CHILDREN'S INTUITION OF PROBABALISTIC CONCEPTS EMERGING FROM FAIR PLAY

Draga Vidakovic, Sarah Berenson, and Jane Brandsma
North Carolina State University, USA

The goal of this study was to examine students' mathematical ideas that are used or developed when they are engaged in playing games, together with their beliefs about fairness. The study was conducted in a classroom of 16 eighth grade students in an urban school. During the four days of instruction, the students were engaged in a series of investigations designed to provide a context for a number of important mathematical ideas relative to the particular concepts. Working in pairs, the students began each activity by constructing personal representations and making conjectures about the tasks. The study revealed that students have a wide spectrum of intuitions and ideas about fairness involving chance, probability of equally likely elementary events (and expectation), and sample space. Some of their ideas appeared to be very deeply embedded in their minds which posed a challenge for us (teacher-researchers) in creating a discourse among the students that would prolong the discussion and development of the new ideas.

INTRODUCTION AND BACKGROUND OF THE STUDY

The introduction of probability and statistics as mathematical topics in grades 4 through 12 has been encouraged by new curriculum documents in the United States (NCTM, 1989). The Standards emphasizes the importance of engaging middle grade students in modeling problems, conducting simulations, and collecting, graphing, and studying data. The main premise is that students learn mathematics through their active engagement in activities that foster learning (Maher, Martino and Pantozzi, 1995). One example of such an activity is engaging students in game play that includes various concepts of fairness. In these new settings, students work with a variety of tools (manipulatives, calculators, computers) which foster their sense making. Commonly, students work in small groups on various problems and share their solutions with the whole class.

Games are powerful activities which can help students bridge theory and intuition (Borovcnik, 1994; Dann, Pantozzi, and Steencken, 1995). The emphasis on communication during game-playing provides students with an opportunity to make sense of mathematics and clarify their thinking while discussing strategies and relationships with their classmates (Dominick and Clark, 1996; Leonard and Tracy, 1993). Additionally, the game-playing situation is a "neutral, non-threatening context in which fairness can be examined" (Bright, Harvey, and Wheeler, 1981).

In recent years there has been a call for further research regarding how students think about probability and how that thinking changes. Specifically there is need for additional research regarding students' ideas of chance and random events (Ahlgren and Garfield, 1991; Shaughnessy 1992). This study was designed to capture students' initial intuitions regarding fairness, sampling, equally likely events, chance, and expectancy. In the context of a game-playing activity, the study gave students an opportunity to confirm or challenge those intuitions, and then allowed those intuitions to be refined or revised.

METHOD

Subjects - description of the setting

The study was conducted in a classroom of 16 eighth-grade students in an urban parochial school in a large county of an south-eastern state. A unit of instruction, Developing Probabilistic Ideas, was taught by the researchers. During the four days of instruction, the students were engaged in a series of investigations. The first lesson was an introduction to combinatorics with subsequent lessons on the development of probabilistic ideas through game play.

Working in pairs, the students began each activity by constructing personal representations and making first conjectures about the tasks. After completing the activity they engaged in conversations with other students to compare and contrast their ideas, which then may, or may not, have been modified.

Data collection

A variety of data was collected for this study: video taping of the whole class as the students engaged in discussion about various problems and issues that arose; audio taping of verbal interactions during pair engagement in problem situations that were given to the students; video taping of targeted pairs as they worked on the same problems; individual written work; and researchers' field notes.

RESULTS AND DISCUSSION

The data analysis started with the researchers' group viewing of video tapes and the transcription of verbal interactions of the pairs, starting with targeted pairs. Discussion and negotiation about the data interpretation among the researchers started from day one of the data collection and continues through the present time. We believe that the results of this study will contribute to the research of students' probability/statistical thinking about chance and sample space, and their beliefs of fairness of the game and will serve as a guideline for the development of instructional material that might encourage probability/statistical discussions.

Because of the limit in size of this paper we are reporting here only the analysis of classroom discussions that occurred.

Students' ideas about chance and sample space

Students seem to have a variety of ideas about chance, most of them coming from their everyday lives. Those ideas appeared in discussions about the fairness of the games in Investigation #1 and Investigation #2 (Maher, 1995)[1].

