KILLING TWO BIRDS WITH ONE STONE: DEVELOPING PRESERVICE TEACHERS' STATISTICAL THINKING THROUGH AN INVESTIGATION INTO LEARNERS' KNOWLEDGE OF STATISTICS

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Preservice Mathematics teachers are faced with the task of learning Mathematics subject content and developing pedagogical knowledge. This paper describes an attempt to address these tasks simultaneously by designing a course in which preservice teachers collect data related to learners' understanding of statistics and fractions, and develop their own statistical understanding and expertise through analysis of this data. The preservice teachers' statistical thinking was assessed by analysis of articles they wrote based on their own data. It is asserted that even in the presence of the requisite raw materials, statistical thinking is not intuitive and requires explicit teaching.

INTRODUCTION

This study is set in the context of a Mathematics subject specialisation course taken in the second year of a Bachelor of Education degree. The Mathematics content area was statistics, but in line with the multiplicity of competences required of teachers (RSA, 2000), the course addressed several other competences including pedagogical knowledge. This paper will discuss the assessment of the level of statistical thinking displayed by the preservice teachers in the analysis and discussion of their own authentic data.

FRAMEWORK

The new South African school curriculum (Department of Education, 2001) requires that school Mathematics activities are set in real world, historical and cultural contexts. However, du Feu (2001) cautions that there has been "a mania to ensure that all mathematics is justified by being set in context" (p.2). He asserts that many of the contexts used are contrived and given a semblance of credibility by the inclusion of names of people and organisations. Learners are often left in doubt as to whether they must consider the practical reality of the situation presented or simply cut to the mathematics problem hidden in the words and/or pictures.

This strong emphasis on context applies directly to statistics education and in particular the notion of statistical thinking. Following Wild and Pfannkuch (1998), I am taking statistical thinking to consist of "the generic thinking that should take place where statistical methodology meets a real-world problem" (p.334). These authors contend that the raw materials on which statistical thinking works are statistical knowledge, context knowledge and the information in the data. The process of statistical thinking involves looking at the data and statistics arising from the data, and then going to the context to interpret the statistics, and possibly finding new issues and questions which will require yet another look at the data. The likely products of this thinking are new insights into the context and predictions relating to similar situations. Many textbook statistics examples are indeed set in authentic contexts with genuine data gleaned from official databanks. The drawback is that the data is in a sense "cleaned" and selected for the students. It is unlikely to interest all students nor fall within their experience. Figure 1 is a case in point. Although the data is real, there is little opportunity for statistical thinking for most of us since our knowledge of the context is limited and so we are missing an essential ingredient for stasitical thinking.

Consequently, it can be seen that the choice of context is a critical factor in providing opportunities for statistical thinking. A good way of ensuring knowledge of the context occurs when the student has personally participated in the collection of the data. A suitable real-world context for preservice teachers is a group of learners in the school in which their teaching practice occurs. I thought instructive and interesting data for preservice teachers to analyse and interpret could be obtained by surveying the statistical and fractional knowledge of these learners, and their personal opinions about learning Mathematics. The rationale for each preservice teacher

collecting their own data was that “students are more easily convinced of the power of statistical reasoning if they see it applied to questions that are interesting and real to them” (Smith, 1998), and knowledge of the context was assured. Snee (1993) concurs that the "collection and analysis of data is at the heart of statistical thinking. Data collection promotes learning by experience and connects the learning process to reality." The data collected was intended to promote teacher competences such as knowledge of learners and pedagogical content knowledge of statistics and fractions. I hoped that the preservice teachers would gain some insight into the knowledge base of the learners in these areas, common misconceptions and areas of weakness. In addition the personal questions would give a window into the attitudes of learners towards Mathematics.

Many scientists who study animal behaviour are interested in the relationship between social dominance and reproductive success. The following data are wins in aggressive encounters, number of cubs born (1978-1982), and number of cubs surviving 1 year for 12 female spotted hyenas observed in the Masai Mara National Reserve in Kenya.

Female	Wins	Cubs born	Cub survival	Female	Wins	Cubs born	Cub survival
04	63	5	5	30	4	5	3
03	45	6	6	22	3	3	2
63	11	2	No data	11	3	1	No data
N2	10	5	1	44	5	2	No data
KB	3	4	2	16	2	3	3
40	9	5	2	31	3	3	1

Source: Frank,L.G.(1986). Social organization of the spotted hyena *Crocuta crocuta*. II. Dominance and reproduction. *Animal Behaviour*, 34, 1510-1527.

- What is the correlation between wins and cubs born?
- What is the correlation between wins and one-year cub survival?

Frank & Althoen (1995) p.123.

Figure 1. Textbook Example Using Genuine Data.

RESEARCH METHOD

This paper describes a case study i.e. a "specific instance that is frequently designed to illustrate a more general principle" (Cohen, Manion & Morrison, 2000). The specific instance I researched was an eight week programme situated in a second year Mathematics specialisation module, hoping that this would indicate the feasibility of the principle of simultaneously developing knowledge of statistics, statistical thinking and pedagogical knowledge.

Prior to the teaching practice period in local schools, I collaborated with the preservice teachers to construct a survey to be administered in the schools in which they would teach. This survey had two sections: firstly a general information section designed to elicit personal data and data on the learners' attitudes to Mathematics, and secondly a section comprising eight statistics type questions and eight fraction type questions. The TIMSS (1996) study was a major source of these questions. This group of preservice teachers had recently covered the content and teaching methdology of both these school Mathematics topics in another course. Each preservice teacher was given thirty questionnaires to administer to either a Grade 7 or Grade 8 class.

