TOWARDS A CHARACTERIZATION AND UNDERSTANDING OF STUDENTS' LEARNING IN AN INTERACTIVE STATISTICS ENVIRONMENT

Dani Ben-Zvi and Abraham Arcavi, The Weizmann Institute of Science, Rehovot Israel

We shall describe episodes of middle school students working on Exploratory Data Analysis (EDA) developed within an innovative curriculum. We outline the program and its rationale, analyze the design of the tasks, present extracts from students' activities and speculate about their learning processes. Finally, from our observations, we propose a new construct -- learning arena, which is suggested as a curriculum design principle, which may also facilitate research.

THE LEARNING ENVIRONMENT

The teaching of statistics is undergoing major changes (Shaughnessy et al., 1996; Moore, 1997), mostly due to: (a) a shift in content at the introductory stage from descriptive statistics to EDA -- i.e., emphasis on organization, description, representations, and the analysis of data, with a considerable use of visual displays and technology; (b) new pedagogy which abandons an "information transfer" model in favor of a *constructivist* view of learning (Davis et al., 1990; Garfield, 1995) -- i.e. students construct their own knowledge by combining present learning experiences with existing conceptions; and (c) the incorporation of new technologies, which facilitate the handling, change, and use of various data representations, and provide tools for making sense of data (Biehler, 1993).

With these perspectives in mind, we initiated a middle school statistics project, which includes curriculum development (Ben-Zvi and Friedlander, 1997a), classroom trials, teacher courses, research on learning processes and the teacher's role in an investigative technological environment (Ben-Zvi and Friedlander, 1997b). The project is part of an innovative program developed at the Weizmann Institute (Hershkowitz and Schwarz, 1997).

We chose to emphasize *statistical content* and *concepts*: for example, types of data, posing questions and collecting data, statistical measures, handling and interpretation of graphical representations, and intuitive notions of inference and correlation. We also emphasize the elements of *statistical thinking*; e.g. the need for data, proof and certainty in statistics, the existence of variability, and the logic underlying processes and methods.

The form of *instruction* for the course was based on the following main principles: (a) the use of extended *real* (or realistic) problem situations; (b) an emphasis on a community of learners communicating with each other; and (c) a new role for the teacher as "a guide on the side" (Hawkins et al., 1992).

The curriculum content and pedagogy is based on the PCAIC cycle (*pose*, *collect*, *analyze*, *interpret, and communicate*) proposed by Graham (1987) and Kader and Perry

(1994), using a spreadsheet. Students work on two parallel strands - (1) classroom activities and (2) a research project.

(1) The *classroom activities* are semi-structured statistical investigations, in which considerations and processes involved are also pursued. The students are encouraged to hypothesize about possible outcomes, choose tools and methods of inquiry, representations, conclusions, and interpretation of results. Most learning is designed to be done in pairs.

(2) The *research project* is an extended activity, in which the students act as independent and responsible learners. Students identify a problem and the question they wish to investigate, suggest hypotheses, design the study, collect and analyze data, interpret the results and draw conclusions. At the end they submit a written report and present their main results to fellow students and parents in a "statistical happening".

During the experimental implementation, the learning materials were field-tested, presented in teachers courses, and then published. In order to study the effects of the new curriculum, we analyzed student behavior using video recordings, classroom observations, interviews, and the assessment of students' notebooks and research projects.

In the following, we "observe" pairs of students, as they engage in two kinds of action: *taking a stand* in a debate on the basis of data analysis, and *designing* (or re-designing) a representation to emphasize a point they want to make. The purpose of the analysis (presented briefly below) is to advance our understanding of (1) how students learn in such an environment, and (2) how can we be more aware of student actions, in order to design "better" tasks. (By "better" we mean, situations in which students engage seriously, work and reflect, and advance their knowledge.)

