Teaching Stochastic Finance in a Multimedia Environment

Christian Mohn
University of Oldenburg, Institute of Mathematics
26111 Oldenburg, Germany
mohn@mathematik.uni-oldenburg.de

Dietmar Pfeifer
University of Oldenburg, Institute of Mathematics
26111 Oldenburg, Germany
pfeifer@mathematik.uni-oldenburg.de

Discrete Asset Models

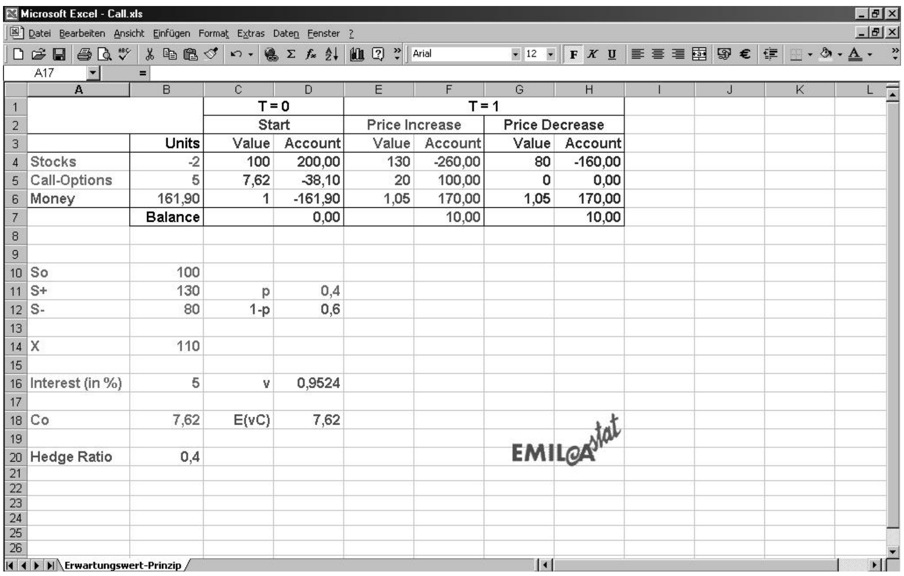
Teaching Stochastic Finance on a basic level, e.g. to undergraduates or even in school is a task which is hard to fulfil with common continuous theory. It takes a lot of mathematical prerequisites, sometimes it is not even possible to introduce them since the level of education among students is too low. This is one of the main reasons for the use of discrete asset models to describe financial markets and their behaviour. With discrete theory it is possible to show the fundamental ideas of asset-pricing without spending much time on mathematical prerequisites.

Besides this, the discrete asset-pricing theory is very suitable for visualization, especially computer-based multimedial visualization, an important fact in teaching a new subject to students. For illustration we use the CRR-model of a discrete financial market by Cox, Ross and Rubinstein (1979) and the examples of Eberlein (1998). The CRR-model describes a financial market as a number of assets mathematically represented by binomial trees where the steps indicate the discrete times of trading and the branches indicate the discrete states of the asset at each time.

In this model there are several interesting facts to visualize. In part we will discuss here the behaviour of the equivalent martingale measure in a one-period model and the no-arbitrage option price in a multi-period model. The software we use for illustration is MS Excel, but other spreadsheets could also be used.

Visualization of the equivalent martingale measure in a one-period CRR-model

In the one-period CRR-model we have a market with a riskless interest rate and a single sort of stocks with movements represented by a Bernoulli-distributed random variable. The goal is to visualize the problem of arbitrage if an option on the stock is considered. The spreadsheet in figure 1 contains the parameters of the market model in column B from line 10 to 18, that are the stock prices in all states, the strike price of the option, the interest rate and



		T = 0		T = 1			
		Start		Price Increase		Price Decrease	
	Units	Value	Account	Value	Account	Value	Account
Stocks	-2	100	200,00	130	-260,00	80	-160,00
Call-Options	5	7,62	-38,10	20	100,00	0	0,00
Money	161,90	1	-161,90	1,05	170,00	1,05	170,00
	Balance		0,00		10,00		10,00
So	100						
S+	130	p	0,4				
S-	80	1-p	0,6				
X	110						
Interest (in %)	5	v	0,9524				
Co	7,62	E(vC)	7,62				
Hedge Ratio	0,4						

