasoning involving random phenomena, interpreting samples, making inferences and communicating results by means of the statistical language" (BRAZIL, 1998:134). Soon, it is evident that all the efforts must be used for aiming an urgent statistics literacy of the pupils.

In the meantime, one of the biggest difficulties in introducing probability and statistics in elementary school is that mathematics teachers had not gotten a good understanding of such subjects during their training. Thus, many times these teachers present these subjects with the exactness, the determinism and the calculation that the mathematical tradition imposes, opposing themselves of this form to the exploration of situations that involve approach, randomness and estimations. This lack of experience in the "statistical way to think" seems to imply not just a wrong way of teaching the statistical methods, but also a lack of interest for the subject by the teachers. For these reasons it is necessary to prepare better the mathematics teachers for working with statistics, which means first to motivate and to prepare not the pupil, but the teacher.

We also have to emphasize that the statistics it is not and never will be an activity as an end, but as a means, so mathematics teachers must strengthen the importance of the adequate use of the concepts and procedures of statistics in the organization, the presentation and the summary of data involving diverse areas of knowledge. It is evident that statistics, for the proper nature of the involved concepts, leads to an interdisciplinary pedagogical treatment.

In front of these facts, some questions are pertinent, specifically: How are statistics and probability taught in this level? Which is the teachers' degree of knowledge of these subjects?

Which pedagogical resources can be used for the qualification and motivation of the teachers?  
How can these contents be faced by means of interdisciplinary projects?

With these questions in mind, we aim to answer them in this article. We initially sketch a general panorama of the statistics education in grade II of the elementary school, in the city of Lavras - MG, by interviews and diagnosis tests, with a group of twenty-one mathematics teachers from public and private schools as well. At a second stage we conducted a pedagogical workshop named: Adequacy of the Statistical Reasoning for Interdisciplinary Works. Three meetings of 4 hours each were held, focusing on the idea of developing critical thought by means of the teaching of probability and statistics without the exaggerated formalisms of formulas and concepts, appealing to the contextualization, the globalization and the interdisciplinarity that these subjects can propitiate.

### **A priori analysis**

As previously said, we initially carried through research aiming to diagnose the teaching of statistics in Lavras, such research, named Evaluation of the teaching of statistics in the elementary school in Lavras, Minas Gerais, Brazil (Rodrigues et al, 2006). We concluded, in this preliminary work, that the education of statistics in our city was restricted only to the manipulation of formulas, tables and graphs. This prioritizing of descriptive statistics does not prepare students for the demands of this century. It seemed to us that the focus of the problem was the lack of preparation and motivation of the teachers, not the pupils.

Based on these conclusions, and following some methodologies oriented by the works of Cordani (2001), Batanero and Godino (2002), Lopes (2003, 2004), Mendes and Brumati (2003), Coutinho et al (2004), we started to elaborate a short-term plan of action that could contribute positively to change this situation. We prepared a pedagogical workshop that was offered for 21 teachers from public and private schools. This qualification was composed of three meetings with the following themes: Exploring the estimators; Making right decisions: a statistical problem; Influence of previous knowledge in a Bayesian treatment. We choose these topics because, as we had previously said, most of the teachers do not work with problems involving statistical inference, restricting education just to the descriptive treatment. Working these topics, the teachers had a chance to carry through diverse activities involving point and interval estimation, observing as well how to use the previous knowledge of a phenomena to infer upon it (Bayesian treatment).

Every meeting had a theoretical component and some activities of practical character, named as didactic sequences, which aimed to a deeper knowledge and to provide integration between the participants, as well as emphasizing the viability of its application for pupils of grade II of the elementary school. There were presented situation-problems where the teachers, individually or collectively, were challenged to surpass the mere descriptive context for a conclusion that involved an inference, always followed by uncertainty.

Before starting the first meeting we applied a survey, made up of three open questions, to evaluate the level of knowledge and significance of these subjects for the teachers.

