EXPRESSIONS OF CONTROL IN STOCHASTIC PROCESSES

Dave Pratt, Mathematics Education Research Centre, University of Warwick, UK
Richard Noss, Institute of Education, University of London, UK

We use the activity of two children, age 10, as they interact with resources in a computer-based domain, to illustrate their initial meanings for controlling stochastic artefacts within that domain. As the children continue their explorations, we observe the emergence of new meanings for control, and discuss the evolution of chldren's understandings and how they were shaped by the computer-based resources.

BACKGROUND

The last two decades of research into the meanings which we hold for randomness have been heavily influenced by a dominant interest in our errors, inconsistencies and irrational decision-making strategies, especially when making judgements of chance. An important contribution to this work has been that of Kahneman and Tversky, who have catalogued a range of heuristics emphasising inherent systematic bias (Kahneman, Slovic, and Tversky, 1982) . The following quotation points to what they see as a central implication behind this work (Kahneman, Slovic, and Tversky, 1982):

> The accumulation of demonstrations in which intelligent people violate elementary rules of logic or statistics has raised doubts about the descriptive adequacy of rational models of judgement and decision models.

The tone of this work, to which we can broadly attach the label, the *misconceptions* approach, has been adopted by most researchers in this field (Konold, 1989; Shaughnessy, 1992) . The misconceptions approach has been challenged because of its weakness in explaining how expert-like mastery of mathematical principles can evolve (Smith, and Rochelle, 1993) . A pedagogic implication of the misconceptions approach is that these irrational ways of thinking must somehow be replaced by new *correct* knowledge. In our view the notion of replacement begs the unanswered question, "What knowledge *is* used as the basis from which the new meanings are constructed?" This question inevitably raises further issues: how do the specific features of the learning environment mediate existing and new meanings for randomness and what is the relationship between these components?

Our long-term aim is to study the use of tools for the expression of meanings for stochastic phenomena. We have been using an iterative design procedure (diSessa, 1995) to cycle between phases of tool design, in which we are building an expressive computational medium, and tool use, in which children, between the ages of 9 and 11 years, interact with and modify tools. In this sense, the computer-based environment, the

Chance-Maker microworld, acts as a window (Noss and Hoyles, 1996) on learners' thinking as they articulate the evolution of their fundamental ideas of chance and probability.

Within the constraints of this paper, we can only offer a glimpse of one aspect of this extensive process. Our intention is to illustrate through a snapshot of the work of two girls, Amanda and Rachael, how meanings for long term (global) stochastic behaviour emerge through the forging of connections between meanings for short term (local) behaviour and the tools within the Chance-Maker microworld. But first we must describe a few of the structures that we have made available within that microworld.

SOME FEATURES OF THE CHANCE-MAKER MICROWORLD

The central objects of the microworld are a series of 'gadgets', computational tools which behave in many identifiable respects like their everyday counterparts.

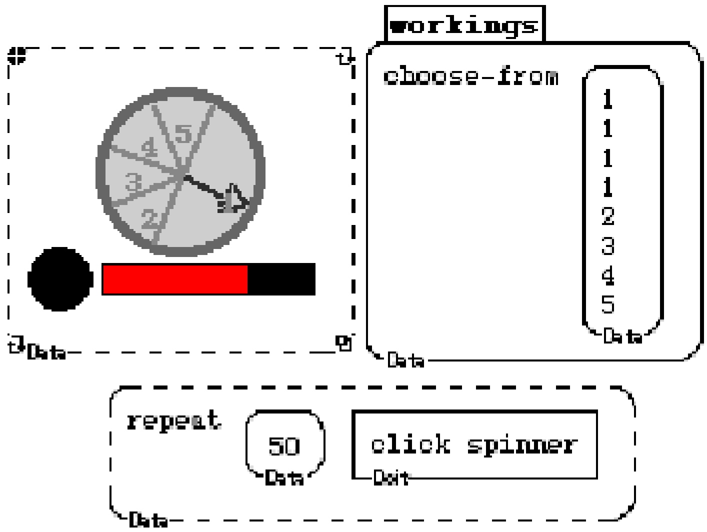

Figure 1. : Three features of the Spinner gadget.

