

Solving Problems is Not Enough : Assessing and Diagnosing the Ways in Which Students Organise Statistical Concepts

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1. Introduction

Educators are reformulating the goals of both teaching and assessment within content domains. Some of the recurring themes include the need for instruction to promote students as thinkers, problem-solvers, and inquirers; the need to conceptualise meaningful understanding as involving active construction; the need for testing for thinking and understanding; and the need for assessment tasks to more closely resemble "real world" learning tasks. According to this view, assessment procedures used in introductory statistics courses should not only assess students' skills in carrying out statistical procedures, but also their understanding and thinking about statistics. Teaching statistical thinking and reasoning would be greatly facilitated, for example, by using instruments capable of assessing the structure and the relationships students impose on their own knowledge of statistical concepts. This paper provides a conceptual framework for generating assessment tasks which provide an instructor with a richer description of students' thinking and reasoning than is possible by just giving students problems to solve. Although the framework is general, its application is illustrated with material from college and beginning graduate level pre-calculus introductory statistics courses which focusses on the following topics: sampling, interval estimation, point estimation, and hypothesis testing.

A fundamental objective of instruction should be the development of students' connections among concepts and the structuring of their knowledge (Glaser, 1989). A beginning student's knowledge base consists of isolated bits of information reflecting a shallow understanding of concepts and their interrelationships. With instruction and

experience, knowledge should become more structured and better integrated with prior knowledge.

Research that examines how students process and structure knowledge has direct implications for the design of more valuable assessments (Glaser, Lesgold, and Lajoie, 1987; Lane, 1989; Marshall, 1988; Nitko, 1989). Tests need to assess not only declarative and procedural knowledge but also the relationships that provide a foundation for the organising of students' knowledge structures. There is a need to assess whether a student can obtain the right answer to a problem and also whether the solutions are based on an understanding of underlying principles and concepts, that is, whether they are responding to the deep structure of the problem rather than its surface structure. Diagnosis of learning should focus on how a learner perceives the structure or organisation of content and processes information and knowledge to solve problems (Nitko, 1989). Because students' structures of knowledge change over the course of instruction, such assessments should also be sensitive to modifications and reformulations in students' knowledge structures.

In this paper, we adapt work done in the mathematics discipline by Putnam, Lampert, and Peterson (1990) to the statistics discipline. We relate statistical *activities* to various ways of describing a person's *understanding* or cognition of these activities. The relationship between activities on the one hand and cognitions which underpin these activities on the other, provides a framework that gives statistics instructors guidance concerning the kinds of assessment tasks to create in order to assess the ways in which their students think about and reason with statistics.

2. Statistical activities

Statistical activities may be divided into three interrelated domains: problem-solving, statistical modelling, and statistical argumentation. Within each of these activity domains instructors can differentiate students in a statistics class. *Statistical problem-solving* is the ability to apply statistical concepts to problem situations. Students who are good problem-solvers are able to abstract from a given situation the relevant bits of information and their relationships, and to perform transformations on the mathematical relationships underlying symbols. Knowledge of conventions, symbols, and systems, and their interrelationships, underlie successful problem-solving activity. Assessment tasks need to capture students' ability to formulate problems, identify principles underlying problems, analyse situations to determine common properties, verify and interpret results, and to generalise solutions.

Statistical modelling involves knowing how to develop a model by representing the relationships in a problem graphically and symbolically. Students should also be able to describe the conditions under which a particular model is appropriate. Problem-solving and statistical modelling are related by the activity of formulating a problem. Assessment tasks focus on students' ability to identify appropriate models, distinguish among competing models, identify the conditions necessary for application of a certain model, and identify how models are interrelated.

Statistical argumentation is the ability to evaluate the truth of propositions and to develop plausible arguments for statistical assertions. Inductive reasoning is required to recognise patterns and to make conjectures. Deductive reasoning is required to verify

conclusions, to judge the validity of arguments by considering counter-examples, and to construct valid arguments. Assessment tasks focus on students' ability to identify false arguments, to explain similarities and differences among competing arguments, and to articulate the relationships among concepts and procedures.