The first question presented the results of survey applied to a sample carried out in the website of the Brazilian Society of Diabetes, showing the opinion of specialists and carriers of diabetes on the use of frozen embryos for cell-trunk attainment to be used in research. We raised the following question:

*Do you think that this result can be taken as the opinion of all Brazilians?*

The idea was for the teachers to realize that there were problems of sampling in this survey, since a huge part of the Brazilian population does not have access to the Internet, beyond the fact of the web site is directed to a specific public, therefore this result could not be extended to represent the opinion of the population (bias in the sampling).

In the second question we presented two electoral studies carried through the IBOPE (Brazilian Institute of Public Opinion and Statistics) for the positions of president and governor of Rio de Janeiro. The two studies presented the margin of error of plus or minus two percentage points, considering a confidence degree of 99%. We figured that the teachers would

realize that in both studies, some intervals overlapped and that was not possible to form a definitive conclusion from them (technical tie).

Finally, in the third question, we present Table 1 with statistics of the State Department of Roads, with the number of accidents with victims, fatal or not, and the conditions of the main driver involved, sober or drunk:

TABLE 1 Statistics from the State Department of roads

Driver/victim	Not fatals	Fatals
Sober	1228	275
Drunk	239	76

After that this questioning was placed:

*Would you say that the fact of the driver to be drunk or not affects the occurrence of fatal victims?*

We expected that the conditional probabilities were calculated, even if informally, and concluded that the drunk drivers had more possibility of being involved in accidents with fatal victims.

We verified that the results of the diagnosis test had been unsatisfactory. Then this test served not just as a diagnosis, but as an orientation for the other two meetings as well.

### First meeting - Exploring estimators

We started the first meeting, exploring the idea of estimate, presenting a parallel between the determinism of mathematics and the uncertainty of statistics. We also approached some basic concepts of sampling, emphasizing the importance of a representative sample to get a more accurate estimation. We finished the meeting by working with descriptive statistics, showing how to get punctual estimates of some population parameters and how can it be used to manipulate information.

Specifically in this meeting three didactic sequences were carried through. In the first one we presented nine scenes showing research published in magazines or newspapers that presented problems of sampling and doubtful inferences or without a scientific base, where the participants had been challenged to identify errors and important omissions, as for example, the research represented in Figure 1.



FIGURA 1 - Voting through web site [www.superinteressante.com.br](http://www.superinteressante.com.br)  
Font: Super Interessante Magazine, ed. 212 (04/2005).

The second didactic sequence aimed to calculate the average time on teaching for the participant teachers. First we choose three of the more aged teachers and then we randomly choose three participants more. Calculating average time of the first and the second groups, we inquired which of the two averages represented a better estimate for the average of the

population (participant teachers), evidencing the importance of the randomness of the sample. After that 10 participants were drafted, and a new average was calculated. Again we inquired on which would be the best estimate for the average of time on teaching for the participant teachers, emphasizing the importance of the size of the sample.

Finally at the third didactic sequence, we presented the process of the capture and recapture technique for estimating the size of the population, using balls in an opaque container. Each participant captured, marked and returned a ball to the container; recapturing after that and calculating the ratio the number of marked balls and the total of removed balls, and thus they estimated the total number of balls. They had repeated the process four times and had the results compared. Each ball in this activity represented an individual of the Royal Sparrowhawk species, a bird of prey that is in the list of animals in extinction risk. The choice of this animal was aiming to illustrate the possibility of carrying out an interdisciplinary project. We emphasize that presenting statistics as a link among the contents to carry out interdisciplinary work was a general theme of all the meetings.