Consequently, on their return to lectures, each preservice teacher had a set of data personally collected from learners they had met. The organization and analysis of this data was done using Microsoft Excel. The preservice teachers were required, as an assessment task, to write two articles. The first article, using simple statistics and graphs, was for an informal school newsletter and the second article, intended to be more academic employing the more complex statistics they had learnt during the course, was for a Mathematics Education journal. This writing task, inspired by work done by English (2001), was an opportunity to develop communication skills and to demonstrate statistical thinking. These articles, and the spreadsheets generated by the preservice teachers became the data for my study.

RESULTS

Continuing the metaphor in the title of this paper, the birds being targeted were statistical thinking and pedagogical knowledge. The course was designed to line the birds up so both these

outcomes could be achieved simultaneously. The pedagogical knowledge was the anticipated result of the statistical thinking and this should have been evident in the articles the preservice teachers wrote. The outcome of this activity can be viewed in terms of the three raw materials for statistical thinking referred to by Wild and Pfannkuch (1998), namely statistical knowledge, contextual knowledge and the information in the data. Of particular interest is the degree to which the preservice teachers were able to synthesise these aspects and develop their pedagogoical knowledge.

Knowledge of statistics: The preservice teachers had adequate statistical knowledge.to utilise. This was confirmed in the standard examination at the conclusion of this course. All the preservice teachers passed the statistics component of this examination, with a mean mark of 66%.

Knowledge of context: The data under consideration was collected personally by each preservice teacher in a school in which they had taught. This personal interaction with the school pupils provided at least a basic familiarity with the context.

Data for analysis: Each preservice teacher had a set of between 25 and 30 cases to analyse. For each case, there were nine personal information and opinion questions, eight fraction questions and eight data questions. This was sufficient data for substantial statistical thinking.

The preservice teachers were in possession of the raw material for reflection and synthesis into examples of statistical thinking. However, the general impression gained from the articles written is of superficial analysis with a disappointing lack of higher order statistical thinking. Many of the articles employed only basic statistics commonly learnt at Year 9 level of schooling, using up to nine statistical measures in a two page article, with little interpretation of the numbers generated and even less effort to link the statistics to the context. Preservice teachers were far more comfortable with the qualitative data (favourite school subject, reasons for studying mathematics etc.) and seemed to prefer to discuss this rather than get into the numerical data generated by the analysis of the pupil responses to the Mathematics questions. Specific examples may be helpful. The first case describes two articles, written by Jill the top scoring student in the group, which I consider display statistical thinking and the second is an example in Gabie's work of a missed opportunity to investigate some interesting data.

Case of Jill: The first article was entitled "The Usefulness of Mathematics" and was a discussion of the question “What parts of mathematics do you think will be most useful to you in your adult life? Give reasons.” The pupil responses were analysed and displayed on a pie graph and interesting aspects highlighted. For example, despite the fact that 44% of the pupils could not give a reason for Mathematics being useful in daily life, they all intended studying the subject past the compulsory level and all achieved high marks in the statistics and fractions competency test. Jill concluded that knowing the importance of the subject and its usefulness is not a prerequisite for success. Jill's second article described an analysis of the questions relating to pupils' feelings towards Mathematics within groups of differing ability as indicated by the competency test. This resulted in good insights into pupils' thinking. For example, all of the pupils in the upper quartile attribute success at mathematics to natural talent, compared to only 17% of those in the lower quartile. These less able pupils attribute success at mathematics to a positive attitude towards the subject. I consider this a good example of using the statistics to increase pedagogical knowledge.

Case of Gabie: Figure 2 is a graph that was included in the work submitted, but was not part of an article. I find it very interesting to note that the twelve pupils scoring highest in fractions scored between 3 and 8 (out of a possible 8) for the data. In fact, several did no better than those scoring the lowest marks in the fractions. This was a prime opportunity for statistical thinking - for relating the statistical results to the context. Gabie remarked that the pupils had no formal experience of data and that the best scores were obtained by boys. This could have been investigated further in more depth. Despite the presence of the raw material for statistical thinking, the opportunity to gain in depth pedagogical knowledge was missed.

It is my contention that, with a few exceptions, the stone missed both birds. Despite the opportunity afforded them, the preservice teachers were not able to demonstrate meaningful statistical thinking, and little pedagogical knowledge resulted.

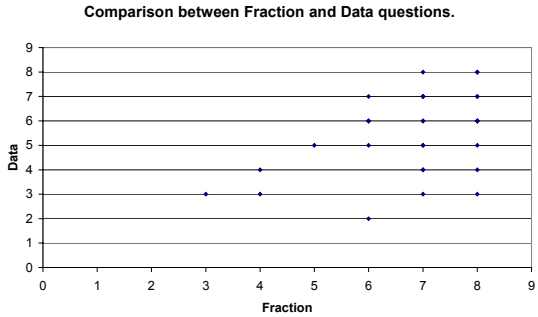

Figure 2. Gabie's Graph to Show the Correlation between Marks for Fraction and Data Questions.

CONCLUDING REMARKS

Firstly, time constraints make it imperative that learning activities are designed to target more than one outcome - preservice teachers cannot afford the time or energy to think statistically about hypothetical cases. It was my experience that the preservice teachers enjoyed working with their own data, and their enthusiasm for this course was evident. The challenge is to identify suitable contexts and issues that are amenable to statistical thinking and result in meaningful learning in other areas. Secondly, this study has indicated that statistical thinking does not occur spontaneously when the correct raw materials are brought together. More specific intervention is required. My first thoughts on promising approaches include:

- Specific modelling of the process of moving between data and context.
- Generation of flowcharts for this process
- Group activities to promote discussion and synergy.
- Individual practice supported by tutoring and opportunities for improvement.

It is essential that the teachers possess the ability to reason statistically if they are to promote this skill in the learners in their care. This requires Mathematics teacher educators to identify suitable contexts for investigation and to develop strategies to promote statistical thinking.

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