

# The Origin of Inconsistencies in Probabilistic Reasoning of Novices

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## 1. Introduction and method

This paper describes a study in which subjects were asked about various aspects of coin flipping. Many gave contradictory answers to closely-related questions. We offer two explanations for such responses: (a) switching among incompatible perspectives of uncertainty, including the outcome approach (Konold, 1989), judgement heuristics (Tversky and Kahneman, 1982), and normative theory; and (b) reasoning via basic beliefs about coin flipping. As an example of the latter, people believe both that a coin is unpredictable and also that certain outcomes of coin flipping are more likely than others. Logically, these beliefs are not contradictory; they are, however, incomplete. Thus, contradictory statements (and statements at variance with probability theory) appear when these beliefs are applied beyond their appropriate domain.

In the study, twenty subjects (twelve women and eight men) were recruited from undergraduate psychology courses at the University of Massachusetts. Eleven of the subjects had taken, or were currently enrolled in, a statistics course. Subjects participated in an hour-long videotaped interview (conducted by Hendrickson) that included several questions concerning various aspects of probability. The questions about coin flipping came in the middle of the interview. These questions are presented below in the order in which they were asked.

*4-Heads Problem:* "A fair coin is flipped 4 times, each time landing with heads up. What is the most likely outcome if the coin is flipped a fifth time? (a) another heads is more likely than a tails; (b) a tails is more likely than another heads; (c) the outcomes (heads and tails) are equally likely?"

*H/T Sequence Problem:* 1. "Which of the following sequences is *most* likely to result from flipping a fair coin 5 times? (a) HHHTT; (b) THHTH; (c) THHTT; (d) HTHTH; (e) all four sequences are equally likely." 2. "Which of the following sequences is *least* likely to result from flipping a fair coin 5 times?" [The options above were repeated, with option (e) worded "... equally unlikely."]

*Probes:* After giving answers to these questions, subjects were asked to explain their answers and to give probability estimates for various sequences. Almost all the interviewer's probes (which are available upon request) were standardised in a series of pilot interviews. However, the interviewer was free to ask questions that were not planned when, in her judgement, further clarification was required.

TABLE 1  
Correct and incorrect responses to the 4-Heads and H/T Sequence problems

Subject	4-Heads Problem		H/T Sequence Problem				
	Equal	P=.5	Most	Least	P =	Psum	
A	6*	+	+	+	+	+	+
	16	+	+	+	+	+	+
	20	+	+	+	+	+	+
	13	+		+	+	+	+
B	17	+	+	+	+	?	?
	2*	+		+	+		
	15	+	+	?	?	?	?
C	12*	+	+	+	+	+	-
	14*	+	+	+	-	+	+
	19	+	+	+	+	-	+
	8*	+		+	+	-	+
	4*	+	+	-	-	+	+
	5	+	+	+	-	+	-
	11*	+	+	+	-	-	+
	18	+	+	+	+	-	-
	9*	+		-	-	+	+
	3	-	+	+	-	-	-
	7*	+	+	-	-	-	-
1*	-	+	-	-	-	-	
D	10*	-	-	-	-	-	+
% Correct		85	94	74	53	53	65

Table 1 codes correct (+) and incorrect (-) responses for each subject on six aspects of the interview questions; (?) means that the subject did not think it was possible to answer, and a blank indicates a missing value. Subject numbers are listed down the left of the table, response categories along the top. The subjects are in groups A, B, C, D. A (\*) indicates prior or concurrent attendance at a probability/statistics course.

The first two columns indicate subjects' answers to the 4-Heads problem. The first column (Equal) indicates whether they correctly answered that the two outcomes

were equally likely, and the second ( $P = .5$ ) whether they gave a probability value of heads (or tails) as .50. The last four columns note correct and incorrect responses to questions that were asked as part of the H/T Sequence problem. Columns three (Most) and four (Least) indicate correct and incorrect responses to the most- and least-likely versions of the problem. A plus in column five ( $P =$ ) indicates that the probabilities a subject gave for the various sequences were equal. A plus in column six (Psum) indicates that the sum of probabilities given for the mutually exclusive sequences was less than 1. The percentage of correct responses to each question are listed along the bottom row of the table. There were no significant differences in the mean number of correct responses based on either gender or prior statistics instruction.

## 2. Salient inconsistencies

Of primary interest in this study was the consistency of a subject's responses over problems. To convey an overall impression of this aspect, subjects are ordered in Table 1 according to total number of correct responses. Subjects listed in the top group, A, of the table answered all of the problems normatively. The subject in the bottom group, D, answered consistently in accord with the representativeness heuristic (Tversky and Kahneman, 1982). Subjects in group B showed no obvious inconsistencies, but did not respond to a number of questions. The 12 subjects in group C showed various inconsistencies in their responses and thus are of greatest interest to us here. The most salient of these inconsistencies are described below.

(i) *H/T Sequence : most- vs least-likely outcomes.* Subjects who respond that all sequences in the H/T Sequence problem are equally likely should then respond, if they are reasoning normatively, that all sequences are equally *unlikely*. However, subjects  $S_3$ ,  $S_5$ ,  $S_{11}$ , and  $S_{14}$  each responded that all the sequences in the H/T Sequence problem were equally likely but went on to choose a sequence as *least* likely.

(ii) *Qualitative answers vs probabilities : H/T Sequence.* Subjects who respond that all of the sequences are equally likely ought then to give equal probabilities to the options. However  $S_8$ ,  $S_{18}$ , and  $S_{19}$  each responded that all of the sequences were equally likely and then assigned them unequal probabilities.

Similarly, subjects who choose a particular sequence as most (or least) likely ought to assign a greater (or lesser) probability to that sequence than to the other sequences. However,  $S_4$  and  $S_9$  selected option "c" as least likely, but then went on to assign the same probability to option "c" as to the other options (10-20% in the case of  $S_4$ ; 20% in the case of  $S_9$ ).

(iii) *Qualitative answers vs probabilities : 4-Heads.* Subjects reasoning normatively about the 4-Heads problem will respond that heads and tails are equally likely and then will assign equal probabilities to each outcome. Although most of the subjects responded accordingly,  $S_1$  and  $S_3$  selected tails as the more likely outcome and then assigned both heads and tails an equal probability of 50%.  $S_{10}$  was the only other subject to select tails as more likely, and the only one to give a higher probability to tails. Thus his reasoning, though incorrect, was consistent.

(iv) *Responses on the 4-Heads vs H/T Sequence problems.* Subjects correctly answering the 4-Heads problem would seem to be exhibiting an understanding of