3. Statistical cognitions or understandings

Underpinning a person's performance in each of the statistical activity domains, is a complex array of cognitions which we generally describe as a person's understanding of statistics. We focus on five interrelated ways of describing a person's understanding which we call cognition domains: understanding as representation, understanding as knowledge structures, understanding as connections among types of knowledge, understanding as entailing active construction of knowledge, and understanding as situated cognition (Putnam, Lampert, and Peterson, 1990).

Understanding as representation means having internalised ideas, symbols, and systems, and being able to move within and between them in ways that allow for successful problem-solving activity. When assessing this aspect of understanding statistics, an instructor determines the relationships between an external portrayal of statistical information and the internal cognitive representations of a student. The representational ability of the student is reflected in the degree of organisation in a student's knowledge structure (Glaser, 1989). Naive representations of subject matter represents only partial understanding by the student. Assessment tasks focus on students' ability to use statistical vocabulary, notation, and structure to represent ideas, solve problems, and model situations.

Understanding may be conceptualised as an appropriately integrated and organised knowledge structure. Understanding statistics also means having access to the knowledge and cognitive processes needed to perform various statistical tasks. Problem-solving is intimately related to knowledge structures because coherent and organised knowledge structures underlie successful problem-solving. Valuable assessments of students' thinking include examining the understandings and models that students construct for themselves during the learning process. Levels of achievement could be viewed as levels of understanding of concepts and their interrelationships that underlie a subject matter domain (Masters and Mislevy, in press). Assessment tasks capable of characterising knowledge structures underlying problem-solving activity provide valuable information about students' statistical understanding.

Characteristics of knowledge structures that can be examined include the number and types of concepts depicted, the number and types of relationships among concepts (including conditional and hierarchical), and the degree of organisation. For diagnostic purposes, congruence between a student's knowledge structure and various knowledge structures of experts may be examined. MicroCAM (described later in this paper) is one technique for assessing knowledge structures.

Understanding may be viewed as connections between different types of knowledge such as between conceptual and procedural knowledge and between formal, symbolic knowledge, and informal knowledge. From this viewpoint, knowing statistics means understanding the concepts underlying the procedures and being fluent at integrating formal knowledge with informal knowledge developed outside of the class

environment. Tasks focus on assessing an ability to create interrelationships across types of knowledge.

Understanding as situated cognition means that understanding statistics involves the interplay between students' cognitive activity and physical and social situations. Learning is the process of constructing understanding out of activities embedded in the social and physical world. Knowledge may be viewed as a product of the activity and situations in which they are produced and used (Brown, Collins, and Duguid, 1988). This involves students' participation in statistical activity or "doing" statistics such as abstracting, convincing, inferring, organising, representing, inventing, generalising, explaining, validating, and conjecturing. Ecologically valid instructional and assessment tasks entail being representative of the ways in which knowledge and skills are used in "real world" contexts (Brown, Collins, and Duguid, 1988). This implies the need for assessment tasks to be embedded in activities that have "real world" meaning. In a classroom assessment situation, these activities may be simulated by giving students background context, research questions, and sets of relevant and irrelevant raw data, and asking them to use the data to interpret and answer the research questions.

Understanding as the active construction of knowledge means the process by which knowledge structures have been developed or acquired by students. Learning entails actively reorganising and integrating new information with existing knowledge. Understanding statistics means having expanded one's own knowledge structures or ways of thinking to incorporate statistical concepts and principles. Assessment procedures need to be sensitive to modifications in students' knowledge structures and requires the assessment of students at various points in time, such as before and after instruction, in order to trace developing knowledge structures. MicroCAM is an example of an assessment procedure that may be used to examine changes in knowledge structures as a result of instruction.

4. Generating assessment procedures and tasks

The foregoing theoretical conceptualisations of statistical activities and statistical understandings may be used to create a framework which gives guidance to statistics instructors for developing assessment procedures. Assessment tasks can be developed which measure students' levels of problem-solving, modelling, and argumentation (reasoning) abilities using as a guide each of the cognition or understanding domains. This will ensure that an instructor obtains a comprehensive assessment of students' thinking that will be especially useful to guide instruction.