### **Second meeting - Making right decisions: a statistical problem**

At the second meeting we promoted discussions on the use of information based on interval estimation. Initially we presented the idea of intervals of confidence ( $[a, b]$ ) and raised the following questions:

*How to construct such intervals?*

*It could be that we can establish arbitrary values for "a" and for "b"?*

*With the intervals established will we be able to make some type of decision?*

To answer to these questions initially the following definition of sampling error was introduced:

*Error is the difference between the estimate of the sample and the true population parameter. When the true value of the population parameter is unknown, the error also will be unknown, having then to be estimated.*

This definition caused a huge discussion, since for some of the teachers it did not make sense to estimate an error, because in this process of estimation another error would be committed and so on. At the end of the discussion the teachers concluded that the sampling error is a random error. In this way we showed that for estimating such error, it is necessary to establish how reliable is the information that is being supplied (degree of confidence). The estimation of the error is important for the construction of the intervals of confidence, and thus from this estimate, we can establish the values of "a" and "b". In this way, the two first questions previously posed had been answered. A new question appeared then:

*What is the relation among reliable degree, error and size of the sample?*

For many teachers the bigger the sample, the smaller would be the error. Another discussion came out, since many researchers present a relatively small number of interviews, which could imply a high value of the error. In this context the teachers again debated topics such as reliable degree, the representativeness of the sample, and methods for determination of a sample size on the basis of a maximum error previously established by the researcher.

A consensus appeared among the teachers: it would not be difficult to make a treatment of this type with pupils, these concepts being feasible to be used in all the series of grade II of elementary school, in accordance with their cognitive level.

After this discussion we started a didactic sequence with the intention to characterize and interpret a confidence interval. We proposed to the teachers to construct confidence intervals for the average time on teaching (didactic sequence of the first meeting) with different degrees of confidence (0%, 5%, 50%, 95% and 100%).

Some ideas on testing hypotheses were discussed only with the intention to show that we can wrongly accept a false hypothesis, or wrongly reject a true hypothesis without being certain, emphasizing the consequences of this decision and the importance to be well based for not committing an error that can cause great damages.

And for the end the meeting, the idea that from the intervals of confidence we could make decisions was strengthened, with the proposal of a didactic sequence with some research of opinion, with the objective of each one of the teachers verify if it was possible, on the basis of the information presented, to construct intervals of confidence and to make some type of decision, as for example, the research represented in Figure 2.

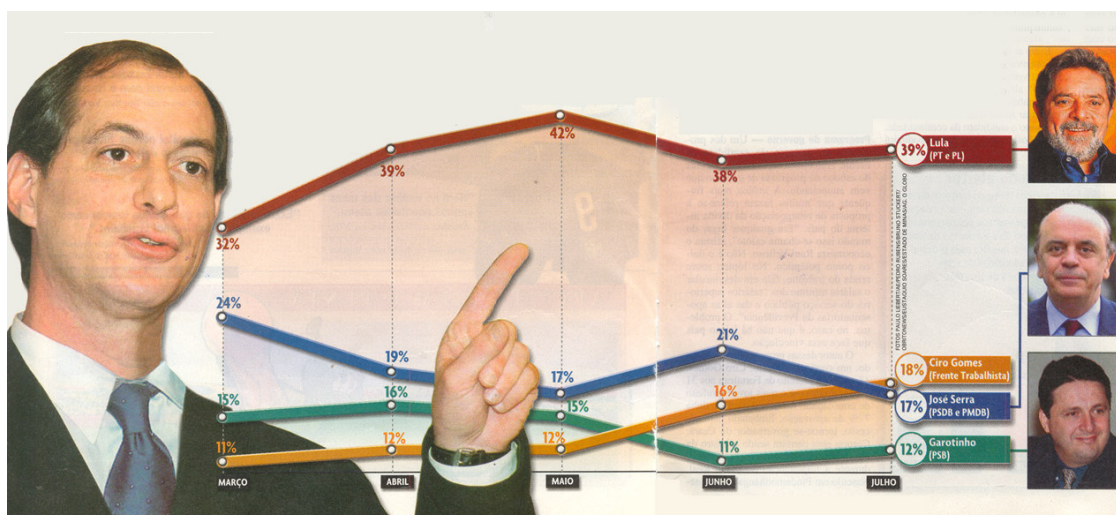


FIGURE 2 – Electoral Research for presidential elections  
 Font: Veja Magazine, year: 35, nº: 27 (2002).

