Counting Noses and Scary Things : Children Construct Their Ideas About Data

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1. Introduction

At every point in their development, students are engaged in serious intellectual work as they attempt to construct their own understanding of the world and their relation to it. As part of this work, they are immersed in mathematical ideas which are just at the edge of their understanding. In this paper, I will first discuss the nature of the mathematics in which the child in the primary grades can engage in the context of data analysis, and then give some examples of children's work in this area to illustrate how young children must construct for themselves key processes which are the building blocks of collecting, describing, and interpreting data.

2. Data analysis in the primary grades

The mathematics within data analysis investigations is different for the primary child than it is for 9-12 year olds. Unlike upper elementary students, 4-8 year olds are not quite ready to pull their attention away from individuals in order to summarise or describe a whole group. What interests these young students is most often the particulars of the data. Therefore, when we look for the critical mathematics in which young students can engage, it is not the central processes of descriptive statistics which are salient to them. Describing and summarising data become more powerful for the students later, around 9-10 years old, as they begin to spontaneously view the whole data set, rather than the individual, as the unit of description.

In what sense, then, can we say that young students can engage in data analysis? In fact, the mathematics which is compelling for them is at the core of data analysis. For primary grade students (kindergarten through third grade in the United States), the focus of data analysis work is in three areas which are closely tied to the central mathematical issues with which students are passionately involved during these years:

(1) What and how do I count? (2) What and how do I measure? and (3) What goes with what? The first two questions focus on the two basic processes for quantifying the physical characteristics of the world - counting and measuring. The third emphasises classifying, which underlies many of the essential processes of mathematical thinking, including defining, comparing, and conjecturing.

Data analysis provides a context in which central mathematical ideas in these three areas - counting, measuring, and classifying - can be played out. It offers problems in which children can construct for themselves such critical pieces of mathematics as one-to-one correspondence or the notion that a single unit can be iterated to measure length. At the same time, students are truly engaging in the fundamental tasks of data analysis.

3. How can we find out who has birthdays in the same months? : connecting compelling topics with compelling mathematics

While much of the data analysis work currently done in primary classrooms successfully taps compelling child-centred, non-mathematical *topics*, it does not connect these topics to the key mathematical issues appropriate for the child. It is not difficult to find questions about which to collect data which interest young children - what you had for breakfast, how you got to school, how old you are, when your birthday is, how many brothers and sisters you have, what pets you have - all these intensely personal topics are of great interest to the young child. Children enjoy collecting these data and posting them in some way in the classroom, so that they can point to a graph and say, "That's me riding my bicycle to school" or "That's me, I'm 5 and 1/2". There is much of educational value here - students develop both oral and written language, learn about social and cultural diversity, and do a bit of counting while undertaking such activities. But these investigations provide an opportunity for much more mathematical content to be accessed through this work.

Making a birthday graph is an example of an activity in data analysis which typically does not go far enough in engaging students to do their own thinking about the processes of organising and describing data. When I taught primary grades, like most other teachers I know, I regularly made a birthday graph with my class. Sometimes students each wrote their name and drew a picture on an index card, and then I lined up the cards on a list of the months from January through December. Sometimes I simply asked students to raise their hands as we read down the list of months, and we wrote the students' names or put checkmarks next to the appropriate month. We posted the graph on the wall and noticed how many students' birthdays were in each month. Students were relatively interested in this activity - after all, students' birthdays are important to them - but very little new thinking about ideas in mathematics or data analysis took place.

How could this ordinary activity be conducted so that students become more engaged with what these data mean and how they might be represented? In one first grade (6-7 year olds) classroom, the teacher began by having each student write his or her name and birthdate on an index card - just the way I began in the old days. But instead of immediately constructing a graph with these in the January through December order, she asked a question: "How can we find out who has birthdays in the same months?"

Students came up with a number of suggestions and finally decided that everyone with the same month should stand together. Although she knew there would be a minute or two of relative chaos, this teacher, with no further instructions, asked the students to group themselves. Some students wandered around with their index cards looking for a match. Finally, some groups established territories in particular corners of the room. They sent out scouts to find others who belonged with them. Sometimes the scouts from one month found other members but forgot to come back to the original group, so that there were two groups for the same month. But, eventually, the students had grouped themselves successfully, and after some time for looking around the room and commenting on what they could see ("I'm the only May", "There's the most in October"), the teacher suggested they make a permanent display. Using the students' suggestions, they clipped their birthday cards, in their groups, to a clothesline.

