Relationships Among Dimensions of Statistical Knowledge

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1. Introduction

A controversy has arisen concerning the relative merits of conceptually-oriented teaching versus calculation-centred teaching. Marks (1989) maintains that concepts are far more important than computations, and that they can be successfully taught without the related computations. In contrast, Khamis (1989) claims that students cannot truly understand statistical information until they have had experience doing calculations by hand. Both authors present persuasive arguments, but no empirical evidence to support their conclusions. The present paper outlines a study which aimed to fill this gap. First, however, we try to place the controversy into the context of wider cognitive issues.

2. Ways of knowing

This debate about the virtues of conceptual and computational training parallels a more general debate in education that has been going on for nearly a century. During the 1920s and 1930s the debate in education focussed, as does the current debate in the teaching of statistics, on the prescription of optimal educational practices. That is, authors claimed one or the other method to be superior, and described how and why it should be used in classroom settings. Today the discussion of conceptual and computational learning in mathematics education has shifted from a focus on prescription to a focus on description. Current investigations are directed at understanding the antecedents, mechanisms, and consequences, of these different ways of knowing and learning (see Hiebert and Lefevre, 1986).

The two contrasting "ways of knowing" have been variously referred to by different authors in the field of mathematics education, but their meanings are essentially the same. Nesher (1986) has contrasted mathematical *skills* with *thinking*, Skemp (1978) uses the terms *instrumental* and *relational* understanding to describe these two different ways of learning and knowing, and Resnick and Omanson (1987) distinguish between *conceptual understanding* and *procedural or computational skill.* In each case the comparison is between knowing how to do something, how to execute a procedure or a skill, and knowing about something in an otherwise meaningful way.

The distinction between procedural skill and conceptual understanding in the field of mathematics education bears some resemblance to a distinction found in the cognitive psychology literature. Anderson (1983) and others have contrasted procedural knowledge (knowing "how") with declarative or propositional knowledge (knowing "that"). Although these two views on ways of knowing may appear similar, they are by no means isomorphic (Nesher, 1986). The significant difference is that mathematics educators contrast procedural ability with meaningful knowledge, while the cognitive psychologists contrast procedural ability with knowledge of facts and propositions. For the mathematics educators the issue is one of meaning rather than specific facts, and they believe that meaning is the result of making links or connections between various facts and propositions.

In order to avoid the confusion caused by these two models that sound similar but are fundamentally different, it is necessary to consider a more elaborate description of the different ways of knowing. Baroody and Ginsburg (1986) have chosen to contrast meaningful knowledge with mechanical knowledge, and to further subdivide mechanical knowledge into knowledge of propositions and knowledge of procedures. This model, while providing a fuller view of the ways in which individuals learn and know, makes it difficult to conceptualise the ways in which relationships or connections can be made. We propose a model which contains facts or propostions at one level, procedures at another level, and in which links (the mechanisms for meaning-making) may be drawn among facts, among procedures, and between facts and procedures.

This distinction among propositional knowledge, procedural knowledge, and meaningful knowledge can help us to think more clearly about the processes of teaching and learning (Hiebert and Lefevre, 1986). Research has shown that knowledge of one type does not guarantee knowledge of another. For example, third grade children who showed a strong conceptual understanding of the process of subtraction with borrowing could not always perform the algorithm for it. In addition, there were children who were successful with the algorithm (the procedure) but did not show evidence of any meaningful understanding of the principles underlying the algorithm (Resnick and Omanson, 1987). In order for procedures and facts to imply one another they must be linked; there must be, by definition, meaningful understanding. In a study on the teaching of probability, Hansen, McCann and Myers (1985) found that students who were taught using an explanatory approach with an emphasis on the connections between various pieces of information and skills were more successful at solving story problems.

Despite the importance of the distinctions among declarative, procedural, and conceptual knowledge, it must be remembered that these categories are not mutually exclusive, nor are they exhaustive. In fact, it is the relationships among these different ways of knowing that create true meaningful understanding, and thus are very important. We must make these distinctions not as an end in itself, but in order to more clearly understand how the pieces of the learning puzzle fit together.

We have partitioned the ways of knowing into three basic categories: calculations (an example of procedural knowledge), propositions (simple facts), and conceptual understandings (linking of two or more propositions). We further distinguish between the ability to understand and articulate propositions and concepts in symbolic (or notational) form and the ability to do so in words. Thus we will deal with a "structure" having five categories:

Calculations	Propositions		Conceptual Understandings	
	Words	Symbols	Words	Symbols

3. An empirical study

The purpose of the study was to investigate numerical inter-relationships of student performance on subsets of test items that were grouped according to the above five categories. That is, we were interested in addressing questions such as: How do students perform on calculation items relative to their performance on proposition items? Relative to total test performance?

The test comprised 31 items, all of the multiple-choice variety. Ten items belonged to Calculations, ten to Propositions (5 Words, 5 Symbols), and 11 to Conceptual Understandings (6 Words, 5 Symbols). Items from the five categories were "scrambled" to arrive at an ordering of the 31 items. In addition to responding to the 31 items, students were requested to provide some demographic information: gender, age, undergraduate major, highest level of mathematics, and number of years since last mathematics course.