STUDENTS TAKING A STAND

One activity was the *'Work dispute'* in a printing company: the workers are in dispute with the management, who has agreed to a total increase in the salary bill of 10 percent. How this is to be divided among the employees is a problem - and thereby hangs the dispute. The students are given the salary list of the one hundred employees, and an instruction booklet to guide them in their work. They are also provided with information about the national average and minimum salaries, Internet sites to look for data on salaries, newspaper articles about work disputes and strikes, and a reading list of background material. In the first part of the activity, students are required to take sides in the dispute, and to clarify their arguments. Then, using the computer, they have to describe the distribution of salaries and use measures (e.g. median, mean, mode, and range) guided by their position in the dispute. The students learn the effects of grouping data and the different uses of statistical measures in arguing their case. In the third part, the students suggest alterations to the salary structure without exceeding the 10 percent limit. They produce their proposal to solve the dispute, and design representations to support their position and refute opposing arguments. Finally the class meets for a general debate and votes for the winning proposal. The time spent on the full activity is about seven class periods.

This task context is familiar to students with interesting, realistic and meaningful data (Cobb and Moore, 1997). The data was "engineered" so that the salary list is manageable, and provides points of departure for raising some key concepts. For example, the various central tendency measures are different from each other, to allow students to choose a representative measure to argue their case. We also arranged that the mean salary (5000 IS) is above the real national averages (4350 IS - all employees, 4500 IS - printers only).

Communication of statistical ideas seems to us an essential part of student experiences. We expect students to clarify their thoughts, learn to listen to each other, and try to make sense of each other's ideas. But, most importantly we ask students to *take sides* in the conflict situation. Their actions, e.g. handling data, choosing statistics, creating displays and arguing are all motivated, guided, and targeted by the stand they have taken. Their actions may also cause them to change their original stand, etc. The selection of the stand, if done thoughtfully, has an important value of its own, in that it transforms the pure learning situation into a meaningful position.

Concept learning thus takes place in context, using and applying the concepts and through exchange of ideas among peers. This obviously includes exercises of the "traditional" textbook type, but also much more, as described above.

To illustrate the use of concepts and arguments that the task promoted, we bring the following short transcript from a video recording of one of the experimental classes. It records a group of students who chose to take the side of the workers. After clarifying their arguments, they described the distribution of the current salaries, guided by their position in the dispute. The student pairs prepared various suggested alterations to the salary structure to favor workers (as opposed to management), and then held a series of meetings with fellow student pairs (about 10 students in all), in which they discussed proposals, and designed graphical representations to support their position, and prepared themselves for the general debate.

The following transcript is taken from the second "workers' meeting".

Id. OK, we have this pie [chart] and we plan to use it [See Figure 1]. Everybody agrees?
Students. Yes, yes.
Id. Let's see what should we say here? Actually we see that... 60 percent of...
It. 60 percent of the workers are under the average wage [4500 IS]. Now, by adding 12 percent... there are far fewer [workers under the national average].
S. OK, but I have a proposal, that brings almost everybody above the average wage. If we add 1000 shekel to the 49 workers, who are under the average...
Id. It's impossible. Can't you understand that?
S. This [my proposal] will leave us with 1000 shekel, that can be divided among the other workers, who are over [the average].
It. Then each of them will get exactly five shekel!...
M. But we don't have any chance to win this way.
Id. What is the matter with you? We'll have a revolt in our own ranks. Do you want that to happen at the final debate?

S. Anyway, this is my opinion! If there are no better proposals...
Id. Of course there are: a rise of 12 percent on each salary [excluding the managers]...
Sh. OK. Show me by how much will your proposal reduce the 60 percent.
D. I am printing now an amazing proposal - everybody will be above the [national] average: NO WORKER WILL BE UNDER THE AVERAGE WAGE! This needs a considerable cut in the managers' salaries...

Figure 1. The "workers'" description of the current salary distribution.

In this exchange, three different proposals for the alteration of the salary structure were presented. The first, offered by It. and Id., suggested an increase of 12% for all workers. The managers' salaries remain unchanged. The second proposal, originated by S., suggested an equal (1000 IS) increase for each of the 49 workers earning less than the national average (4350 IS), the small remainder to be divided among the other workers. Again the managers' salaries remain unchanged. The third proposal, presented by D., suggested a considerable cut in managers' salaries, and an increase for all workers under the national average, to bring them above the average.

We observe that central to students' actions and motives is the stand to be taken by the workers. Putting themselves in the position of workers in a dispute against the managers, motivated the students to find a "winning" strategy, namely to have their proposal approved by the majority. For example, Figure 1 above, is grouped to emphasize the large proportion of salaries below the printers' national average. Moreover, predicting that the management will boast about the relatively high company average (5000 IS), the workers chose to emphasize the relatively low median and mode. Thus, the workers' explanations for choosing representative measures and graphical displays, emerged from their stand in the dispute.

