On Basic Principles of Teaching Statistics for Engineering Students

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1. Introduction

The principles of teaching probability theory (PT) and mathematical statistics (MS) formulated here are based on my personal experience of 15 years' work in the Tashkent Motor Highway Institute. Since the teaching of mathematics in general and PT and MS in particular has considerable invariance with respect to local conditions, I shall concentrate on what seems to me the invariant component of my experience, although I cannot fully avoid some local features.

2. General purpose of teaching mathematics for engineers

An enlightened approach to teaching statistics to non-mathematicians begins with reflections on the purpose of such teaching, and which ways best serve that purpose. Final attainment of that purpose may be an unachievable ideal. Nevertheless, even a few steps in the right direction make teaching worthwhile. It is these first steps which guarantee that individuals with creative abilities will never rest content but will continue on their way to extending their statistical knowledge and techniques.

The purpose of teaching PT and MS in a technical college might be crudely described as enabling students to consciously select and use suitable statistical methods for solving engineering problems. For me the key word here is "consciously". I take it to mean, on the one hand, having a more or less precise description of the mathematical model used, however adequate to the engineering problem it might be. The description must include an enumeration of all the mathematical model's elements, and must establish the correspondence between them and elements of the engineering problem. On the other hand, it is equally important that the engineer should understand the inadequacy of any mathematical model to represent fully the engineering problem being solved, and the possibility of constructing different models for the same problem.

It is also very important to warn students against the tendency to draw absolute engineering inferences from mathematical solutions. In general, it is the engineer's nature to shift the responsibility for the correctness of his conclusions on to mathematics, ignoring the fact that any mathematical statement is valid only under certain precisely specified conditions which are rarely satisfied in practice.

3. Ideals and reality in teaching applied mathematics

In the Soviet Union, our higher education organisers have determined the optimal scheme for teaching mathematics to engineers to be the following. Simultaneously with general engineering subjects, or a little earlier, a student is taught an axiomatic course in pure mathematics. Later, while special topics are being studied, he is taught to understand and master mathematical models as one of the tools for the solution of engineering problems.

Such schemes were created at times when higher education was rather more élitist than mass-oriented. Mathematics at both the universities and the technical colleges was taught at a practically identical level of rigour. The high mathematical qualifications of persons who taught special engineering subjects allowed applied mathematics to be taught as mathematics with an engineering interpretation.

My personal experience suggests that the scheme described above for engineers can hardly be implemented for the vast majority of modern technical colleges today.

Firstly, it is difficult to imagine that axiomatic principles of mathematics can be taught successfully in such a college with its huge number of students, most of whom have had inadequate mathematical schooling. Then, it is difficult to believe that a combination of methods stimulating both abstract thinking and professional intuition might prove to be effective. And finally, most of the current special subjects teachers have at best a superficial idea of the deeper aspects of applied mathematics. That is why they prefer to retain mainly its routine sections and its dogmatic character. Curiously, they are assisted in this by modern computers with their powerful software.

Under existing conditions, the inculcation of applied mathematics ideology becomes a duty of mathematicians. Unfortunately, in our country, genuine applied mathematics is not taught, though there exists a whole network of applied mathematics faculties. That is why their graduates are not ready for teaching in technical colleges.

The general difficulties which they face from the very beginning of their activities in such a college are:

(i) uncertainty of the final purpose of teaching mathematics;
(ii) the necessity of a compromise in lectures between rigour and accessibility ;
(iii) outdated university traditions in the choice of material and course construction;
(iv) pragmatism of engineering thinking;
(v) students' scepticism about the practical value of mathematics.

Apparently some of these difficulties are universal. As to the students' scepticism about the practical utility of mathematics, in our country it is due to their knowledge of the low level of technological culture and production management, which allows the engineering staff to get by with minimal knowledge.

To help young teachers adjust to technical college conditions, one may suggest various principles for the construction of mathematical courses. The greater their variety, the easier for the new teacher to choose that which best suits their individuality. The main aim of my report is to describe a possible principle of PT and MS course construction in an ordinary technical college, a principle which I think will prove to be feasible both for students with inadequate mathematical schooling, and for more advanced students.

4. Logical foundations of a PT and MS course

It is convenient to begin the course with an introductory discussion including the analysis of a number of examples, which can be solved with the help of PT and MS. The end point of this discussion should be: how to describe quantitatively a set of objects (finite or infinite), later to be characterised as a statistical population. The aim of the following three lectures is to outline the general principles of such a description.

Lecture 1 - Universal Model

Contents: A population, as the fundamental model, and its visual counterpart; 100% listing of objects as a method of describing finite populations; frequency of a chosen property; distribution of a variable and its characteristics.

Commentary: As a visual counterpart of population one can use an urn filled with small balls. Each ball has a number written on it, which is the value of the variable considered.

