Classroom Practicals Using the Binomial Distribution

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1. Introduction

The essence of the practical approach is to start with a *problem*, collect *data*, *analyse and discuss*, and develop a *model*. In this way, pupils are involved actively in the learning process and will enjoy and remember what they have done. Much work done by older pupils on probability tends to consist of paper and pencil exercises (usually tree diagrams). If they are lucky, younger pupils may be allowed to experiment with dice, cards, and coins. Hopefully, further experiments will continue that progression and allow the pupil to make decisions on the basis of probabilistic arguments.

In this paper, practical experiments are used not only to illustrate the binomial distribution, but to make it relevant and important to the pupils. The aim is to take statistics and probability out of the textbook and into the pupils' direct experience.

The practical activities described here are by no means all new; but they have been tried and tested, by myself and by teachers involved in the *Practical Statistics* project (Rouncefield and Holmes, 1989).

While most teachers of statistics teach the binomial distribution to pupils aged 16+, I would like to suggest that this topic can be introduced at a slightly younger age, to enliven lessons on probability. Providing pupils have some understanding of the multiplication and addition laws of probability, it is not a great step then to go on to develop the binomial distribution using tree diagrams.

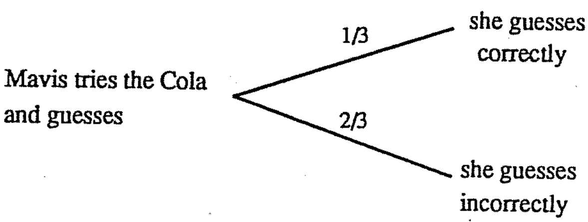
2. Can we really tell the difference?

My child claims not only to be able to tell the difference between brands of cola drink, but also that he positively dislikes the cheaper supermarket brand.

Is this prejudice based on fact? Can people really tell the difference?

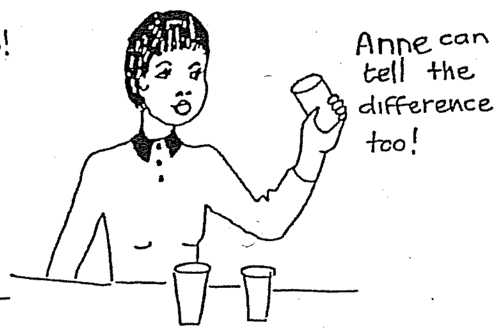
Mavis is given three cups of cola. Two cups contain one brand of cola. A third cup contains a different brand, but she is not told which one this is. Can Mavis distinguish between them and correctly identify the different brand?

Can she really tell the difference though? Or did she guess?



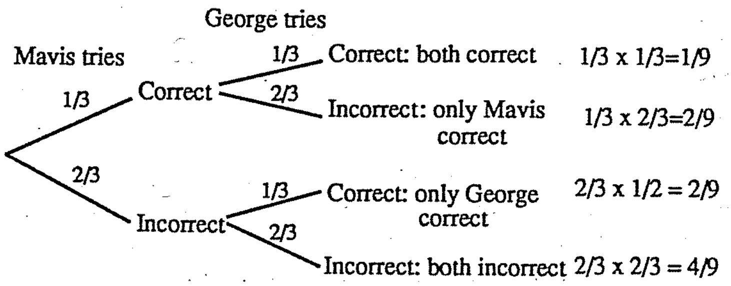
If Mavis really cannot tell the difference, the probability of her guessing correctly is 1/3.

The probability of her guessing correctly is quite high then. Suppose we let someone else try. George has a go. What is the probability that they could both be guessing?



If two people can correctly distinguish between the brands of cola this is much more convincing. There is only a probability of 1/9 of them both guessing correctly. So it does seem much more likely that their correct answers are based on a real difference.

Is this convincing enough?



How many people do we need to make a correct identification for us to be convinced?

Is this sufficient evidence?

A tree diagram is beginning to look rather complicated.

TABLE 1

None correct	1 correct	2 correct	3 correct
I I I	C I I I C I I I C	I C C C I C C C I	C C C
One outcome	Three outcomes	Three outcomes	One outcome
2/3x2/3x2/3 = 8/27	3x2/3x2/3x1/3 = 12/27	3x2/3x1/3x1/3 = 6/27	1/3x1/3x1/3 = 1/27

Here we can notice the symmetry of the arrangements for one and two correct identifications, and develop strategies for ensuring that all possible outcomes are listed (2^3 in total).

