CASE STUDIES FOR THE FRAMEWORK ERIE INITIAL VALIDATION

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ABSTRACT

ERIE (acronym for the Spanish term Esquema de Razonamiento en Inferencia Estadística, Framework for Inferential Statistical Reasoning) is a framework for teaching and assessing elemental inferential statistical reasoning, Moreno (2003), Vallecillos and Moreno (2005). In this paper we describe the case studies consisting of interviews with selected students. The interviews had the aim to validate the ERIE in a laboratory study. The analysis of the interviews made it possible to situate the groups of students (by grade) in their response levels and place each student at a level for each construct, thus validating the constructs and levels, and overall, the ERIE as a whole. Although this is the main objective for conducting and analyzing the interviews, this study also provided valuable information on basic aspects of learning elemental statistical inference such as previous conceptions of the students or the meaning attributed to certain concepts, such as randomness, fundamental to research.

INTRODUCTION

Statistical and probabilistic training in the modern world is a recognized need today in educational systems at all stages of teaching, not exclusively for higher education (NCTM, 2000). In Andalucía (Spain), the latest curricular reforms for primary and secondary school have included statistical inference as a new subject. The recommendations for ESO (curriculum for students 12-16 years old; Junta de Andalucía, 1992; 2002) refer to methods of data gathering and analysis as well as the intuitive approach to representativity of samples and to statements that can be made from their study. The Bachillerato (curriculum for students 16-18 years old; Junta de Andalucía, 1994, 1997), includes sampling, problems related to the choice of samples, conditions for their representativity, and the analysis of the conclusions that can be drawn from their study, as well as introduction to different types of sampling.

Although valuable works are already contributing our knowledge concerning the learning processes of students at all grades great effort is still needed in theoretic research to provide learning models developed in the Statistical Education own field. Antecedents in this line are the works of Wild and Pfannkuch (1999) on statistical reasoning and Jones et al. (2000) on statistical reasoning. Moreno (2003) also follows this line, seeking to formulate, refine and validate an initial theoretic framework, ERIE, to describe the reasoning in elemental statistical inference in secondary students. Table 1 summarizes the complete ERIE.

Moreno (2003) describes two empirical studies and a cases study. The present work describes the case studies consisting of interviews with selected students. The interviews had the aim to validate the ERIE, formulated by the two previous empirical studies results, in a laboratory study. The analysis of the interviews made it possible to situate the groups of students (by grade) in their response levels and place each student at a level for each construct, thus validating the constructs and levels, and overall, the ERIE as a whole.

THE INTERVIEWS: AIMS, SAMPLE AND METHOD

The interviews were carried out with the aims to test the ERIE in a laboratory study and can be classified as semi-structured. The analysis of the interviews, furthermore to provide an in-depth analysis of the student responses in order to explain them, made it possible, as we say before, to situate the groups of students in their response levels and place each student at a level for each construct. The questionnaire used in the second experimental study, Moreno (2003), served as the script and it contains several questions about the four constructs considered. On this questionnaire, completed by the student before the interview, the researcher had made a series of questions that

