EXPLORING IN SAMPLE SPACE: DEVELOPING YOUNG CHILDREN'S KNOWLEDGE OF RANDOMNESS

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This paper focuses on 6-8 year-old children's thinking about randomness. It reports the findings of a study in which the children engaged with a game-like environment to construct for themselves spatial representations of sample space. The system was designed so that the rules governing the relationships between the selection of elements of the sample space and the outcomes of the game were available for inspection and reconstruction by the children. In response to a range of tasks, the children manipulated the sample space in ways that generated corresponding outcomes in the game. We present a case study of children's activities, which illustrates how the novel medium mediates the children's expression and understanding of chance events.

INTRODUCTION

The definitive text on the development of probabilistic cognition was written by Piaget and Inhelder (1951, translated 1975). One of their experiments concerned a tilt box, where the child was given a rectangular box, resting on a transversal pivot, allowing seesawing. They concluded that children from four to seven years fail to understand the random nature of mixture. Further, they maintained that the idea of chance is not acquired before the stage of concrete operations, and that the understanding of chance presupposes an understanding of the irreversibility of mixture and requires the possession of a combinatorial schema. Their main argument is that a fundamental property of operations is that they are reversible, so that even when a child is a concrete operational thinker, randomness is still a strange idea that does not fit into the way they normally see the world.

The literature on randomness overflows with references to children's and adults' incompetence in dealing with judgements of chance (for just one of many references, see Kahneman et al., 1982, who set out a series of heuristics that, because of their inherent bias, have been termed by other authors as "misconceptions"). Given this emphasis on incompetence, the 'Piagetian' argument appears to have strong explicatory value. However, Fischbein (1975) has reported how children have intuitions for relative frequency from a very young age, and this leads us to search for other evidence that even very young children have emergent cognitive resources for making sense of randomness. Papert (1996) has argued that educators should 'look' in pragmatic ways for connections between pieces of knowledge and that, on a theoretical level, the metaphor of learning by construction leads to a range of interesting questions about connectivity of knowledge. Learning consists of making connections between mental entities that already exist. Pratt (2000) has reported on how children (aged 10/11) were observed to use four separable resources for articulating randomness namely: unsteerability, irregularity, unpredictability and fairness. Children constructed meanings for randomness in the context of a computational microworld, visualising how the random behaviour of objects actually worked. In this paper we describe the activities of even younger children (aged between 6 and 8) who constructed for themselves random behaviour by making spatial representations of sample space in a game-like environment. One special feature of this research is that the children were expressing their ideas for randomness in two-dimensional continuous space, through tools for directing and redirecting the simulated movement of balls.

THE SOFTWARE

The game was written in a rule-based medium, called 'Pathways' (funded by the European Union as part of the *Playground Project*, Grant No. 29329: http://www.ioe.ac.uk/playground/). Pathways was designed to afford children the opportunity to talk and think about probability in the context of quasi-concrete manipulations. The game had the following appearance (Figure 1).

Figure 1. The Game

In the right-hand corner, the reader sees a rectangle. A small ball bounces around the walls of the square and occasionally collides with and bounces off other balls. In Figure 1, one red and one blue ball are depicted (they are the other two larger balls inside the rectangle), but the child can control the number, size and position of the balls. Each time there is a collision with a red ball, one point is added to the red score (just above the yellow rectangle), and the "space kid" (the yellow triangular creature) moves one step up the screen. Similarly, collisions with the blue balls add one point to the blue score and move the space kid one step down the screen. Whilst individual collisions can be seen as single trials in a stochastic experiment the totality of these movements gives an aggregated view of the long-term probability. The game itself is defined in terms of simple iconically represented rules, which are designed so that the children can easily change the nature of the game itself (see Goldstein & Pratt, 2001, for a fuller description of the Pathways environment).

The task was for children to construct for themselves representations of the sample space. In order to explore children's connections between fairness and randomness, we began with a situation in which the children had to try to make the space kid move around a centre line in order to construct a 'fair sample space'.