It should be noted that the assessment framework we provide requires you to utilise a wide range of assessment techniques; many more techniques, in fact, than are used for assessment in a typical introductory statistics course. Among the assessment techniques needed to adequately utilise our framework are the following question types: essays, short answer, yes-no with student-supplied justifications, concept-oriented multiple-choice items, masterlist, analogical reasoning, graphic inference, concept mapping, and computer-guided. Longer-term individual and group projects may be required also.

TABLE 1
 Framework for the Generation of the Assessment Tasks

Domain of Understanding Statistics	Problem Solving	Domain of Statistical Activity	Statistical Arguments
Representation	<ul style="list-style-type: none"> a. Formulate problems using statistical concepts and ideas b. Express solutions in terms of appropriate concepts, symbols and systems c. Translate words into statistical symbols and relationships and vice versa d. Use statistical vocabulary, notation, and structure to represent statistical problems, ideas, and situations 	<ul style="list-style-type: none"> a. Use geometric, algebraic, and statistical models to represent data, problems and situations b. Describe how practical problems can be restated to fit one or more statistical models c. Identify data needed to support the fit of a statistical model 	<ul style="list-style-type: none"> a. Recognise false arguments b. Use inductive reasoning to make and validate conjectures c. Critically evaluate conclusions and claims made using data obtained from samples and surveys d. Justify selection and use of specific statistical indices in particular research and/or decision contexts
Knowledge Structure	<ul style="list-style-type: none"> a. Identify principles that underlie problems b. Organise and classify problems 	<ul style="list-style-type: none"> a. Show how models are derived and interrelated b. Describe conditions under which statistical models are upheld 	<ul style="list-style-type: none"> a. Explain similarities, differences, advantages, and disadvantages among competing statistical arguments b. Explain how statistical concepts are linked to specific statistical arguments
Connections Among Types of Knowledge	<ul style="list-style-type: none"> a. Explain how statistical concepts are used to solve a problem b. Use statistical vocabulary, notation, and structure to describe relationships c. Analyse situations to determine common properties, structures, and patterns 	<ul style="list-style-type: none"> a. Articulate systems or models, linking several models into a unitary framework b. Articulate how statistical models may be used to answer research and practical questions c. Articulate the relationship of statistical modelling to other forms of knowledge acquisition 	<ul style="list-style-type: none"> a. Articulate the relationships among concepts, indices, and procedures b. Articulate relationships among formal and informal statistical knowledge c. Articulate how statistical arguments link to other epistemological frameworks

Table 1 provides a framework for generating assessment. Each of the three activity domains is crossed with three understanding domains. The body of the table contains examples of the types of *student* performances which an instructor may assess. As stated previously, the domains are interrelated, so the table might be better conceptualised as seamless fabric, rather than as discrete cells.

One way to view this table is to read downward within a statistical activity type and across types of understanding. For example, within problem-solving, you can see the wide range of performances a student should display if he or she understands the nature of a statistical problem and its solution. Simply obtaining the correct answer to a problem is insufficient to display deep understanding.

We have illustrations of a variety of assessment tasks that correspond to each cell in the theoretical framework captured by Table 1. These tasks focus on sampling, interval estimation, point estimation, and hypothesis testing, content areas typically taught to majors in education or the social sciences in a college-level pre-calculus introductory statistics course. Because of space limitations, these sample assessment tasks cannot be reproduced here. They may be obtained from the authors.

Assessments in the cognition domains of active construction of knowledge and as situated cognition, do not readily fit into the framework of Table 1, but these domains have been shown to underpin a students' understanding of problem-solving, modelling, and argumentation (Lave, Smith, and Butler, 1988; Marshall, 1988; Schoenfeld, 1988). A variety of techniques have been developed for assessing knowledge structures and their construction (for reviews see Nitko, 1989; Shavelson and Stanton, 1975). Here we discuss one technique, MicroCAM, which uses a Macintosh computer.

5. MicroCAM

MicroCAM (Ju, 1989) is a microcomputer-assisted measurement tool that assesses students' knowledge structure and interrelationships among different types of knowledge in various subject matter domains. The program was developed in HyperTalk for the Macintosh microcomputer. Using MicroCAM, students provide a spatial display on the computer screen of their knowledge structures including the concepts, connections among concepts, and the relationships underlying the connections. Figure 1 shows one student's representation of concepts related to basic statistics. To obtain this figure, the student used a mouse to move the concepts in the boxes around the screen to depict how the student associated them. The student then connected the concepts by lines to show the linkages the student perceives. The student then entered words on the lines to describe the nature of the linkages or relationships.