Table 1: Theoretic framework ERIE

CONSTRUCTS	LEVELS			
	N1: IDIOSYNCRATIC	N2: TRANSITION	N3: QUANTITATIVE	N4: ANALYTICAL
POPULATION AND SAMPLE (PM)	The student uses the normal concept of population. Does not use the statistical terms of population or sample. Does not identify the population or sample in most contexts. Does not recognize sample variability. Does not construct sample space.	The student uses the normal concept of population. May present difficulties using the statistical term. Focuses on simple elements to define sample, a single characteristic or an example. Sometimes can refer to more than one but without relating them. Identifies the population but not the sample in most contexts. Recognizes the variation between the different samples but attributes it to chance, as if it were a supernatural force.	The student describes the population based on more than one characteristic although not in a complete way. Defines the concept of population in statistical terms. Describes the sample in several contexts and does so using all the elements necessary although without relating them. Recognizes sample variability, although not in all contexts, and explains it in statistical terms. Constructs the sample space in the concrete context and does so systematically.	The student completely describes the population and refers to it in statistical terms. Identifies the sample in all contexts and describes it using all the elements necessary, relating them appropriately. Recognizes sample variability and explains it in statistical terms. Identifies the sample space and constructs it systematically.
INFERENCE PROCESS (PI)	The student undertakes the task but does not complete it. The responses follow personal expectations concerning the composition of the population in most contexts. Shows prior conception.	The student gives responses conditioned by a single aspect: variability (determinist conception), equal-probability bias, or the nature of the population. Shows a determinist conception.	The student describes the population composition as similar to that of the sample in most contexts. Uses numerical criteria for calculations. Shows an identity conception.	The student shows that the composition of the population studied cannot be known by studying one of its samples. Uses numerical criteria and formal language to express this. Shows inferential conception.
SAMPLE SIZE (TAM)	The student shows indifference to the sample size. The estimation is based on personal criteria. Gives no criteria to establish an appropriate sample size in most contexts.	The student selects the sample size based on aspects of little relevance such as expectations concerning the population or ease of calculation. Expresses determinist ideas concerning the sample. Selects a sample size based on a single relevant aspect in most contexts.	The student recognizes the influence of sample size in the estimation but does not relate them, or does so subjectively. Uses criteria for establishing an appropriate sample size in most contexts.	The student recognizes the influence of sample size and uses detailed statistical criteria. Analyses the suitability of the sample size of the population under study.
TYPES OF SAMPLING (TIM)	The student’s choice of the sampling method is subject to irrelevant personal expectations. Does not distinguish between random methods. Does not recognize sources of bias.	The student distinguishes some types of sampling using statistical terms but does not have criteria to distinguish between random samplings. Does not consider the sampling methods adequate, and prefers censuses. Does not value random methods nor identifies sources of bias.	The student recognizes bias in the non-random sample although does not always identify the source. Recognizes that the non-random samplings present more bias than do the random ones. Distinguishes between random sampling methods but not always correctly.	The student recognizes the possibility of the presence of bias in the non-random sampling and identifies the sources. Prefers random sampling over the non-random to conduct a poll. Finds differences between the random methods.

could be modified in some cases according the students answers. Each interview, taped in audio and afterwards transcribed, lasted some 20 min. An attempt was made to present each question in the terms, language and perspective of each student. The sample was intentional and 8 students were interviewed—three from 3rd and 4th of ESO, and two from 2nd of Bachillerato.

RESULTS: LEVELS OF RESPONSE STRUCTURING BY GRADE

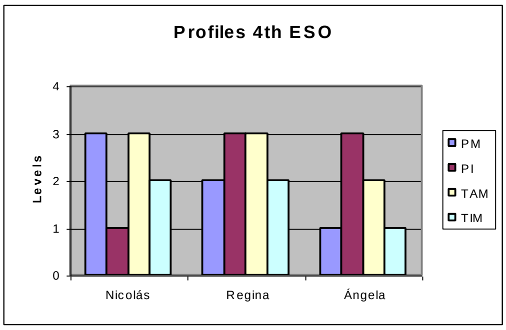
The students interviewed from 4th of ESO were Nicolás, Regina and Ángela. None of them gave responses at the Analytical level in any of the constructs, although, in most cases, the students were situated at the Transition and Quantitative levels in at least two constructs. One student gave answers at the Idiosyncratic level in two of the constructs. The three students in 3rd of ESO were Alejandro, Myrian and Ismael. These students, overall, gave responses of higher levels than did the students of 4th, despite the difference in age. The responses at Idiosyncratic level corresponded to the TIM construct, which refers to the types of sampling, in which we found generally greater difficulties and erroneous basic conceptions of randomness, etc.

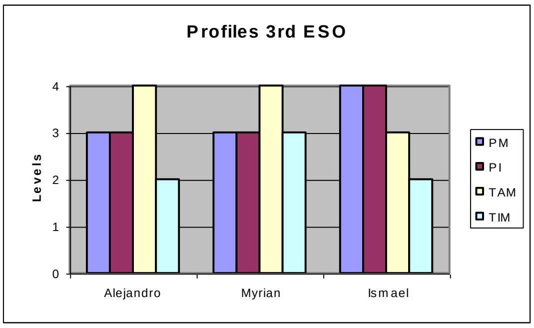
The two students of 2nd of Bachillerato were Cristina and Jesús. The responses of the Bachillerato students contrasted sharply with the previous results, with no response in the Analytical level, and most in the first two levels, Idiosyncratic and Transition. For these students, it is necessary to look for additional explanations related to curriculum or another that would complement the information gathered to date.