### Third meeting: Influence of the previous knowledge in a Bayesian treatment

At the third meeting the following topics were presented: a priori information; conditional probability and Bayes' Theorem. The first challenge was to determine the form of dealing with these subjects on the grade II of the elementary school.

Initially we worked with some examples showing how information or knowledge a priori can assist at the moment of a person making some affirmation, or estimating a value, or "to guess" the probability of occurrence of one event. One of the questions was:

*What is the probability of rain on December the 25th (Christmas) on the city of Lavras?  
 And on the city of Rabat, Morocco?*

The idea was to discuss with the teachers who, being resident in Lavras and knowing about the meteorological conditions of previous years in the same period, had better conditions to estimate a probability, as opposed to an estimate for a city which at best they've heard about already (Rabat).

We started then from simple situations such as this one to some more elaborate, only with the intention to show that the concept of a priori information can assist in situations with different degrees of complexity.

To present conditional probability, we adopted initially a didactic sequence that worked with simpler properties of probability, the frequency concept, union and intersection. The teachers individually answered a set of five questions, with the intention to verify their previous knowledge.

After this we presented another didactic sequence, distributing a die for each pair, to answer two questions, this being one an example of one of them:

*Given that the number that came out is even (event A), which is the probability of it being a prime number (event B)? Calculate the probability in the launching of the die of getting a prime number also?*

They had carried through this activity without the formula of the conditional probability being presented, just for the teachers to perceive that it is possible for the pupil to answer questions involving this subject based on intuition, which means without necessarily knowing the theoretical formularization. The results presented for them had been considered satisfactory. We have to emphasize also that the concepts of dependent and independent events had been managed. This adopted procedure is in accordance with the ideas presented by Coutinho et al (2004).

To present Bayes' Theorem, a cross table with the following context was used as example:

*Suppose that 300 women, who suspect to be pregnant, had taken a test to diagnose the pregnancy, with the following results (Table 2):*

TABLE 2 Results of the test of pregnancy of 300 women

Pregnant	Diagnosis		Total
	Positive	Negative	
Yes	216	24	240
No	18	42	60
Total	234	66	300

We presented to the teachers the following questioning:

*For our example, given that you received the result from the test and it is positive, what is **the probability of that finding is wrong?***

Using only the information of the table the probabilities were all calculated in an intuitive form, as for example, the probability of that the result gives positive when the woman is actually pregnant. And that was the procedure until the determination of the probability of interest. Just at the end Bayes' Theorem was presented, culminating with a new didactic sequence with a similar problem, this one on breast cancer.

To finish we discussed the real possibilities of the application of these concepts for grade II of elementary school, as well as the development of interdisciplinary work. In front of what was showed, most of the teachers concluded that the subjects are pertinent and depending on the group the use of them in the practice of the classroom is possible, without the presentation of formulas for the initial series.

It is worth pointing out that, to the end of the first and the second meetings, two extra didactic sequences had been carried through, involving the concept of geometric probability, as a confrontation of the results with the frequentist probability: the classic problem of the Needle of Buffon (Tunala, 1995) and the problem of the Pasta (Wagner, 1997). We also took advantage to strengthen and explore concepts as sample, estimate, randomness, simulation.

### **A posteriori analysis**

At the end of each of the meetings, the teachers had answered the same question of reference of the test of initial diagnosis. Beyond this formal verification, we had verbal stories and writings of self-evaluation, being possible to verify a sensible improvement in the results.

As an example, the 1st question of the diagnosis test, it was observed that at the first moment many teachers did not have well defined concepts of sampling, since out of the 21 teachers, 7 had answered that the research could be extended as opinion of the Brazilian population, and 14 had answered that it could not, and among these the great majority had justified that they had answered "not" for reasons completely different from the sampling concepts. In the second application it was observed an improvement, since only 1 teacher

answered yes and 20 had answered no, among which 16 had answered that it was biased sample.