Three correct identifications is pretty convincing then. The probability of three people getting this result by guesswork is only 1 in 27. What if four people took the test and three of them were correct? Would we be as convinced by that result?

The tree diagram for this situation is beginning to become rather fearsome. Let's try the other method:

TABLE 2

None correct	1 correct	2 correct	3 correct	4 correct
I I I I	C I I I I C I I I I C I I I I C	I I C C C C I I C I I C I C C I C I C I I C I C	I C C C C I C C C C I C C C C I	C C C C
One outcome	Four outcomes	Six outcomes	Four outcomes	One outcome
2/3x2/3x2/3 x2/3 = 16/81	4x1/3x2/3x2/3 x2/3 = 32/81	6x2/3x2/3x1/3 x1/3 = 24/81	4x2/3x1/3x1/3 x1/3 = 8/81	1/3x1/3x1/3 x1/3 = 1/81

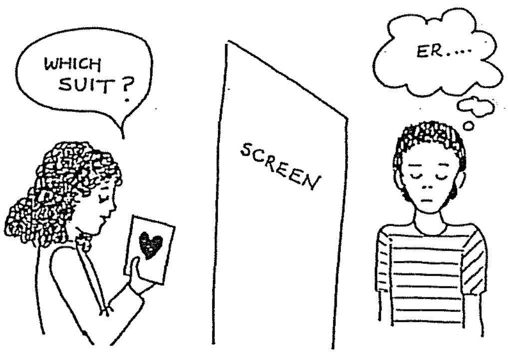
In this situation we would certainly be convinced by all four people giving correct answers. But what about three?

Here, three correct out of four is actually less convincing than three correct out of only three.

By now our pupils are calculating binomial probabilities almost without realising it. As listing all possible outcomes perhaps becomes more difficult for larger numbers of trials, pupils will be fascinated by Pascal's triangle and enjoy using it.

3. "Are you a mind-reader?"

This is another experiment popular with all pupils. Again, results can be analysed in the same way using the binomial model. Ordinary playing cards can be used.



If the experiment consists of the subject having to identify (or guess?) the suit of 10 cards, how many must they get right in order for us to believe that they can read the experimenter's mind?

The binomial probabilities for someone guessing correctly can be calculated using Pascal's triangle for the binomial coefficients, or read off from a table.

The probabilities for 10, 9, 8, or 7 correct responses are all very close to zero. If a subject gets this many right it is extremely unlikely that they are guessing. These results all seem very convincing. Just how far down can we go?

Is an accumulated probability of 0.08 (8%) sufficiently impressive? In that case we can accept results down to five correct responses. If we insist on a probability of 0.05 we can only accept results of six correct or better.

4. Other problems and the sign test

The possibilities for experimentation and hypothesis testing can be opened up still further by introducing pupils to the sign test. This test can be used to answer any of the following questions (and any others which involve comparing two measurements for the same person).

(i) Can people catch a ball better with their right hand or left hand?
(ii) Are people better at remembering sequences of numbers or sequences of letters?
(iii) Are females better at verbal tests or spatial tests?
(iv) Does a period of practice improve people's skill at a simple task, e.g. tracing a simple shape or sorting objects?
(v) Is a person's reaction time faster if they use their writing hand as compared to their non-writing hand?

The sign test is easy to use and pupils who can calculate binomial probabilities can progress on to this, providing the underlying logic is explained. These are the results of a group of girls who tried two tasks: one testing spatial ability, and one testing verbal ability. In order to test whether girls are faster at the verbal task we need only count how many do have faster times, and then decide whether this is a convincing result.

TABLE 3

Girl	Time on verbal test	Time on spatial test	Faster at verbal test?
A	14	16	√
B	25	24	x
C	18	19	√
D	30	31	√
E	27	32	√
F	18	20	√
G	17	18	√
H	24	23	x
I	17	20	√
J	25	29	√

The model to be used to analyse the results here is the binomial distribution n = 10 and p = 1/2. The reader may like to check that eight girls would need to have faster results on the verbal test for us to be convinced.

Reference

Rouncefield, M and Holmes, P (1989) *Practical Statistics.* Macmillan.