MicroCAM can be used to evaluate students' initial understanding prior to instruction as well as changes in understanding resulting from instruction. Understanding of students' initial organisation of knowledge and the relationships underlying their knowledge structures allows the teacher to tailor instruction to promote individual student learning. Students' explicit representations of their knowledge structures prior to and after instruction provides a means to knowing students' misconceptions, weaknesses, or problems, and this information may be used as a basis for remediation. Moreover, comparisons of students' knowledge structures before and after instruction can be used to determine the effectiveness of instruction.

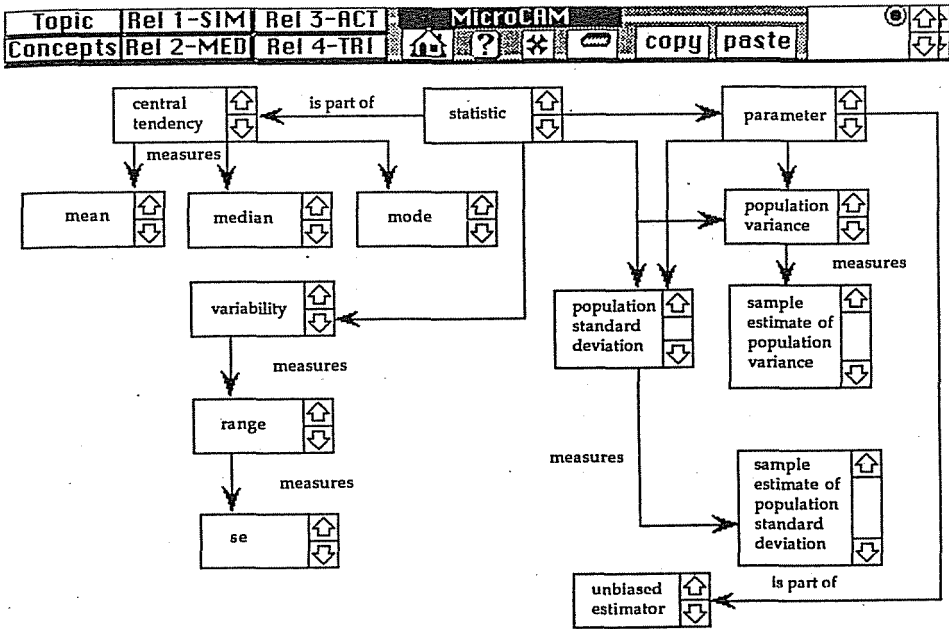


FIGURE 1
Knowledge structures of student G

Teachers or researchers can develop their own assessment instruments using MicroCAM. The following types of assessments can be developed: (a) the teacher specifies concepts and presents a taxonomy of concept relations to the students; (b) the teacher specifies concepts, but does not present a taxonomy of relations to the students; (c) the teacher does not specify concepts, but does present a taxonomy of relations to the students; and (d) the teacher does not specify concepts, and does not present a taxonomy of relations to the students. The taxonomy of relations that was built into MicroCAM was derived from three sources (Armbruster and Anderson, 1984; Ballstaedt and Mandl, 1985; Fisher, 1988). Ju investigated MicroCAM's stability reliability, concurrent validity, and sensitivity to change during statistics instruction. Results were positive in each of these areas and lend support to the reliability and the validity of using MicroCAM to obtain useful information about students' knowledge structures.

6. Implications for integrating assessment and instruction

The assessment framework presented in this paper can be a basis for providing valuable information regarding students' learning difficulties. Assessments developed using the ideas in Table 1 are capable of identifying misconceptions, and missing concepts and links in students' knowledge provide information to guide the instruction. As another example, MicroCAM can be used prior to instruction to determine the statistical knowledge that students bring to a course, including their misconceptions, concepts, and their interrelationships. Instruction can be aimed at building and

organising students' knowledge structures to facilitate statistical problem-solving and reasoning. MicroCAM can also be used after instruction to assess the degree to which students have organised and integrated new information with existing knowledge.

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