Taking a stand also made students check their methods, arguments and conclusions with extreme care. They felt it natural to face criticism and counter-arguments made by peers and teacher, and to answer them. Sometimes, when the results of their work were not in line with their stand, they were forced to persevere and search for more congenial evidence and proof. Finally, after much refining they formulated the following proposal.

> To divide most of the money among the workers, because they currently earn very low salaries; i.e., an increase of 13.35% for workers under the printers' average salary; an increase of 8.85% for the remaining workers. The managers salaries to be increased by 1000, 900, 800, ..., 400 IS, inversely related to the size of their present income. The small remainder to be contributed to orphanage homes in town.

They also prepared the following arguments for the final debate.

> If we remove five managers from the salary list, the company average decreases dramatically below the national average. We shall display the significant improvement in the workers' salaries, made by our proposal, using a pie chart [Figure 2] and statistical measures [median and mode]. We wish to create a "balance" in the company. No longer will 66% of the employees earn below the national average. Most of the employees will earn a salary close to the company average.

Figure 2. The workers' final proposal for the new salaries.

These observations suggest that students' actions and interactions were partially due to the design of the problem situation, which includes *taking a stand*. They

a) dealt with a complex situation and the relevant statistical concepts (averages, percentages, charts, etc.);
b) used critical arguments to confront conflicting alternatives;
c) used statistical procedures and concepts with a purpose and within a context, to solve problems, relying heavily on visual representations and computer;
d) demonstrated involvement, interest, enthusiasm, and motivation in their learning;
e) were able to create their own products (proposals and their representations).

We turn now to a second brief example from another activity.

STUDENTS AS DESIGNERS (OR RE-DESIGNERS)

The activity - *"The same song, with a different tune"* - occurs early in the curriculum. The problem is presented to students in the following way.

> Two sports journalists argue about the record times in the 100 meters. One of them claims that there seems to be no limit to human ability to improve the record. The other argues that sometime there will be a record, which will never be broken. To support their positions, both journalists use graphs.

One task of this investigation asks students to design a representation, using a computer, to support different statements, such as:

(a) During the years, the times recorded in the Olympic 100 meters improved considerably.

(b) Throughout the years, the changes in the Olympic times for the 100 meters were insignificant.

(c) Between 1948 and 1956, the times in the 100 meters worsened considerably.

This task context is familiar to students with interesting and meaningful data, providing opportunities to search for patterns, centers, variations, and gaps in the data. Students go beyond reading, interpreting and understanding data representations - they are encouraged to *design* (or *re-design*) *representations* (Harel, 1991).

The task directed students attention to the problem of scaling in the design and interpretation of a representation. Thus, they developed “scale sensitivity”, became critical of the false impressions that may be induced (see, for example, the graphs in Figures 3-6).

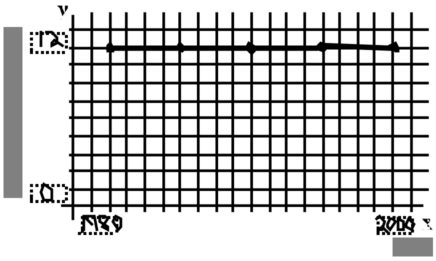

Figure 3. Design to support statement (b) above.

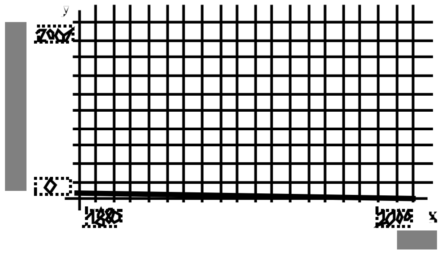

Figure 4. Design to support statement (b) above.

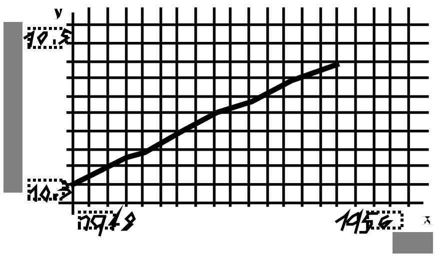

Figure 5. Design to support statement (c) above.