Investigation #1. The game is played by two players and involves rolling one die. Player A gets a point if 1, 2, 3, or 4 occurs; Player B gets a point if 5 or 6 occurs**.** The first player to get to 10 points is the winner. [*Note to the reader: The chance of Player A getting a point is 4 out of 6 while the chance of Player B getting a point is 2 out of 6.*]

Classroom discussion

After the students finished their paired investigations, a classroom discussion concerning the fairness of the game took place. There were no disagreements among the students on the first two questions. All students predicted that the original game was not fair and all of them confirmed their conjectures after playing the game. The most interesting discussion occurred when the class discussed how to make the game fair. All students agreed that both players should have the same number of events. That is, that three out of six numbers should be assigned to one player and the remaining three should be assigned to the other player. The disagreement among the students occurred while discussing how to distribute those numbers. A large number of students believed that distribution of numbers 1, 2, and 3 to player A and 4, 5, and 6 to player B would give player A an advantage as the numbers 1, 2, and 3 were, in their opinions, more likely to occur. Other students believed that the distribution of numbers is not important as long as the numbers of possibilities to get a point for both players is the same. Jill and Linda, for example, seemed to share Bruce and Jack's belief mentioned in the previous section. That is, the girls believed that the assignment of one-two-three vs. four-five-six would not be

[1] The tasks (investigations) were developed in the study supported by a grant from National Science Foundations #MDR-9053597 to Rutgers, the State University of New Jersey, directed by Robert B. Davis and Carolyn A. Maher

fair. When the interviewer[2] asked them to explain why they thought the game was not fair, Jill replied:

Jill: Even though we gave each person three, um numbers, it still landed on the numbers one, two, and three the most so we thought it was the placement on the die ...
T/R: So you're finding ... say that again nice and loud.
Jill: Um, that when we split the numbers and gave like Player A one, two, and three and Player B four, five, and six, Player A still won when the numbers were one, two, three, and we thought um, the placement of the numbers on the die ...

Probing further we concluded that "placement of the numbers on the die" meant to Jill that the consecutive numbers are placed on the adjacent sides of the die. Continuing discussion, Ricardo and Mike added that when they "gave Player A two, four, and six, and Player B one, three, and five to even them out" the results were pretty even. Linda defended her pair's explanation by adding:

Linda: Um, we split in a different way so A had one, four, and six and B had two, three and five, and it was much more fair. Because in the game A had six and B had five and we rolled the die one more time to see who would win and the next roll was a five for player B, and so player B would have won if there was one more, so it was much more fair than the way that they asked you to do [it] because the numbers were mixed up.

As we notice, some students, including Bruce, Ricardo and Mike, thought that selecting odds and evens will make the game fair. Others, such as Linda wanted to "mix up" or "spread out" the numbers as much as possible between the two players. A few students perceived that any three numbers assigned to players, regardless of order or pattern, would make the game fair. It is among these students that probabilistic thinking in relation to fairness begins to emerge. We present the next excerpt from a classroom discussion to illustrate how those ideas developed.

T/R: OK, so we have a theory here. We have Linda and Jill's theory, alright? And their theory is that the game is fair if you give the numbers mixed up. But if you give the numbers one, two, three to one player and four, five, and six to the other, not mixed up, the game isn't fair. Now that's a theory, right? What do you think about that theory? Other people? Does that make sense to you? Is that a reasonable theory?
Bruce: Yes.
T/R: Bruce? [Jack answers instead]

[2] The interviewer in this study is refered to the researcher-interviewer (T/R).

Jack: The odds are still 50-50.
T/R: Jack, nice and loud.
Jack [louder]: The odds are still 50-50.
Researcher: The odds are still 50-50 when?
Jack: If you give like any player half of the numbers and the other another half.
T/R: Did you hear him? Could you say it nice and loud, Jack, so back there ...
Jack: They're still even if you give one player half of the numbers and the other player another half.
Researcher: So we have a competing theory here. Now, Jack's theory says it shouldn't matter as long as you give half the numbers to one player and the other half the numbers to the other player, OK? He said something about odds being 50-50. Can you tell me what that means, odds being 50-50? What does that mean? Um, yes.
Ricardo: It means that you have a fifty percent chance of winning and you have a fifty percent chance of loosing.

From the above excerpt it is obvious that Jack, and some of his classmates, had a strong sense about equally likely events when rolling a die. Using proportional reasoning he was able to articulate his arguments ("odds are 50-50").

Investigation #2. The game is played by two players and involves rolling two dice. Player A gets a point if the sum of the two numbers is 2, 3, 4, 10, 11, or 12; Player B gets a point if the sum is 5, 6, 7, 8, or 9. The first player to get to 10 points is the winner. [*Note to the reader: The chance of Player A getting a point is 12 out of 36 while the chance of Player B getting a point is 24 out of 36*].