METHODOLOGY

The findings reported here are part of a broader study that adopted a strategy of iterative design, in which the computer-based tool was developed alongside the gathering of evidence for children's use of the tool. The children were interviewed before and during their interaction with the computer: all interviews of each iteration were videotaped and transcribed. The role of the researcher was that of participant observer, interacting with the children in order to probe the reasons behind their answers and their actions. In the final iteration 22 children, aged between 6.5 and 8 years, were involved. The children worked with the software individually for between 2 and 3 hours. The semi-structured interview began with children expressing their ideas about Piaget and Inhelder's (1975) tilt task. The children were then asked to inspect the rules of the game through the Pathways tools, and finally worked with a range of game-oriented problematic situations.

EXPRESSIONS OF RANDOM MIXTURE

The results from the interviews on the tilting task were consistent with those of Piaget and Inhelder (1975). Most of the children responded to the 'Piagetian' task by expressing what the order of the colours should be when the balls move to the other side of the tilt box. For example, Jane said, 'The reds will move here, then the yellows, the greens here and the oranges here'. The children envisaged that the balls would finish up in an organised manner, rather than being mixed up in the tilting process.

In this context, the children were generally unable to express notions of random mixture. In contrast, they seemed to be able to express the notion of random mixture in the game-like environment. The computer environment supported several ways in which the idea of mixture could be expressed. The children changed the position, size or number of the red and blue stable balls in the sample space, or they described in words the 'uncontrolled' continual movement of the white ball. We describe four categories of children's strategies below.

1. Haphazard movement

Jane (age 6 7/12) characterized the white balls as follows: 'It moves by itself. It goes different places…' She also described this mixture in a very simple way: 'It (*the white ball*) moves up and down, right and left and when it touches one ball and gets a point it then goes *everywhere* in the yellow square'. Mixture for Jane meant to move everywhere, without any obvious pattern or pre-ordained positions. The children's awareness of a lack of pattern and lack of controls over the movement within the two-dimensional space is evocative of the local resources for making sense of randomness as reported by Pratt (2000). This apparent consistency suggests that the children were using similar internal resources or intuitions as one might expect in a more conventional study of randomness, and so lends credence to the idea that these very young children were indeed constructing meaning for a random mixture, in a way that was not evident in Piaget and Inhelder's (1975) experiment.

2. Complex movement

Paul (P below) expressed his idea of random mixture in a different way. He was influenced by the manipulations of the task, and he attempted to make all the balls bounce around, collecting points. (E is the researcher.)

P: Ok now! Ah! I know what to do!
He changes the balls and the speed again.

E: The blues have more points than the reds.

P: Do you know what to do? We can take out all the white balls and give a speed to the red and blue balls and when the blue touches to a blue or a red touches to a red ball to get two points otherwise to get one point.

Paul gave movement to the red and blue balls and decided to change the rules under which points were scored and the mixture of the balls was made more complex.

3. Symmetry of placement

a. symmetrical teams

Most of the children tried to achieve a fair game by placing the balls symmetrically. They didn't care so much where each ball was placed, but were concerned about the positioning of a team of the balls.

E: Can you do something to stay near the yellow line?

J: Yes! I'll put near all the red balls and then all the blue ones. Then, I will take these two teams and I will put them near to each other (Figure 2).

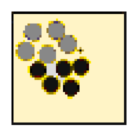

Figure 2

E: What about these balls here?

L: I can make something… I will do something not to be a big difference between them. To be near. I will do the same (Figure 3).

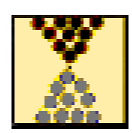

Figure 3

They saw each colour as one team. For example, Jane expressed randomness by first separating the two colours to create two different teams of equal numbers.

J: Because I will put them near to each other in the middle. So, when the ball goes to touch the one, it will touch also the other that's near it. So, it will touch both of them and we will have equal points. We do not know where it (*the white ball*) will go, but if it touches one ball it will touch the other as well.