The 57 students in this investigation had taken introductory statistical methods in a College of Education. (The level of study may be indicated by the textbook used, viz, Moore and McCabe (1989).) All students were enrolled in doctoral programmes.

Before presenting results of relationships among various student scores, some numerical descriptions of test performance will be presented. Eleven different sets of test items were considered: Total Test (31 items), Calculations (10), Propositions (10), Propositions with Words (5), Propositions with Symbols (5), Conceptual Understandings (11), Conceptual Understandings with Words (6), Conceptual Understandings with Symbols (5), Words (11), Symbols (10), and Noncalculations (21). The internal consistency of the 11 sets of test items was assessed using a Kuder-Richardson formula. The total test coefficient value of .734 is respectable, indeed. The subtests involving Symbols yielded three of the four lowest reliabilities (.331, .087, .327). In general, the items were not too difficult since about one-half of the items were correctly answered by at least 60% (34/57) of the students. The number of correct items for the total test ranged from 9 to 28 with a median of 17. An examination of responses to individual items indicated that the Symbols items were more difficult than others. High difficulty and low variability are two contributors to the attenuation of the reliability of the Symbols subtests.

Two groups of students were formed from student feedback on "highest level of mathematics": college algebra or less (32 students) versus introductory calculus or beyond (23 students). Mean differences for these two groups were examined with respect to the 11 test scores. The 11 mean differences were assessed via the squared point-biserial correlation coefficient, which ranged from .007 (Conceptual Understandings in Symbols) to .112 (Calculations) with a median of .022. The relatively large difference (5.87-4.44) in the Calculations means is, perhaps, not too surprising. But mathematics background (as defined here) accounting for about 11% of the variance of Calculations scores is not too impressive, and does not yield strong support for student concern often expressed. (Similar results were found when performances of students with only high school algebra were compared with all others.)

Two groups of students were also formed from feedback on "number of years since last mathematics course": 14 or fewer years versus 15 or more years. Squared point-biserial correlation coefficient values ranged from .001 (Total Test) to .041 (Words) with a median of .019. These data clearly do not support student concern for lack of recent mathematics study. (Similar results were found when performances of students who studied mathematics less than nine years ago were compared with those away from mathematics for 15 or more years.)

It was thus decided to examine relationships among the 11 test scores utilising the total group of 57 students. Of the 55 correlations, 21 reflect relationships between scores on subtests designed to be non-overlapping in terms of knowledge or skill being measured. The non-overlapping character appears to have numerical support, at least to some extent, since the percent of shared variance ranged from only 1% (Propostions with Words versus Conceptual Understandings with Symbols) to 37% (Calculations versus Propositions) with a median of 13%.

There is a particularly interesting trio of correlations, namely those among Calculations, Propositions, and Conceptual Understandings. The relationships among these three subtests may perhaps be understood in light of the model proposed by Baroody and Ginsberg (1986). The Calculation-Propositions correlation is .61, while the Calculations-Conceptual Understandings correlation (.40) and the Propositions-Conceptual Understandings correlation (.37) are at a lower (and comparable) level. This evidence, albeit somewhat tentative, lends support to the Baroody-Ginsberg notion that meaningful knowledge (what we label Conceptual Understandings) is "different" from mechanical knowledge (what we would label Calculations and Propositions).

Of course, caution must be taken in making direct comparisons across correlation values since the number of items (i.e. test-length) is not constant for all values. The subtests with smaller numbers of items would have somewhat restricted score variability which would attenuate the correlations. The five 10- or 11-item subtests are approximately equally good discriminators. The 21-item subtest on Noncalculations is approximately equivalent in terms of discrimination to the 10-item subtest on Calculations.

4. Discussion

The conceptual-computational issue in learning mathematics/statistics was alluded to earlier in this paper. This is an issue, too, in the teaching of statistical methods and in the assessment of statistical knowledge acquired by students. From a pedagogical standpoint, it seems reasonable to the current authors that students should be tested in more than one "domain". In fact, we suggest three such domains: calculations, propositions, and conceptual understandings. Of course, these domains are not independent, in any sense of the word. But, in the current preliminary study we obtained some empirical support for the notion that these domains overlap to a rather limited degree. (Heretofore, empirical evidence to the inter-relationships of such domains has been virtually nonexistent.) That is to say, it doesn't appear that focussing student testing on any one domain will result in a very thorough assessment of statistical knowledge.

At first glance, one might be tempted to conjecture that testing on calculations alone would yield a fairly basic assessment of learning (since the correlation between Calculations and Total Test was .85). But, looking further one sees that Noncalculation scores correlate to the same extent with Total Test scores ($r = .93$).

It is also suggested the instructors of statistical methods classes think seriously about encouraging (and expecting) students to think in terms of multiple ideas and making connections among them; i.e. to develop conceptual understandings from their study. And, of course, instructors are encouraged to *test* for student conceptual understanding.

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