Why Mathematicians may have Problems Teaching Model Assumptions in Statistics

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Introduction

I first became aware of the problems encountered when mathematicians with insufficient experience with statistical data are teaching statistics when I met Georg Schrage of the university of Dortmund. He was invited to Kristiansand and gave an excellent guest lecture which was translated to norwegian (Schrage 1985).

In 1988 David S.Moore had the target article in The College Mathematics Journal under the heading: “Should Mathematicians Teach Statistics?” There are several reasons why I agree with David S.Moore, but I will mainly stress the following: Mathematicians teaching statistics often lack hands-on experience with real data. Neighter is assessment of model assumptions a part of the training of mathematicians. Instead they often rely on certain ”principles” to compensate for their lack of experience.

Principle of Insufficient Reason

This principle was introduced by the reverend Thomas Bayes and published posthumusly in 1763. It was accepted by the majority of mathematicians before 1900, even by giants like Laplace. However around 1900 several examples were given indicating that “The Principle of Insufficient Reason” could be misleading. One famous example is Bertrand’s Chord Paradox: In a circle of radius 1 a random chord is drawn. What is the probability that the length of this chord exceeds $\sqrt{3}$? Three different answers: 1/2, 1/3, 1/4 can be given with different interpretations of what “random chord” means. In the 20th century R.A.Fisher was prominent in criticising “The Priciple of Insufficient Reason”. He pointed out that re-labelling of the sample space would change the probabilities if the principle is used. By 2003 we ought to throw this principle into the wastebasket of history. It comes as a mild surprise, therefore, to find a version of this principle in a recent book on mathematics intended for children!

Carol Wordeman: How Mathematics Work, Dorling 1996 In this book “The Principle of Insufficient Reason” is presented like this: **“Some events are random and can not be predicted. All possible outcomes events are then equally likely”.** To demonstrate how this is used in a “practical” situation the following problem is given in the text: “(1) A bag contains a ball known to be either black or white. A black ball is put into the bag, it is shaken and a ball is drawn and appears to be black. What is the probability that the remaining ball is black?” The answer to this question is given including a really nebulous version of the Principle of Insufficient Reason: “If we draw a black ball, we can not know whether the remaining ball is the original (which can be either black or white) or the new one (which is black). This gives three possibilities: Black, black white. Accordingly the probability of black is 2/3.” It seems to be taken for granted that the first sentence, (1) means: “P(Black) = ½, P(White) = ½”. But lack of knowledge is surely not the same as knowledge of probability.

The principle of insufficient reason is often used to argue that samples are “random” even when very little is known about their origin. Sometimes persons are characterised as “random” even when the population they are taken from are not clear. A popular “illustration” of probability where monkeys use typewriters may strengthen belief in “The Principle of insufficient reason”. I have always thought this example to be a joke, but recently I have learnt that at least some mathematicians think that it is obvious that all the buttons have equal probability every time a monkey makes a letter. To be able to compute the probability of a monkey writing some sensible text it is also necessary to use the concept of independence, which is even more far fetched than uniform probability in this highly artificial situation.

Principle of Insufficient Dependence

This name is an invention of my own. The principle expresses the misguided perception that if the researcher is unable to evaluate the amount of dependence between a set of random variables, they can be considered to be independent. Surely it will usually be hard to investigate the amount of dependence in a given situation. Statisticians can then tentatively choose to do an analysis using independence. But this analysis must be supplemented by an evaluation of the impact of dependence on the result, that is: A sensitivity analysis.

An example showing that grave misunderstanding can result from this principle was presented at the Conference of the Royal Statistical Society in Sheffield in 1992 and subsequently printed in Teaching Statistics 1994. The Airbus 320 has five flight computers working in parallel. This means that if one of the flight computers breaks down, the next takes over, and so on. It is claimed that the breakdown probability of each of the flight computers is 0.00001 per hour flight. From this it is concluded that the probability of computer breakdown is 0.00001^5 per hour flight. Apparently someone has thought it appropriate to assume independence between breakdowns of the five computers. The fact that two A-320 aeroplanes have crashed, probably because of computer failure, does raise serious doubts concerning the independence of the breakdown of different flight computers!

Even if monkeys have limited brain capacity it is completely unrealistic to think that they choose letters independently. As can be seen I agree with David S. Moore to a certain extent, but I think his introduction to his article is somewhat too categorical: “No! Statitics is no more a branch of mathematics than is econometrics, and should no more be taught by mathematicians. It is a separate discipline that makes heavy and essential use of mathematical tools, but has origins, subject matter, foundational questions and standards that are distinct from those of mathematics.”

In Norway we educate so few statisticians that we really need someone to take part of the teaching. I agree with David S.Moore that is may be better to use the best qualified people in introductory courses, but these people also need to be able to communicate such notoriously difficult topics as equiprobability and independence. Next year we will try one of the textbooks by David S.Moore and perhaps things will improve. In 1989 I presented a translation of some of David S. Moore’s article to Norwegian in TG, the newsletter of the Norwegian Statistical Association. It was met by thundering silence except for a colleague mathematician who felt personally offended.

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