RESULTS: PROFILES OF THE STUDENTS INTERVIEWED

We finally examined the profile of each student interviewed. In each case, we studied the stability of their responses at the levels of each construct.

In 4th of ESO, Nicolás and Regina were situated at the Transition and Quantitative levels in three to four constructs, while Ángela was placed at the Quantitative and Idiosyncratic levels. This latter student was classified as predominantly Idiosyncratic, Nicolás as mainly Quantitative, and Regina as between Transition and Quantitative.





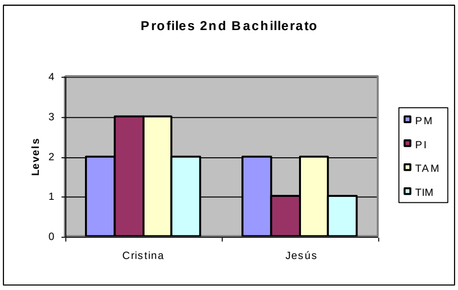
Graph 1: Profiles of students from 4th of ESO   Graph 2: Profiles of the students of 3rd of ESO

The three students of 3rd of ESO were clearly situated between the Quantitative and Analytical levels. Only Alejandro and Ismael were classified at the Transition level in the case of the construct referring to the different types of sampling. In the case of Ismael, Analytical reasoning clearly predominated, while Quantitative predominated in the other two.

The students of 2nd of Bachillerato were both situated between the Transition and Quantitative levels, Cristina being predominantly Quantitative while Jesús oscillated between Idiosyncratic and Transition.

The most stable profile corresponded to the students of 3rd of ESO, who maintained the Quantitative level in the three constructs PM, PI, and TAM. They reached a lower level of structuring in the construct TIM, which also proved more difficult for the students. Most of the students were stable at two levels and in two or three constructs. In no case did we find responses of different levels in each of the constructs.

The responses of the students interviewed did not fit any clear pattern of stability that would enable us to determine response levels that we could consider as a reference for students at this academic stage.



Graph 3: Profiles of students of 2nd de Bachillerato

For the moment, we have no reasons to venture possible causes for this fact, which may not even be necessary. At this point, we have evidence only of the need to continue the research to test ERIE and possible also the theoretic framework of SOLO.

CONCLUSIONS

The results from the interviews conducted afterwards enabled us to apply ERIE, in a laboratory situation, to selected students, and we found that it is possible to classify the students at their respective levels according to their responses on the questionnaire and in the interview. Therefore, we consider ERIE valid for characterizing the learning of elemental inferential statistics at the secondary-school stage. ERIE also enabled us to evaluate the overall learning of inferential statistics: the passage from one level to another also implied learning. Subsequently, we can establish the differences between the structuring levels of the response in terms of learning, such as the increase in the capability of handling concepts—for example, to relate the concept of sample variability and sample size, but also relate this with population variability and size. In relation to the application of the framework ERIE, we observed that the students in the highest grades had less difficulty in answering the questionnaires. In fact, the responses of these students were distributed primarily among the highest levels of response in ERIE, in agreement with the distribution by levels that would be expected for these students. The responses of the students of the earlier grades were distributed in the lowest levels of the framework ERIE, showing a greater number of difficulties, overall, to establish relationships between concepts involved in the process of making and evaluating inferences. Also, we found, in general terms, many cases of *décalage*—that is, the same student giving responses at different levels in different constructs, and sometimes in different contexts of presentation of the question. This result is of great pedagogical interest, as it advises against possible simplification of teaching, clearly revealing the difficulty of instruction in this subject matter at the secondary-school stage.

In this situation, together with the social need to improve it, the researchers in Educational Statistics bear the responsibility of in-depth research on this issue. Teachers needed their contributions for improve both teaching and learning statistics in all teaching level.

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