Figure 1. Arbitrage-Table in a One-Period CRR-Model

the option price. Depending on these values the balances of portfolios at time of maturity (*T=1*) in case of an increasing and a decreasing stock price are calculated. The credit is chosen in such a way, that the balance of the portfolio at time zero is *0*. In line 7 of figure 1 these balances are displayed. Now, if one parameter of the model is changed - e.g. the option price - the balance is automatically recalculated. Thus, using the correct hedge ratio, the idea of arbitrage can easily be shown as well as the behaviour of the equivalent martingale measure p .

Visualization of option pricing in a multi-period CRR-model

In contrast to the one-period model we now have not only one trading period, but several. In the example in figure 2 there are 7 times of trading, the last one is the time of maturity of the option. As in figure 1 the parameters of the model are editable in column B (line 1 to 7). The stock price changes with an upwards volatility k^+ or a down-wards volatility k^-, where S_t is the stock price and C_t the option price. Depending on the parameters of the model the stock prices are calculated from time zero to time of maturity, and in a reverse way the option prices at each time by using the equivalent martingale measure p^* .

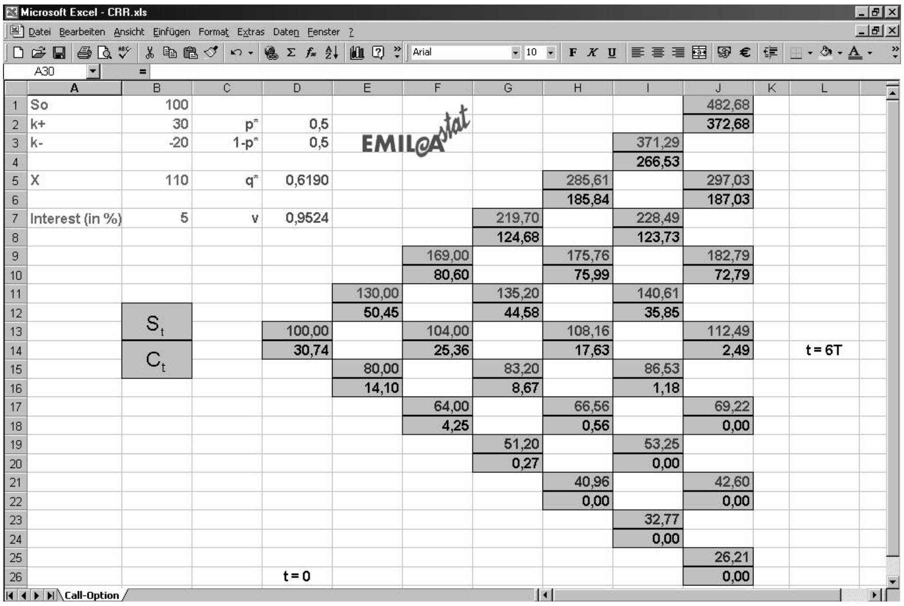


Figure 2. Screenshot of a 6-Period CRR Binomial Tree

Benefit from a multimedia environment

Significant benefits are for example the close connection between input and output. The result of changes in the parameters of the model can be observed immediately. Furthermore, the whole spreadsheet is editable, thus not only parameters can be changed, but it is also possible to change, add or delete any other value to test its influence on the result, a fact that is strongly recommended in explorative learning. Besides this, all formulas used for calculating can be viewed directly which is helpful to understand the connection between theory and visualisation.

REFERENCES

Cox, J. C. and Ross, S. A. and Rubinstein, M. E. (1979). Option Pricing: A Simplified Approach. Journal of Financial Economics 7, p. 229-263.

Eberlein, E. (1998). Grundideen moderner Finanzmathematik. Mitteilungen der Deutschen Mathematiker-Vereinigung, issue 3, 10-20.

RESUMÉ

La visualisation par ordinateur permet d'accéder simplement aux modèles de mathématiques financières stochastiques discrètes. Deux exemples de programmes de calcul sur des tableaux vont être présentés dans cet exposé