In a second session, the teacher asked a new question: "If we wanted to know whose birthday is coming next, and then what's the next one and the next one, what could we do to our display so that we could tell?" This question led to a discussion of how many months there are, the order of the months, and the fact that some months were missing from the display because no students had birthdays in that month. The students eventually decided to reorder the display so that it began with the current month (October), with each month following in order, and to move October to the "end of the line" when it was over, and so forth, so that the current month would always be first and they would know whose birthdays were in that month. As one student observed, "You can't stop the months. The months just keep going, going, going, going." Following students' directions, the teacher then re-ordered the groups in each month by their dates. As they did this, she probed students' reasoning about the orders they suggested and encouraged comments from the students. Because the teacher guided the students in deciding the nature of the chart for themselves, they were able to grapple with the issues of classification and sequencing that we might have, unthinkingly, decided for them.

4. What are we scared of? : making decisions about data

We have two choices in undertaking data analysis work with students: we can lead them to organising and representing their data in a way which makes sense to us, or we can support them as they organise and represent their data in a way which makes sense to them. In the first case, they learn some rules - and they learn to second-guess what they are supposed to do. In the second case, they learn to think about their data.

Students need to construct their own representations and their own ways of understanding, even when their decisions do not seem correct to adults. Students in the elementary grades usually have few experiences, such as the construction of the birthday chart discussed above, in which they develop and discuss their own representations of data. We should not, then, be surprised when, by the age of 9 or 10, many students have great difficulty constructing even a simple graph for themselves. They do not know how to choose an appropriate representation, are unable to plan a reasonable scale so that their graph will fit on their paper, and have no sense of how to sketch their data to get a quick, informal look at them. They spend inordinate amounts of time tediously colouring in squared paper to make elaborate bar graphs which tell them nothing that they did not already know about data which might have been represented more appropriately in a table (e.g. dogs 12, cats 14, turtles 3).

Classification of data is another area in which students make choices which are different from those adults might make. In both upper elementary and primary classrooms, we have observed teachers' attempts to protect students from difficulty and complexity by suggesting to them ways of simplifying their categories even before they have collected their data (Russell and Corwin, in press). When students work out strategies for classifying data, their categories reflect their own ways of viewing and understanding their world. For example, several groups of second grade students (7-8 year olds) undertook an investigation of scary things. They collected data about what was scary for them and what had been scary for their parents when they were the same age. In order to compare their own data with the data they had collected from their parents, the students sorted their data into categories. They talked intensely about what "scary things" should be grouped together. In one group of 25 or so students, there was a surprisingly high degree of agreement about some categories which seemed quite strange to the adults listening to their discussion. For example, they agreed to include ghosts, skeletons, rats, demons, goblins, bats, maggots, and Dracula in a category which they called "Haunted House", hardly a grouping we adults would be likely to make, yet it has a compelling integrity from a child's eye view. Students found that some data seemed to fit in more than one category or that a particular piece of data did not seem to fit with anything else. They grappled with these common and legitimate problems in classification, not always solving them in what we might consider the "right" way, but they were thoroughly engaged in thinking hard, at their own level, about the process and difficulties of classification.

5. Do our chairs fit? : supporting confusion

Allowing students to construct their own understanding requires a willingness to hold back from providing a structure which we know will solve the student's problem. We must be willing to tolerate the discomfort of confusion and frustration in the midst of solving a problem and must give students time to work through their own confusion (Duckworth, 1987). For example, in one third grade classroom (8-9 year olds) in a large urban school district, students had usually encountered measuring activities of a well-structured nature: "How many inches long is this line?" "How much longer is Line A than Line B?" They had little experience with problems in which they themselves had to decide on the relevant measurement. As a result, when they became involved in the problem, *Do Our Chairs Fit Us?* (Corwin and Russell, 1990), they were quite unsure of how to approach it. Their teacher asked several students to try different-sized chairs and asked the class to think about what data are important in deciding whether their chairs fit:

> *We've found that Maria does real well in this chair ... What does it have to do with, making the chair fit and you fit in the chair?*
> Wanda: The body.
> *What do you mean about the body?*
> Wanda: She's small.
> *She's small. But I could say Jose's small, too, but he doesn't fit well in that chair.*

[Jose sits in Maria's chair. He's too big. Everybody giggles.]
Students: He doesn't fit.
Why, but why?
Students: He's bigger.
But what does that mean, he's bigger?
Carmen: Because his legs and his knees are sticking up.