An evaluation in form of questionnaires of some pedagogical aspects of the workshop and the monitors was still requested, the results being presented in Table 3.

TABLE 3 Results of the evaluation of the given pedagogical workshop for 21 professors of Mathematics.

Questions	Excellent (%)	Good (%)	Regular (%)
Practical applications of content, motivation.	66.67	33.33	-
Review of the content with clarity.	71.43	28.57	-
Domain of content by the monitors.	80.95	19.05	-
Concern on the learning of the group.	85.71	14.29	-
Communication and relationship with the group.	80.95	19.05	-
Indicate your degree of learning with the lectures.	38.10	57.14	4.76
The choice of the contents was adequate.	66.67	33.33	-

In this same evaluation, beyond the questions presented in Table 3, the teachers had given some suggestions. Most of them asked for new lectures with related contents to statistics, prioritizing the character of interdisciplinarity, strengthening the necessity of updating and qualification of the mathematics teachers. As example of a suggestion:

*"The course was very interesting, to alert about received information within the day-by-day, through magazines and medias, making us to become more participative citizens and with critical conscience. The most important was the update to use the results as teachers".*

### Conclusions and future Works

In a global evaluation of these three meetings we perceive initially a great demand of qualifications for the mathematics teachers, in particular in the area statistics, since they present a strict deterministic conception, with a formal and Cartesian mathematical reasoning. During the meetings teachers developed a maturity and adequacy of the statistical reasoning and an increasing motivation for interdisciplinary works, that is, we saw to the possibility to develop activities with disciplines as biology, history, geography, Portuguese, philosophy, and sociology, among others.

We aimed to awaken the idea that it is possible to work the statistical contents in grade II of the elementary school without an exaggerated formalism (many times without meaning), privileging the reflection and the understanding of the managed concepts.

Although we received a positive evaluation of this work, we see the necessity to plan more pedagogical workshops aiming to promote a stronger foundation for the teachers in service. In addition, we expect to initiate a deeper debate on the diverse ways to motivate and to prepare the teachers, especially on the course of majoring in mathematics.

Working with the teachers on all these discussed aspects, we expect to reach our greatest objective: to urgently promote the statistics literacy of the pupils on grade II of elementary school.

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## ABSTRACT

*In the first stage of this work an evaluation of teaching statistics in grade II of elementary school was carried through, by means of interviews with 24 mathematics teachers of public and private schools in Lavras - MG. The results showed that most of the teachers attended a course of the disciplines of statistics in its process of formation, but, despite this fact, when faced, they teach it in a strict and descriptive way, with excess of formalism and without context. In a second stage, with base in this research and aiming to contribute with the improvement of education, three pedagogical meetings were conducted (of 4 hours each), in the format of workshops with 21 other teachers. The objective was to make possible an extrapolation of formal and Cartesian mathematical reasoning to statistical reasoning, focusing on uncertainty. A diagnostic test was applied initially to make a previous analysis of the degree of knowledge of the teachers with the subjects which would be proposed to work with: concepts of sampling and descriptive statistics, with emphasis in the punctual estimation; interval estimation with a purpose to make decisions; influence of the previous knowledge in a Bayesian treatment. During the meetings, some didactic sequences were proposed, always involved with a treatment to interdisciplinary work. At the end of each meeting, the same question of reference of the initial diagnostic test was applied, with a reasonable improvement in the results. In this way, associated to the verbal stories and writings, a change in attitude to statistics was perceived, followed by a maturity and adequacy of statistical reasoning. Although a positive evaluation of this work, in a collective reflection, we see the need of planning, in the short and medium term, more courses and workshops of basic concepts in statistics for the teachers, as well as an urgent statistics literacy on the pupils.*