For example, in the case of the spinner gadget (see Figure 1), the learner can 'throw' a spinner with varying strengths, as well as inspect and change the 'workings' which control the behaviour of the spinner through the primitive choose-from. The spinner is shown with its default setting for the workings. We will also refer later to the coin and dice gadgets, each of which operates in similar fashion to the spinner. The coin has one head and one tail in its default workings box, whilst the dice gadget holds one of each outcome from 1 to 5 and four 6's in its workings box. Each gadget, then, incorporates a workings box, which embodies quite explicitly and accessibly a mathematical representation of how the gadget works.

The gadgets are activated using the strength bar, depicted in Figure 1 as a solid black bar with a circular switch at one end. We can imagine the child controlling the strength by allowing a tube (the black bar) to fill with a red fluid until the switch is clicked. The strength of the throw is represented by the amount of red fluid. When the spinner is activated, the arm rotates several times before stopping at the result. The other gadgets also dynamically simulate the action of their everyday counterparts before generating a result.

In order to study the results of many spins, the child can enter a number into the repeat line. Figure 1 illustrates how 50 trials can be easily generated. The historical results of trials with each gadget can be inspected in the form of an ongoing list in the results box or as a chart (see Figures 2 and 3).

THE CASE OF AMANDA AND RACHAEL

In order to illustrate the construction of meanings for randomness through interaction with the tools in the Chance-Maker microworld, we now turn our attention to the work of two girls, Amanda and Rachael, each ten years old.

Meanings for Local Behaviour

During interviews held prior to the computer-based work and in the early interactions with the Chance-maker microworld, Amanda and Rachael articulated three meanings for the local behaviour of stochastic phenomena:

- unpredictability: ‘you can not say what will happen next - it is random’
- fairness: ‘it is fair so it is random’
- lack of sequencing in previous results: ‘there is no pattern in the results - it is random’

Rachael was initially concerned that the coin was not random because the computer controlled the output.

> I ask if the coin is random. Rachael: “Not really ... it’s probably been programmed to do it, in a loop.” I ask Rachael what she meant by ‘in a loop’. Rachael: “Well, it’s programmed to do heads, then maybe heads again and then tails.” I clarify, “In some sort of pattern?” Rachael: “Yes.”

On the one hand, their attempts to determine the output through the strength control appeared to be ineffective and so the outcome was unpredictable. On the other hand, the computer *must* be exerting control, so Rachael thought, through the programming code. There are then two types of control here. The first, for which we have coined the term, *joystick control*, describes the control that the children exert through their own efforts, such as when using the strength control. The second, *computer-in-control*, expresses the idea that the computer controls the result through the programmer’s code.

Rachael later offered the opinion that she could click on the choose-from primitive and then the computer would choose.

> “I suppose if you used the choose-from, you could probably choose it by clicking on that and telling it to throw ...”

Rachael co-ordinated the two meanings of control by recognising that, when she used the choose-from primitive, she relinquished control, rendering the gadget's behaviour unpredictable. In this sense, she exerted her control by electing to use the choose-from primitive. We shall see how this recognition of the rôle of choose-item was a first step towards constructing new meanings for the idea of control.

Meanings for Global Behaviour

There was evidence that Amanda and Rachael did not discriminate initially between local and global behaviour.

Res: If you are trying to decide whether the coin is working properly or not, what do you think a good number of times to throw it would be?
R: About ten.
Res: Do you think there would be any advantage in doing it more than ten?
R: Yes because you get like a clearer answer.
Res: How many times would be better than ten.
R: Twenty. *Amanda agrees*
Res: Would more than 20 would be better?
R: Yes.
A: I don't think so.

