Paradigm and Paradox in Teaching Applied Probability

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1. Introduction

In South Australia mathematics students typically have had considerable exposure to calculus but little to probability on entering university. The amount of probability in school syllabuses has decreased, and our first year university students often associate the subject with fatuous but intricate examples. Some undertake a first year statistics subject, but many will come to a first course in applied probability/ stochastic processes in second year applied mathematics without that background. They are ill at lease with any second year applied mathematics courses not based on calculus (forgetting the effort with which the comfort with calculus was won!) and will often demand evidence of meaningful applicability at an early stage. This creates some challenge and necessitates an examination of one's teaching philosophy.

With a first course, I aim to have the class absorb a knowledge of some elementary ideas and, more particularly, techniques at an automatic response level. This is the stage of familiarity with basic paradigms. It is important that the student try his/ her hand at many exercises, imitating the process whereby familiarity with calculus was achieved at school. The underlying philosophy shares features with that behind the Suzuki teaching system for music (see Mill and Murphy, 1973). Shinichi Suzuki observed that whilst quite ordinary children in Japan learn the language without difficulty, learning to play an instrument is commonly regarded as something for a gifted élite. He argued that by imitating the process whereby a child learns its mother tongue, any child can learn to play an instrument. There has been a Suzuki explosion in the West since 1958 following a film which showed an ensemble of a thousand Japanese children playing Bach's Double Concerto. This thinking is also related to the (Japanese) Kumon system for mathematics which, like the Suzuki system, aims at *mastery* rather than merely *competence* as a goal, and utilises repeated practice to give easy familiarity.

In the first instance, mastery of basic paradigms by practice has little to do with understanding; indeed, attempting explanation can produce an impediment to learning. Studying a course on theoretical mechanics is not a good way to learn how to ride a bicycle. This point was also raised at this Conference by John Taffe (1991), who commented that "explanation is a poor teaching strategy". One learns by practice until a skill comes automatically, when the attention is freed from the mechanics of the procedure and can be devoted to special aspects of the problem and to an overview.

Everyday observation shows us that humans like acquiring, exercising, and honing skills. This natural tendency can be harnessed in the paradigm phase of learning through practising examples.

Understanding takes its proper place at the level of a second course. With some background in place, an additional feature enters, namely sharpening understanding of concepts by jolting the intuition through the introduction of paradoxes. Interest is easy to arouse via curiosity. This begins with simple paradoxes which can involve such ideas as

(i) The mean: The mean is an attempt to capture a typical value. However, the mean of a random variable that is finite with probability one can itself be infinite.

(ii) The renewal paradox, wherein the average time an arrival at a random epoch has to wait for an event in a renewal stream can exceed the average time between consecutive events in the stream.

Eventually, of course, one has to face the substantial body of theory involved in stochastic processes and the pitfall of "too much structure and too little insight" alluded to by Marcel Neuts (1991) and by Roger Mead (1991) in their talks. It is easy to become enmeshed in too much theory at an early stage. Restructuring textbook approaches can mitigate this problem. Thus, hitting probabilities and first passage times can be considered for Markov chains *ab initio* before there is any mention of transience or recurrence of states or of communicating classes. There are graphic applications to examples such as tennis matches. The discussion involved then makes the subsequent treatment of necessary and sufficient conditions for transience appreciably easier and clearer.

Roger Mead and Marcel Neuts have also commented on the use of stochastic processes for experiential teaching.

2. An example

In this section a simple result with a real-life application (in Australasia) is noted which relates to the paradigm aspect of teaching. This result is recent and has not yet reached the textbooks. I am indebted to Hugh Morton for bringing some recent articles (particularly Dawkins and Thomson, 1989; Morton, 1987 and 1989) to my attention.

Suppose $n \geq 1$ distinct numbers are chosen from $\{1,2,...,m\}$ at random. Then we have the striking result that the probability p_r of exactly r consecutive pairs of numbers being selected follows a hypergeometric distribution, that is,

$$p_r = {}^{n-1}C_r \; {}^{m-n+1}C_{n-r} / {}^{m}C_n \quad (0 \le r < n \le m).$$

This is quite different from the standard occurrence of the hypergeometric distribution involving the choice of r elements at random from a population of n, of which n_1 are red and the remainder black (cf. Feller, 1964, Chapter 2, Section 6).

The result follows from elementary considerations of a permutations and combinations type. A mathematical discussion of the problem with a generalisation will be presented in a separate article. The problem is also susceptible to treatment by a variety of other methods, such as the use of generating functions, which can uncover further structural information. However, these are less effective in that they do not as fully exploit the independence structure of the problem. As noted by Marcel Neuts in his talk, the development of this sort of structural perception in the student is crucial.

The example arises from modelling the game of Cross Lotto, in which each of a number of players marks eight numbers from {1,2,...,45} on his/her sheet. The house chooses a set of eight numbers by chance without the knowledge of the players. Any player who has chosen this set achieves a win. In practice, players tend to choose no consecutive pairs less frequently than occurs by chance, which gives a winning margin to the house.

References

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