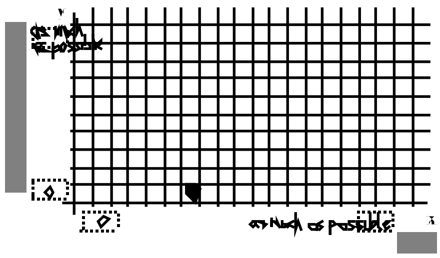

Figure 6. Design to support statement (b) above.

This task provides an opportunity for students to represent ideas creatively in various ways, and also understand some meta-representational issues (diSessa et al., 1991).

DISCUSSION

The examples we have described illustrate how curriculum design can take into account *both*: (1) new trends in subject matter (EDA), its needs, values, and tools; and (2) student learning. By staging and encouraging students - e.g., to *take sides* and *design* (and *re-design*), we pushed them towards levels of discussion, which we have not observed in the traditional statistics classroom. They showed themselves able to understand and judge the complexities of the situation, engaged in preparing a proposal which in their view was acceptable, rational and just, and were able to defend it. The second example involved more than just being able to read and interpret, it focused attention on the crucial role of scales in creating a visual impression for good or ill.

We suggest that we can usefully characterize *taking sides* and *designing* as *learning, arenas* and propose a definition of the term. *Learning arenas are problem situations based on careful research design, aimed at promoting and supporting certain student actions, which encourage meaningful learning of statistics (mathematics).*

We propose that *learning arenas* can be a helpful guide in the development of tasks for student learning. We intend to pursue this work further in order to identify, develop, and research more arenas similar to those above. We hope that the refinement of this idea, and the accumulation of examples, will contribute towards a theory of principled development of learning materials.

REFERENCES

Biehler, R. (1993). Software tools and mathematics education: The case of statistics. In C. Keitel and K. Ruthven (Eds.), *Learning from Computers: Mathematics Education and Technology* (pp. 68-100). Berlin: Springer.

Ben-Zvi, D., and Friedlander, A. (1997a). *Statistical Investigations with Spreadsheets* (in Hebrew). Rehovot, Israel: Weizmann Inst. of Sci.

Ben-Zvi, D., and Friedlander, A. (1997b). Statistical thinking in a technological environment. In J. B. Garfield., and G. Burrill (Eds.), *Proc. of the IASE Round Table Conf. on Research on the Role of Technology in Teaching and Learning Statistics* (pp. 45-55). Voorburg, The Netherlands: ISl.

Cobb, G. W., and Moore, D. S. (1997). Mathematics, statistics, and teaching. *The Am. Math. Mont. 104(9),* 801-823.

Davis, R. B., Maher, C. A., and Noddings, N. (Eds.) (1990). *Constructivist Views on the Teaching and Learning of Mathematics*. Reston, VA: NCTM.

diSessa, A. A., Hammer, D., Sherin, B., and Kolpakowski, T. (1991). Inventing graphing: Meta-representational expertise in children. *J. of Math. Behavior, 10,* 117-160.

Garfield, J. (1995). How students learn statistics. *Inter. Statistical Rev. 63(1),* 25-34.

Graham, A. (1987). *Statistical Investigations in the Secondary School*. Cambridge, UK: Camb. Univ. P.

Harel, I. (1991). *Children Designers*. Norwood, NJ : Ablex.

Hawkins, A., Jolliffe, F., and Glickman, L. (1992). *Teaching Statistical Concepts*. Harlow, UK: Longman.

Hershkowitz, R., and Schwarz, B. B. (1997). Unifying cognitive and sociocultural aspects in research on learning the function concept. In E. Pehkonen (Ed.), *Proc. of the 21st Conf. of the Int. Group for the Psychol. of Math. Education* (Vol. 1, pp. 148-164). Lahti, Finland: Univ. of Helsinky.

Kader, G., and Perry, M. (1994). Learning statistics with technology. *Math. T.in the M. S. 1(2),* 130-136.

Moore, D. S. (1997). New pedagogy and new content: the case of statistics. *Int. Stat. Rev. 65(2),* 123-165.

Shaughnessy, J. M., Garfield, J., and Greer, B. (1996). Data Handling. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick and C. Laborde (Eds.), *Int. Handbook of Math. Education* (part 1, pp. 205-237). Dordrecht, Netherlands: Kluwer.