Rachael intuitively expected that more information would be beneficial though this intuition was still constrained to small numbers of trials. Amanda was less inclined to intuit any potential for the use of a large number of trials in the behaviour of coins. Amanda and Rachael carried out 50 trials with the coin gadget; the pictogram tool depicted a predominance of tails (Figure 2).

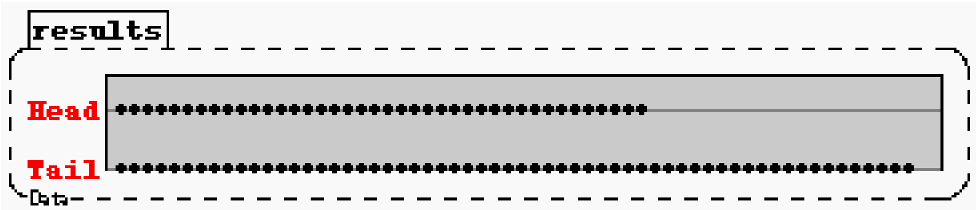

Figure 2. The pictogram shows more tails

Amanda conjectured that the coin gadget was inclined to generate tails. Another experiment of 100 trials, where the pictogram indicated a predominance of heads, caused Amanda to change her mind. We see, in their reaction to these pictograms, that Amanda and Rachael focus their sense-making on the unpredictability of local behaviour.

A: Oh .... I don't think you can really estimate which one.
R: You can't be too sure really.
A: The first time there were loads of tails, so I thought it was going to be tails again. But probably after a couple of goes, it will probably do tons of heads again.

We trace below the evolution of children's meanings during a period of about an hour in which Amanda and Rachael interacted with the coin, spinner and dice gadgets.

These meanings, which were articulated in terms of the resources and tools within the Chance-Maker microworld, empowered Amanda and Rachael to make more effective use of the tools. We therefore refer to these meanings as *situated abstractions* (SA) (Noss and Hoyles, 1996) .

A and R repeat 200 new trials. The pictogram shows roughly equal rows.
R: It may be just chance that it came out even.
A: I think there was a reason.
A and R repeat another 1000 trials (intending to do 500). The pie chart is even.
A: The higher the number of throws, the more even the pie chart gets. (SA)
R: With less trials, the pie chart is only half even. (SA)
A and R familiarise themselves with the spinner and find that there are a lot of 1's. They edit the workings to read: choose-from [1 2 3 4 5]. The pie chart is uneven. They edit the workings to include two of each number. After 50 trials, the pie chart is still uneven (Figure 3). R suggests that they remove a 1 from the workings. They edit the workings and do 50 new trials. The pie chart is still uneven. A edits the workings back to choose-from [1 2 3 4 5]. and they repeat 50 new trials. The pie chart is fairly even but not entirely convincing.
R: Maybe we should throw it more times like the coin.
They repeat 150 new trials. The pie chart is more even.
R: The higher the number of trials, the more even the pie chart gets, I think.
They repeat 200 trials, and the pie chart is a little uneven. They repeat 1000 new trials and the pie chart shows even sectors.
R: The more times you spin it, the more even the pie chart gets. (SA)
A and R begin to use the dice gadget. There appear to be too many 6's.
A and R: If we do it a lot of times, the pie chart will be even.
A and R repeat 100 trials and find that the 6's are much more common.
A: There are more 6's in the results because there are more 6's in the workings box. (SA)
R edits the workings to contain just one 6. They repeat 1000 trials. The pie chart shows even sectors. I ask them to make a dice which favours 1's and 2's. They edit the workings to read: choose-from [1 1 2 2 3 4 5 6] and repeat 1000 new trials. The pie chart shows most 1's and 2's and least 3, 4, 5, and 6's. I ask them to make the 1's more likely than the 2's. They edit the workings to read: choose-from [1 1 1 2 2 3 4 5 6].
A and R: Higher numbers of trials generate even pie charts when the workings are fair. (SA)
A and R: Smaller numbers of trials result in the pie chart being less even.(SA)

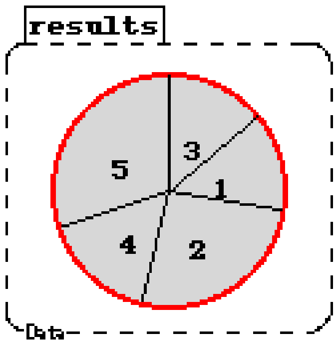

Figure. 3. The pie chart is uneven

There were three phases within this episode; each phase marks the emergence of a new meaning for the idea of control.