E: That sounds a nice idea…

J: …I will put the two lines in the middle. Now, I need another two balls. I'll get the magic wand to get more balls. It's the good fairy that gave it to us…

E: Do you know how many balls do we have?

J: Yes…they are equal. I know they are equal, but I don't know how many balls I have (Figure 4).

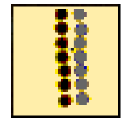

Figure 4

E: How do you know that they are equal?

J: I copied one red and one blue each time. It doesn't matter actually how many they are. They are equal.

This excerpt emphasizes that what mattered to Jane was the equality of the teams, rather than precisely how many there were. She seemed to equate equality with fairness, and presumably fairness in some way with randomness.

b. making circles

The circle was made to trap the white ball in order to touch the balls in the circle the same number of times, and sometimes it turned out to be a start for a symmetrical development, as well. Celia here started by having the white ball in a circle and then constructed another random situation by copying more balls.

A: …I'm going to make all the balls have the same size.
I'll do another arrangement.

E: So, what are you doing now? (Figure 5)

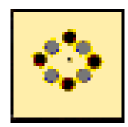

Figure 5

A: I'll make more copies of them.
She switches the game on.

E: Oh…does it work?

A: Yeah…It keeps going up, down, up, down….

E: Great!

A: I'll make more copies…

E: What's the arrangement now?

A: That one (the blue ball) is facing that one (the red ball) and that one is facing that one and so on… I've got also a better idea! They (the red balls) will face a blue one. There! (Figure 6)

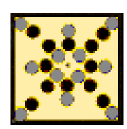

Figure 6

E: What did you do?

A: The blue ones are facing the red ones and the red ones the blue ones.

E: Ok! What number will you have here (on the scorers)?

A: I don't know, I'll try it out!
She starts the game.

Anne started here by constructing a circle and developed that into a symmetrical picture.

c. making a pattern

The pattern was very useful for Jane to create random environment.

J: I know why it went to the red planet. The red balls were more and it touched them more times…you see…I destroyed so many red balls and there are still more…Ok! I will leave two blue balls and two red ones. I will put one red, one blue, one red, one blue.

The logic behind it was for one colour to be near the other, so that when the white ball was going to touch one colour it would touch the other as well. The pattern was also used to have an equal number of balls without counting.

4. Equal size of balls

The spatial environment played a major role in helping Jane to look at whether two balls were equal in size or not. As she said, one of the balls 'is bigger and it (*the white ball*) will touch the most of the time, because the ball takes up more space in the yellow square'. She looked at the effects of the game and she used the global event, the movement of the space kid, to see whether her environment was fair.

J: I think the red will win.
E: Why is that?
J: I think I made it a little bigger than the other… We can open the game and if the scorers are the same that means that they have the same size, otherwise the one is bigger than the other (Figure 7).

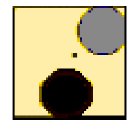

Figure 7

E: What about the space kid?
J: If it is as now that means our balls have the same size…

Jane was making a connection between the spatial appearance of the sample space and the possible outcome from the game in the longer term.

CONCLUSIONS

The study seems to indicate interesting differences between the way that children responded to Piaget and Inhelder's (1975) experiment and how children in this non-conventional context were able to express ideas for random mixture. A main difference from the Piagetian study was perhaps that, in this study, the children constructed randomness, not just as a cognitive act or thought experiment, but in a quasi-concrete way. Children's thinking moved from finding outcomes and describing the random behaviour to constructing for themselves a random behaviour. The study shows that children have various cognitive resources for constructing randomness, beyond those that might be expected from classical experiments. We believe that a possible reason for this is that the tool offers them the opportunity to use these resources for random mixture in a two-dimensional continuum. Moreover, children's culture involves many experiences of random movement, and the context for such experiences is changing from playing board games towards playing video games. We conjecture that cognitive resources for random mixture may be more likely to find a means of expression in continuous two-dimensional movement than in more conventional contexts that involve discrete number.

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