LEARNING AND TRANSFER IN ISOMORPHIC UNCERTAINTY SITUATIONS: THE ROLE OF THE SUBJECT'S COGNITIVE ACTIVITY

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Our previous researches showed the difficulties encountered by adult subjects in elementary "purely random" situations, although they are very simple. Indeed, those imply either the throws of two dice, or the simultaneous drawing of two poker chips among three, and thus do not require to be expert in probability. We hypothesized that prior knowledge guides the spontaneous representations and that the "errors" observed may be explained by the activation of "tacit models". More precisely, we hypothesized that the correct combinatorial representations are available for most subjects, but that experimental tricks are required to activate them. This paper will focus on the role of the subject's cognitive activity who is led to construct by himself a correct representation. So, we used a learning paradigm and we tested the transfer on isomorphic problems. The results showed the positive role of the subject's cognitive activity , and above all its strong effect in the transfer of correct representations to isomorphic situations.

INTRODUCTION

In our previous studies we considered the following simple situation: three poker chips in a jar, two red and one white, two chips are drawn together, and the following two results "a red chip and a white chip are obtained" and "two red chips are obtained" are to be compared. Most of the adult subjects (about 60 per cent) answered erroneously that the two results are equiprobable *"because it's a matter of chance"*. Thus random events are thought to be equiprobable *"by nature"*. This representation, called "chance model", has been seen as a "tacit model" in the sense used by Fischbein (1989). It is in the context of elementary arithmetic tasks by children that Fischbein described the role of these tacit models, which are based on various everyday-life experiences. Those models were considered by the author as some base of the interpretation of mathematical concepts which are resistant despite the acquisition of formal concepts. In the situations of drawing poker chips in a jar, it seems reasonable to assume that the tacit "chance model" guides the interpretation and explains the difficulties to perceive the situation as an example of elementary combinatorial problems. In the present research, the difficulties encountered by the subjects when solving a "drawing poker chips in a jar" situation are assumed to be due to the fact that the chance context of the situation spontaneously evokes an inadequate tacit model which is not adequate. More precisely, in elementary uncertainty situations, the adequate combinatorial or logical models are assumed to be available to most subjects, but they are not spontaneously evoked by situations involving a “chance” factor. Two main experimental paradigms were characterized for testing this hypothesis. (1) In a first series of experiments, the combinatorial features of the situation were given , either by an explicit enumeration of all the possible cases (by the subject himself or by the experimenter), or by animated computer display. About 600 students took part in these experiments from which it appeared that none of the considered cognitive aids had a strong positive effect. Indeed, still more than 50 per cent of the responses were erroneous, and due to the activation of the chance model which appeared strongly resistant (Lecoutre, 1984). (2) In another series of experiments, we set up new situations isomorphic to the preceding ones, but in which the chance aspect of the problem was masked by formulating the problem in practical terms, and bringing the subjects’ attention to a geometric figure, and then to the combinatorial features of the situation. The findings showed that such an experimental trick was able to induce the utilization of appropriate combinatorial representations (the rate of correct responses was equal to 75 per cent). However, an unexpected result was a very little transfer between this situation and the subsequent isomorphic classical “Poker chips” situation (in less than 60 per cent of the cases). Furthermore, for more than 20 per cent of the subjects, the two considered problems were quite different (Lecoutre, 1992). This lack of transfer can be explained by the fact that the subjects did not succeed in perceiving the abstract schema of this kind of problems.

Our claim here is that when the subject has to construct himself/herself an abstract schema of the situation, the cognitive activity involved in this construction plays a crucial role in the evocation and the stabilization of the representations. To test this hypothesis, a learning paradigm supposed to help the subject to perceive the global structure of the situation was constructed. So the subjects had to solve a series of problems which pointed out all the relevant properties of the situation. In random situations in which three elements are concerned, with two identical ones, and one different, four events linked one to each other by elementary operative relations can be considered. These four events respectively concerned : (1) one drawn element. (2) the two remaining elements after having drawn one (complementary event of the first one). (3) the remaining element after having drawn two (symmetric event of the first one). (4) two drawn elements (complementary event of the third one, and symmetric of the second one) i.e. the “classical” standard problem. In the present study, the problems in the learning phase were defined so that each of them concerned one of these four possible events. The main idea was that if the subject constructed by himself/herself the representation of the structural properties of the situation, the tacit “chance model” would fail, and the result of such a cognitive activity would be a more frequent and a more stable transfer to isomorphic situations.

METHOD

Materials and Design

Two conditions were defined. In the first condition, the “Geometric figures” situation was used as the base situation in the learning phase, and the “Poker chips” situation was used as the target situation in the transfer phase. In the second condition, it was the reverse.

The learning phase

Four problems were successively presented, in the same order for all the subjects. P1: one object was drawn and the question was related to this object. P2: one object was drawn and the question was related to the two remaining objects. P3: two objects were drawn and the question was related to the remaining object. P4 (“classical” standard problem): two objects were drawn and the question was related to these two objects. So, the problems were either symmetrical (P1 and P3, P2 and P4), or complementary (P1 and P2; P3 and P4). For each problem, the subjects were asked to give their response by choosing one of the four possibilities: (1) more chance of obtaining R1; (2) more chance of obtaining R2 (3) an equal chance of obtaining R1 and R2 (4) it’s impossible to give an answer.

The transfer phase

The problem P5 used in the transfer phase is isomorphic to the last problem P4 of the learning phase, presented in the reverse situation.