Classroom discussion

In the classroom discussion the students were divided about the fairness of the game. Some students believed that the game was fair while others thought it was not fair. Those who thought the game was unfair were divided in their thinking about which player has the better chance of winning. The argument of the students who believed that B has a better chance was that "Player B has more numbers that are easier to get." And again, the sample space that students considered in making their decision was the one constructed over the sums of numbers, not over the ordered pairs. In order to bring the appropriate sample space to the students' attention, we asked them to play the game again and to record the numbers as they appear. The discussion about the results took place at the beginning of the next day's class.

We started the next class by showing the students a transparency of a selected recording that one of the pairs used. The students were asked to observe the listing of data and give some interpretation of the commutative ordered pairs, say one-six and six-one or five-four and four-five. Not all students saw the two events as being different. The students who ‘centered’ on the sum of numbers argued that “they’re the same numbers, just written down in a different order ... ‘cause five and four both equal nine. They’re the same numbers.” There were only a few students who indicated that they were thinking of the two events as possibly being different. For example, Ronald interpreted the situation in the following way: “... if it’s six and one, the six is going on another ... like if they had a green and a white one, green and white, like um, if it landed on a green one with a six and a white one with a six one time.” Although Ronald had difficulty expressing his thoughts, it appeared as the conversation continued that he was suggesting that a six appearing on a green die with a one appearing on a white die was a different roll than a six appearing on a white die with a one appearing on a green die. Regardless of Ronald’s effort to rationalize his thinking it seems that the class was not ready to accept this interpretation as many students still argued that it does not matter. We ended the discussion about the sample space at this point leaving the students in disequilibrium. We are very curious to hear what the students have to say about the same question when we return to their class. Will they still hold their belief that one-six and six-one are the same event?

CONCLUSION AND PEDAGOGICAL IMPLICATIONS

In developing probabilistic ideas through games and simulations, it is very important to keep in mind that students’ conclusions are usually drawn from a limited number of observations. Fischbein and Gazit (1984) refer to this kind of intuition as extrapolation. For example, some students in our study made a conclusion that if one assigns 1, 2, and 3 to one player he/she would have the advantage over the player which has 4, 5, and 6. This conclusion was a result of students’ observation when they played the game.

This study gave us insights into students’ ideas about various probabilistic concepts. The development of these ideas in the context of playing games in classroom settings appears to be powerful and at the same time very complex task. In addition, this study provided us with some insights into the complexity of teacher’s role in facilitating the learning process and enabling students to make their personal constructions of knowledge in the context.

REFERENCES

Ahlgren A. and Garfield J. (1991). Analysis of the probability curriculum. In R. Kapadia and M. Borovcnik (Eds.) *Chance encounters: Probability in education*. (pp. 107-134). Dordrecht, The Netherlands: Kluwer.

Borovcnik, M. (1994). Intuitive strategies for teaching statistics. In L. Brunelli and G. Cicchitelli (Eds.) *Proceedings of the First Scientific Meeting of the International Association for Statistical Education*. (pp. 355-366). Perugia, Italy: University of Perugia.

Borovcnik, M. and Peard, R. (1996). Probability. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick. and C. Laborde (Eds.). *International handbook of mathematics education* (pp. 239-287). Dordrecht, The Netherlands: Kluwer.

Bright, G. W., Harvey J. G. and Wheeler, M. M.(1981). Fair games, unfair games. In A. P. Shulte and J. R. Smart (Eds.), *Teaching Statistics and Probability* (pp. 90-100). Reston, VA: National Council of Teachers of Mathematics.

Dann, E., Pantozzi, R. S, and Steencken, E. (1995). Unconsciously learning something. A focus on teacher questioning. *Proceedings of the Seventeenth Annual Meeting of the Psychology of Mathematics Education, North American Chapter*, Columbus, Ohio.

Dominick A. and Clark F.B. (1996). Using games to understand children's understanding. *Childhood Education*, 72(5), 286-288.

Fischbein, E. and Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? *Educational Studies in Mathematics*, 15(1), 1-24.

Leonard, L. M. and Tracey, D. M. (1993). Using games to meet the standards for middle school students. *Arithmetic Teacher*, *40*(9), 499-503.

Maher, C. A. (1995). Children's development of ideas in probability and statistics: Studies from classroom research. *Bulletin of the International Statistical Institute, 2,* Beijing.

Maher, C. A. , Martino, A. M., and Pantozzi, R. S. (1995). Listening better and questioning better: A case study. *Proceedings of the 19th International Conference for the Psychology of Mathematics Education.* Recife-Brazil.

National Council of Teacher of Mathematics [NCTM] (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Shaughnessy, M. J. (1992). Research in probability and statistics: Reflections and directions. In D. Grouws (Ed.), *Handbook of research in mathematics teaching and learning,* (pp. 465-494). New York: Macmillan.