HOW TO TEACH SOME BASIC CONCEPTS IN TIME SERIES ANALYSIS

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Time series teaching needs some specific concepts that are not intuitive for most students. In this paper we consider an approach to guide the early stages of time series learning. First we introduce the time series definition and some examples of the sort of series which arise in practice. Here, we comment about their components: trend and/or seasonality. After this we talk about some useful transformations to stabilize the variance, to make the data normally distributed, to eliminate trends and/or seasonal components. These transformations are necessary to reach stationarity, which underlies an important class of stochastic models for describing time series. Finally we introduce the sample autocorrelation function, which measures the linear relationship between observations at different distances apart and provides an idea about a model for generation of the data.

INTRODUCTION

A time series is an ordered sequence of n observations, denoted by $\{x_t, t = 1, 2, ..., n\}$; the ordering is usually through time. Some real examples of time series are (see Figure 1):

Series A – daily humidity at Sao Paulo – Brazil from 03 / 23 / 1997 to 06 / 21/ 1997.
Series B – monthly inflation index in Brazil from January, 1970 to June, 1980.
Series C – monthly average air temperature at Ubatuba – Brazil from January, 1976 to June, 1985.

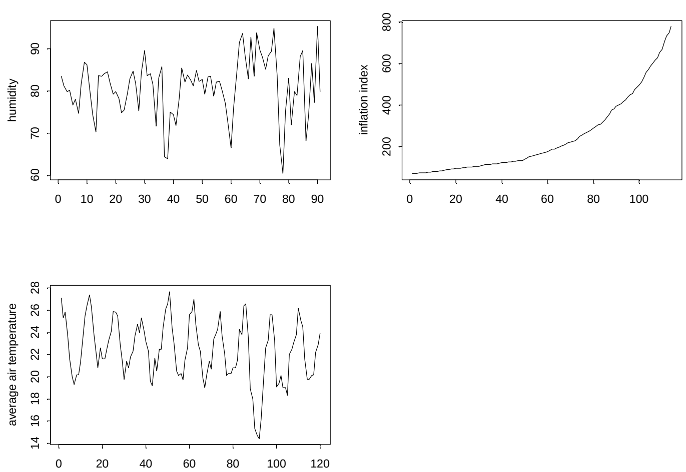

Figure 1: Examples of time series

OBJECTIVES OF TIME SERIES ANALYSIS

There are several possible objectives in analyzing a time series. For example:

a) Description – the first step is to plot the data to look for the possible presence of trends, seasonal variation, outliers (observations which do not appear to be consistent with the rest of the data) and turning points.

b) Modeling – to investigate the generating process of the time series; for example, analyzing a series of monthly values of sales of automobiles in Brazil, we can want to know as these values of sales had been generated.

c) Prediction – it may be useful forecast future values of an observed time series. This is an important task in analysis of economic time series.

COMPONENTS OF A TIME SERIES AND TRANSFORMATION

Most of time series theory is concerned with stationary time series, which assume that the series remains in equilibrium around a constant mean level. Nonstationary time series can occur in many different ways: they could have nonconstant mean (trend and/or seasonal variation), nonconstant variance or have both of these properties.

Through careful examination of its graph, we usually get a good idea about the components of a series. For instance, in Figure 1:

Series A – is a stationary time series.

Series B – has a positive trend and nonconstant variance; this means that we have a nonstationary time series.

Series C – is a seasonal time series with period s = 12 months; this series shows outliers on July and August, 1983.

Most nonstationary time series can be reduced to stationary series by proper transformations:

a) A logarithmic transformation is indicated to stabilize the variance in most of the cases.

b) A first-order differencing is usually sufficient to remove a trend. The new series $\{y_1, y_2, ..., y_n\}$ is given by

$$y_t = x_t - x_{t-1} = (1 - B)x_t = \Delta x_t \ , t = 2, 3, \ldots, n$$

where

B is the lag operator;

$\Delta = (1 - B)$ is the first order difference operator.

c) A seasonal difference is usually sufficient to remove seasonality. The new series $\{y_1, y_2, ..., y_{n-s}\}$ is given by

$$y_t = x_t - x_{t-s} = (1 - B^s)x_t, \ t = 13, 14, \ldots, n$$

AUTOCORRELATION

For a given observed time series $\{x_1, x_2, ..., x_n\}$ the sample autocorrelation function (ACF) is defined as

$$r_j = \frac{\sum_{t=1}^{n-j} \left[(x_t - \bar{x})(x_{t+j} - \bar{x})\right]}{\sum_{t=1}^{n-j} (x_t - \bar{x})^2}$$

where

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, is the sample mean of the series.

The ACF is an important guide to the properties of a time series. It measures the correlation between observations at different distances apart. This behavior is a powerful tool to identify a preliminary model for the time series. Figure 2 shows the ACF plots for the time series studied.

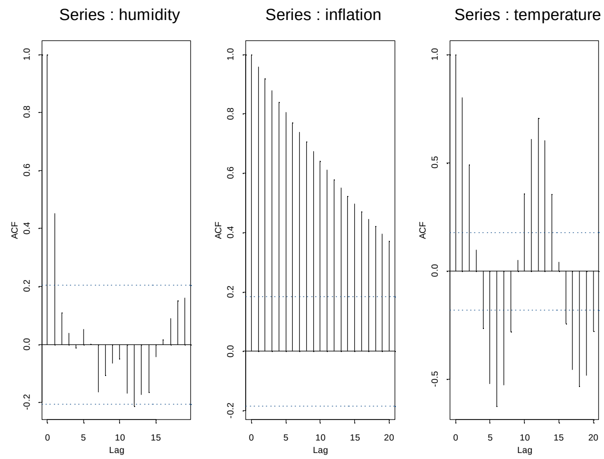

Figure 2: Autocorrelation function (ACF)

Figure 2 shows that:

a) The ACF of the humidity series decays to zero rapidly. This is the ACF behavior of a stationary time series.
b) The ACF of inflation index exhibits slow decay as j increases. This is the ACF behavior for data containing a trend.
c) The ACF behavior of temperature shows a periodic component with the same periodicity of the data (s=12). This always happens with the ACF of seasonal series.

CONCLUSION

Using real data in the learning process of basic time series has been very successful, because this awakes the students to the learning approach of time series.

It is very important to strengthen the main concepts such as stationarity, trend and seasonality under a very intuitive point of view. We know how difficult it is for beginners to understand the autocorrelation function, which is a tool that reflects these three aspects.

In this paper we define this function, showing graphically how each aspect can be described, to make learning easier for the students.

REFERENCES

Brockwell, P. J. and Davis, R. A. (1996). *Introduction to Time Series and Forecasting*. New York: Springer-Verlag.

Chatfield, C. (1996). *The Analysis of Time Series: An Introduction* (5th edition). London: Chapman and Hall.

Morettin, P. A. and Toloi, C. M. C. (2004). *Análise de Séries Temporais*. Sao Paulo: Editora Edgard Blücher.