Let the population Ω contain the objects $w_1, w_2, \ldots, w_N$, and $\xi = \xi(w)$ be some quantitative characteristic of the objects. Denote by $x_1, x_2, \ldots, x_r$, $r \le N$, the possible values of $\xi(w_j)$, $j = 1,\ldots,N$. Using 100% listing of objects we obtain r groups $\{w : \xi(w) = x_1\}$, $\{w : \xi(w) = x_2\}$, ..., $\{w : \xi(w) = x_r\}$.

The frequency (not probability!) of the k-th group is defined as the ratio

$$\frac{\text{card}\,\{w : \xi(w) = \xi_\kappa\}}{N} = p_k, \qquad k = 1,2,\ldots,r$$

So, we have a distribution of ξ in Ω.

Special Points:

(i) It is a triple - an object, a population, and a variable - that constitutes the universal model.

(ii) A population is considered to be described with the help of given variable if its distribution is known.

(iii) With 100% listing of objects, one does not need a probabilistic terminology.

Lecture 2 - Sampling from finite population

Contents: Sampling with replacement. Sample space. Number of occurrences of the chosen property. "Good" samples. Frequency of "good" samples. Law of large numbers. Sampling without replacement. Proximity conditions for sample spaces corresponding to both sample procedures.

Commentary: If replacement takes place then the sample space Ω', considered as a new finite population, contains N^n objects

$$w' = (w_{i_1}, w_{i_2}, \ldots, w_{i_n}),$$

where n is a sample size. So, we can again use the 100% listing to derive the distribution of any quantitative characteristic of w′. Choose some property A or, what is the same, let $A \subseteq \Omega$.

Let further $v = v(w')$ be the number of occurrences of A in w′. It is easy to explain that

$$\frac{\text{card}(w' : v(w') = k)}{N^n} = \frac{n!}{k!(u-k)!}(P(A))^k (1 - P(A))^{n-k}$$

So, the variable v has the well-known binomial distribution. It is important that it is derived *outside* the traditional framework of the Bernoulli trials scheme.

Then one shows that the mean and variance of v/n are P(A) and $n^{-1}P(A)(1- P(A))$ respectively. We call the sample w′ "good", it for given $\varepsilon > 0$, $|v/n - P(A)| < \varepsilon$. Now it is not a problem to explain that as n increases the overwhelming majority of samples becomes "good". So, we have got an idea about the frequentist structure of the sample space.

Special Points:

(i) Sampling is a reasonable procedure which enables us to derive an approximate description.

(ii) The problem is to organize sampling so that its results reflect the true frequency properties of a population.

Lecture 3 - Description of infinite population

Contents: Simple random sampling from infinite population. Law of large numbers as a law of nature. Probability. Probabilistic terminology. Discrete and continuous distributions. Practical description of infinite population. Sample distribution and its characteristics. Consistency. General question of estimation theory.

Commentary: While dealing with infinite populations, random sampling becomes the only practical method of population description. The law of large numbers postulated as a law of nature gives us an opportunity to provide a theoretical description of infinite populations by means of the distribution of a variable. Like every natural law, the law of large numbers can be realized only under certain conditions. That is why an engineer must organise the sampling so that these necessary conditions will obtain. The practical accuracy of the population description is a general question of MS.

Special point: The exhaustive description of an infinite population is impossible.

After these first three lectures, various directions may be chosen. The choice is dictated both by the students' mathematical knowledge and by the final purpose of the lectures. One may continue with formal random sampling schemes (Bernoulli trials, i.i.d. variable sequence etc.), or one may pass to concrete distribution models (normal, exponential etc.) as, for example, in Nagaev (1987).

5. Concluding remarks

We conclude with remarks on some distinguishing features of the suggested approach to PT and MS course construction in a technological college.

(i) PT and MS studied simultaneously are considered as a collection of models and methods designed to solve specific engineering problems.
(ii) Introduction and consistent utilisation of the notion of population and its visual counterpart are used as a universal model.
(iii) Frequency principle is used for population description.
(iv) The population description (PT) is distinguished from the practical one (MS).
(v) Demonstration of the essence of the law of large numbers under conditions of a finite population, using no such notions as "probability", "random", etc.
(vi) The law of large numbers is postulated as a law of nature.

I suppose that this approach allows us to realise some of the general principles of applied mathematics in teaching statistics. Taken as a whole, it differs considerably from the traditional approach adopted, say, in Gmurman (1972), Caracyev (1977), and Ivchenko and Medvedev (1984). The logical foundations of PT and MS within the framework of our approach are rather strict. The visual demonstration of the universal population model, and the general methods of its description (such as sampling and 100% sorting of data) make these foundations accessible even to those individuals who cannot follow purely abstract mathematical arguments.

References

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