(i) The number of trials controls the evenness of the pie chart

After using the coin gadget, the girls articulated situated abstractions, in which the number of trials is seen as a control for the appearance of the pie chart ("I think it's the highest the number, the even more it gets" and "when we did less numbers, it was half even").

These situated abstractions were primarily constructed through purposeful use of the repeat primitive and the graphing tool, especially the pie chart. The new meaning for control was deeply connected with those tools, constraining its domain of applicability, as apparent in the next phase.

(ii) The workings box controls the evenness of the pie chart

Unhappy that the pie chart was uneven, the girls tried to redress the imbalance by corresponding amendments to the workings box. Each time they used only 50 trials to test the workings. Through this iterative amending of the workings box, Amanda and Rachael articulated the meaning that there was a direct relationship between the entries in the workings box and the appearance of the pie chart.

The extraordinary thing about this episode is that they appear to have 'forgotten' the situated abstractions from the previous phase. How might we explain Amanda's and Rachael's actions? Our interpretation is that the unfairness of the spinner cued well-established intuitions of deterministic behaviour, a cause-effect relationship between the workings box and the appearance of the pie chart.

(iii) The number of trials AND the workings box control the evenness of the pie chart

It would be easy to dismiss Amanda and Rachael's tinkering with the workings box as misconceived because they ignored the influence of the small number of trials but this observation would ignore the potential of this activity.

Rachael, having edited the workings to read choose-from [1 1 2 2 3 4 5 6], predicted that the pie-chart would contain, "More 2's, more 1's and less of the others" for 1000 trials, whereas, for 50 trials, Amanda explained that the pie chart would be, "A bit more uneven." Each of these statements was greeted with approval by the other girl.
By using the tools within the microworld, a situated abstraction for the controlling influence of the number of trials was connected with the situated abstraction for the controlling influence of the workings box; Amanda and Rachael constructed a new meaning for control which co-ordinated these two meanings for the global behaviour of the dice gadget.

FINAL NOTE

Amanda and Rachael expressed four separable meanings for control: joystick control, computer-in-control, control through the number of trials, control through the workings box. Joystick control and computer-in-control were articulated prior to, or early in, the interactions with the Chance-Maker microworld. We believe that meanings for local stochastic behaviour are abstracted directly from experience, perhaps through playing games with coins, dice and spinners.

We observed how Amanda and Rachael used these meanings for local stochastic and deterministic behaviour to try to make sense of the global behaviour of the dice gadget – local meanings were their only available internal resources. The poverty of simply labelling these meanings as misconceptions was brought into sharp focus by the subsequent activity, when they became the raw material for the construction of new meanings for the global behaviour of the dice gadget. This construction process did not though happen by chance. Primitives such as repeat and choose-from, and structures such as the workings box and the graphing tool empowered the girls to make effective use of these meanings.

In so doing, Amanda and Rachael constructed new meanings for global behaviour. However, these meanings have not replaced the earlier ones. On the contrary the meanings for local behaviour remain, perhaps not in their original state, but restructured through connections with causal meanings like *the workings box controls the evenness of the pie chart* and *the number of trials controls the evenness of the pie chart*. These new meanings are, for the time being at least, constrained to the domain of the Chance-Maker microworld, but we conjecture that further carefully designed experiences may enable Amanda and Rachael to construct new meanings with wider domains of applicability.

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