The Psychology of Mathematics and Mathematics for Psychologists

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The International Academy of Education (IAE: a scientific association that promotes educational research) published a booklet on improving students' achievements in mathematics (Grouw, D.A. and Cebulla, K.J, 2000). It focuses on aspects of learning that appear to be universal in much formal schooling, although the research it is based on mainly comes from primary and middle school education. Nevertheless, the practices that are recommended in the booklet seem likely to be generally applicable. Even so, the authors caution that the principles they advocate should be assessed with reference to local conditions, and adapted accordingly.

It will be argued that these principles can be generalized to the teaching of psychometrics, statistics and mathematical modeling to psychology students, but that indeed adaptation to the context of teaching mathematics-related subjects to psychology students will be in order.

According to Meier (1993) the results of Aiken et al.'s (1990) survey of U.S. graduate departments of psychology amply documented the lack of emphasis placed by those departments on psychological measurement. Only one fourth rated their graduate students as skilled with psychometric methods and concepts. Only 13 % offered test construction courses. Rated competencies declined with newer approaches in psychometrics: students are judged as most proficient with classical methods of reliability and validity measurement, but less so with factor and item analysis methods, and nearly ignorant of item response and generalizability theories. It led him to urge for a "revitalizing" of the measurement curriculum. Things may have improved during the last decade and may look somewhat better in Europe. However, no strong evidence is available to justify these optimistic expectations.

Meyer points out that this state of affairs can be characterized as surprising. On the basis of history of science it is generally accepted that new measurement techniques are driving forces in scientific development. It is therefore indeed surprising that contemporary psychological training seems to fail to recognize measurement's significance. But as psychometrics is increasingly intertwined with mathematical modeling, it can be conjectured that rather than a failure of recognition we are dealing with a kind of natural phenomenon. Hence, without neglecting the importance of measurement in psychology, we should try to carefully adopt a strategic approach when introducing courses in psychometrics (and mathematics related courses in general) in the psychology study curriculum. Considering the general principles on improving students' achievements in mathematics, we may try to derive a realistic approach.

1. Opportunity to learn

The term 'opportunity to learn' (OTL) refers to what is studied or embodied in the tasks that students perform. In mathematics, OTL includes (1) the scope of the mathematics presented, (2) how the mathematics is taught, and (3) the match between students' entry skills and new material.

Evidence: strong correlation was found between student OTL scores and mean student achievement scores in mathematics. Some of these correlations can be made more explicit as follows:

a) correlation between instructional time and student performance,
b) a strong relationship between mathematics-course taking at the secondary school level and student achievement (reports based on large scale studies show that 'the number of advanced mathematics courses taken was the most powerful predictor of students' mathematics performance after adjusting for variations in home background'),
c) textbooks are also related to student OTL, because many textbooks tend to review too many topics and do not contain sufficient content that is new to the student,
d) students must be given the opportunity to learn important content *and skills*: explicit attention must be given to problem solving on a regular and sustained basis.

In Flanders, and probably in most countries, the entering group of first year psychology students is quite heterogeneous with respect to the number of (advanced) mathematics courses taken in secondary school. This implies that a fair amount of variability with respect to mathematics performance can and should be expected. One way of remedying this situation is usually sought by adding additional course content covering notions from probability theory, calculus, geometry and linear algebra. This, however, creates problems with respect to (c): too much overlap for the more mathematically proficient students. Hence, preference should be given to a more flexible system where students have the option for courses that are taught on different levels. But this should be combined with sufficient course time (study credit points) to be invested in the more mathematically oriented courses so as to allow these students not only to absorb the new contents, but also to acquire the necessary skills that go together with the modeling approach in psychological research.

As a consequence of the Bologna declaration, most European universities are in the process of transforming their study curricula within the framework of the bachelor-master system implementation. For several reasons, a modular approach in the curriculum development has been recommended. If and when this approach is adopted, it seems natural to do it as well to allow for flexibility in study trajectories for students with different types of interests and backgrounds in mathematics education.

2. Focus on meaning

A second principle formulated by Grouw and Cebulla: focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning. By this they mean that students should see how abstract concepts are interrelated with each other and how they relate to every day experience. In generalizing this principle to teaching mathematical models related to psychometrics one should take advantage of the fact that modern psychometrics is much more concerned about construct validity than this used to be the case in classical testing where predictive validity was the main concern. Modern test theory (such as item response theory, cognitive decomposition theory) on the contrary provides methods for investigating more directly factors that influence the response behavior of the subjects and for classifying different types of items. These kinds of insights correspond in a natural way to the kind of questions as psychologists ask them.