Subjects and procedure

Eighty four students in Psychology at Rouen University, about 20 to 25 years old, and without any deep background in probability theory, were tested individually. The problems were displayed with a computer. The five problems were presented with a feedback: in the case of an incorrect response, the subject was asked to give another response, and a help was given by the possibility to come back to the precedent correct responses of the previous problems. So the subjects could not solve the next problem until they correctly responded to the current problem. Furthermore, for each problem, the subjects were asked to justify their response, by speaking aloud.

RESULTS

Two indicators of behavior were considered: the first response given for each problem with the associated justification, and the pattern of responses for the successive problems. A preliminary comparison of the two experimental conditions pointed out a negligible difference between the two conditions, so that the results and comments given below concern all 84 subjects.

The Learning phase.

Problem 1 which was related to the draw of one object among three, and considered, *a priori*, as the easiest one, led to the most important rate of correct responses (78%). The model activated in this case, was a correct model in which the number of objects was considered. However, the surprisingly quite high proportion of equiprobability responses (19%) related to the activation of the “chance model” must be outlined. *Problem 2* showed a quite similar distribution of the responses. All the subjects who correctly responded, used explicitly the complementarity property of P2 and P1. As in the problem 1, the rate of equiprobability responses was quite high (23%), and also linked to the activation of the chance model. Hence, it can be concluded that showing the correct response in the problem 1 is not sufficient to inhibit the activation of this model. In *Problem 3* (symmetrical of the problem 1), the rate of correct responses was quite low (45%), and notably smaller than in the other problems. The symmetry property was used by only a very few subjects. The equiprobability response was frequently given (44%). In this case, most subjects answered this problem by using the complementarity property with the “ classical ” standard problem which had not yet been presented (P4). These findings showed that for most subjects it is difficult (or even impossible) to reason about what it remains after a draw without reasoning about what is drawn. It will be noted here that such a reasoning appeared to be “ecological”: indeed, in most every-day life situations we are conducted to reason about what has been drawn (in lotteries, for example) and practically never about what has not been drawn. This result can be interpreted as the expression of a well-known analogical transfer process. Indeed, in this case, the general knowledge constructed by an experience of lotteries is imported in the new situation, and consequently the subjects are not involved in the analysis of the situation. *Problem 4* (standard-problem) showed a quite high rate of correct responses (62%) which were linked to the use of the complementarity property of this problem with the preceding one.

Thus, at the end of this phase, a positive learning effect was observed. Yet it was not so high as one could expect taking into account the extreme simplicity of the problems. Some interpretative elements were found in the analysis of the patterns of responses. These patterns were very heterogeneous from one subject to another: 36 different patterns were observed, and 30 per cent of these patterns were obtained from only one subject. The most frequent pattern was associated to a series of four correct answers: it was given by 19 per cent of the subjects. In fact, the majority of the patterns were associated with a reasoning in which each problem was treated as a new problem, independently of the preceding ones, and in particular, the symmetry property of two problems was very rarely used. In the case of a first incorrect answer, very few subjects asked to come back to the correct responses of previous problems, which were very rarely spontaneously perceived as pertinent for solving a new problem. All these findings were convergent and supported the conclusion of an absence of a global representation of the structural properties of the situation in most subjects at the end of the learning phase.

The Transfer Phase

Two main points should be noted here. (1) A very high transfer rate was observed, since 82 per cent of the subjects having correctly answered the problem P4, gave a first correct response to the problem P5. Furthermore, it can be observed here that among the subjects who didn’t transfer, some of them spontaneously stated that they made a mistake by answering too quickly (they gave the correct response in the second trial). So the transfer rate can be considered as about 90 per cent. (2) However, our attention was also attracted by the fact that only 65 per cent of the subjects gave the correct answer in the problem P5. It’s important to recall here that in the learning phase the problems were systematically repeated until the subjects gave the correct response. So all the subjects knew the correct response of each problem. These findings showed that such a knowledge was not as efficient as expected for constructing a correct representation. Indeed, a real cognitive activity is required to the subject when constructing a representation, in order to make it correct and efficient.

CONCLUSION

Our main purpose was to investigate how it is possible to act upon the representations spontaneously developed in elementary uncertainty situations by involving the subjects in an active learning of the structural properties of these situations. In the present study, a learning paradigm was defined in order to involve the subject in a deep analysis of the properties of the situation which would lead him/her, to construct an adequate and durable representation of this kind of situation. Results showed a positive effect of the learning phase. indeed, at the end of this phase, more than half of the subjects (57%) correctly solved the classical standard problem (P4). However, this success rate was not as high as expected. This first result, jointly considered with the particularly low rate of subjects who succeed in solving all the four problems of the learning phase (less than 20 per cent of the subjects), once again brought out the difficulties of these problems although they are very simple, and even often considered as obvious ones. But a new exciting result was the high rate of transfer of the correct representation observed in the isomorphic problem (P5) which was about 90 per cent. This result demonstrated that when the subject was implied in the discovery and the understanding of the relevant properties of the situation, the resulting representation was stable and transferable to an isomorphic situation. The subjects who correctly understood the schema of the base situation were able to generalize and transfer their specific knowledge to a new situation. This result may be interpreted as the result of what Holyoak and Thagard (1995) called the abstraction of the schema of a set of problems. Our the findings appeared to have significant implications for teaching mathematical concepts. Indeed, they suggested the value of attempting to act upon the cognitive models used by the subjects. Conditions in which the subjects are able to construct themselves and consequently to “become aware” of the structural properties of the situation are to be privileged. Such an “awareness” appears to be a determinant factor in the stabilization of a cognitive representation.

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