The way Grouw and Cebulla exemplify this principle when talking about the teaching of mathematics on the elementary level shows a remarkable parallelism with how Ramsay (2001) justifies the use of mathematics in psychometrics. Grouw and Cebulla recommend to "emphasize the mathematical meanings of ideas, including how the idea, concept or skill is connected in multiple ways to other mathematical ideas in a logically consistent and sensible manner. Thus, for subtraction, emphasize the inverse, or 'undoing', relationship between it and addition." When talking about psychometrics, Ramsay confronts the reader with the following question: “What characteristics of the people taking tests do test scores and the data that are used to compute them reflect?” He then goes on with the remark: “Test scores are numbers, but not all structure of numbers are meaningful representations of something about examinees.” He proceeds by showing that test scores define *nearness* or *proximity* of examinees, and that by taking off from this basic (and easily understood) concept even algebraic operations of summing and differencing on test scores may be meaningful.

3. Learning new concepts and skills while solving problems

Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later. Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students without conceptual understanding are able to acquire procedural knowledge when the skill is taught, but research suggests that students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge. Therefore, it seems that conceptual understanding is a key factor, while problem solving is essential for self-control of the student with respect to the understanding of concepts and at the same time with respect to acquiring procedural skills. Teaching quantitatively oriented courses in psychology always should include problem-solving sessions.

4. Opportunities for both invention and practice

Research suggests that students need opportunities for *both* practice and invention: when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas. To increase opportunities for invention, teachers should frequently use non-routine problems, periodically introduce a lesson involving a new skill by posing it as a problem to be solved.

5. Openness to student solution methods and student interaction

Student achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics. One way to organize such instruction is to have students work in small groups initially and then share ideas and solutions in a whole-class discussion. This approach has been used extensively and with extremely good results within the framework of the Erasmus Intensive Program Seminars in Mathematical Psychology, where group problem solving and paper discussion sessions with interactions between students coming from different European universities are an essential ingredient of the teaching process.

6. Concrete materials

Long-term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes towards mathematics. In adapting this principle to teaching quantitatively oriented courses in psychology one could re-interpret concreteness in terms of relatedness to psychological observational data and to the questions from which these data originated. In his overview on the history of mathematical psychology Estes (2001) recalls that early scientific milestones of quantification go as far back as 1738 (risk measurement). But it took until the 1950s before mathematical psychology really caught on. According to Estes, this is due to developments in other scientific fields: the advent of computers, measurement theory, cybernetics, communication and information theory, formal models of grammar and signal detection. Again, according to Estes: “These events contributed directly to the expansion of mathematical psychology by supplying concepts and methods that could enter directly into the formulation of *models for psychological phenomena*. But perhaps at least as important, they created a milieu in which there was *heightened motivation for students to acquire the knowledge of mathematics* that would allow them to participate in the exciting lines of research opened up for the fledgling discipline of mathematical psychology.” (our Italics).

7. Use of calculators, computer software, computer based learning systems

They *can* result in increased achievement and improved student attitudes with respect to learning mathematics or mathematically oriented study material. Studies have also shown that additional positive effects can be expected with respect to students’ graphing ability, conceptual understanding of graphs and their ability to relate graphical representations to other representations, such as tables and symbolic representations (e.g. biplots and correlation coefficients). It is evident, that, where relevant (data analysis, psychodiagnostics…), the use of computer software is an additional skill that has to be acquired. It is, however, not evident that computer based learning systems always lead to improved understanding of basic concepts. They may be helpful in solving practical problems (distance learning, overcrowded classrooms), or when they are targeted at the acquisition of specific skills (e.g. with respect to visualization), but they do not as such guarantee better learning and/or better achievement (Morris, 2001).

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RESUME

La proposition est soutenue que l’enseignement de la statistique, la psychométrie et la modellisation mathématique pour étudiants en psychologie peut se fonder sur les mêmes principes qui sont avancées pour l’enseignement de la mathématique au niveau de l’education primaire et secondaire. Nombreuses recherches empiriques sur ces principes ont accordé un role très important à la motivation et l’intérêt des étudiants. Il faut donc stimuler l’intérêt dans l’emploie de la quantification dans la psychologie, mais en même temps, il faut aussi différentier les étudiants selon leur intérêt à étudier la psychologie.