As the discussion continued, the teacher challenged students to develop clear notions about size. This discussion was at first difficult for the students; they did not experience immediate success and satisfaction. However, they gradually began to articulate pieces of what seemed important to them about matching people with chairs: "When your knees are not sticking up", "When your feet are right on the floor". By allowing time for confusion and uncertainty, this teacher allowed her students to come to a consensus, constructed out of their own thinking rather than copied from an adult, about how to collect and organise their data.

6. How many noses do we have? : the complexity of simple ideas

In order to support students in doing their own mathematical thinking, it is critical to listen to students carefully without assumptions about what is "obvious" or "simple" in order to understand what mathematical ideas they are working on. In our work with students and teachers, we are constantly reminded that the simplest mathematical idea is wonderfully complex and surprising when viewed through the child's mind. Keeping open to the views of the child so that her thinking is not invalidated by the correct, adult view, is difficult and challenging. A group of 4-6 year olds collect data on the number of various body parts in the room (Russell and Stone, in press): "How many noses are there?" "How many mouths are there?" "How many eyes are there?" To us it is obvious and certain that if there are 21 students in the room, there are 21 noses; and, of course, there are 42 eyes, legs, and so forth. The questions are foolish from an adult perspective. But the one-to-one and two-to-one correspondences embodied in these questions are, for some of these 4-6 year olds, still in question and, for others, even if the relationships are clear, that very clarity is fresh and new and interesting. Here is a piece of the conversation in this classroom which followed a session in which each child made a nose out of clay to represent his or her own nose:

And how many noses did we have?
Anne: 21.
Ricky: Two of these ... two nostrils.
Two nostrils. OK. But how many actual noses?
Pat: 21.
And what did that tell us about how many kids are in the class?
Chris: 21.
Why? Why were there 21 kids if there were 21 noses?
Adam: Because every one in the whole class made one and that is the same number as the kids in the class.
What a great way of wording it. Did someone else have something they wanted to add to that?
Paul: That we counted them up and we added them.

We counted them. So we had them out here and we counted them and we came to 21? And why did that mean that there were 21 children in the class?
Paul: Because they each had one nose and all the noses from 21 kids would equal 21 noses.
Would equal 21 noses. Now Ricky just said - remember what these are called?
Children: Nostrils.
Nostrils. Does that mean that there were 21 nostrils altogether?

This brief excerpt provides a sense of the kind of thinking, explaining, and describing that students are doing for themselves. The teacher was careful not to turn the students' discussion into a search for the right answer, as in the following exchange:

And did you count all the noses?
Chrissy: Yes.
And did this show that there are 23 of us?
Carlos: Yes.
How do you know?
Carlos: Because we counted them.
Show me.
Carlos: 1,2,3,... [continues counting] ... 21,22,23,24 ... no that's too many.
Let's count again together [points and counts with the students] *1,2,3,..., 21,22,23! So does this show there are 23 of us?*
Chrissy: Yes.

There is a subtle, but critical difference between the two episodes. In the first, students are articulating the ideas which they have come to after their counting activities. In the second, the students may not be at all sure that the number of noses matches the number of students in the class. They do know that the count comes out to 23 when someone helps them count, and that 23 is "supposed" to be the answer. The teacher seems convinced that by showing them that the count is 23, they will understand the seemingly obvious one-to-one correspondence involved in this situation.

Despite the potential for the pursuit of mathematical understanding in the context of data analysis in the primary grades, simply including the collection and representation of data as activities in primary classrooms may not lead students to become engaged in mathematical thinking. As adults, we make two kinds of mistakes. First, we fail to recognise the complexity of the apparently obvious in the mathematics of young children. And because it is so hard for us to see why the child cannot see, we make our second mistake: we tell the child, we lead her, we make her understand the obvious instead of honouring the child's need to do her own intellectual work and construct her own truth.

7. Review

From our work in the primary grades, we have learned or re-learned key principles about data analysis for these students:

(i) For primary students, data analysis activities are closely connected to key mathematical ideas involved in the processes of counting, measuring, and classifying.
(ii) Students learn about data collection and analysis when they construct their own categories, create their own representations, and talk about their own interpretations.
(iii) Tolerating confusion is part of thinking about data.
(iv) Nothing is simple: the more we listen to students, the more we understand about the complexity of the most basic mathematical ideas.

What has been fascinating to us as we work with young children to integrate data analysis into their mathematics learning, is the extent to which Bruner's (1960) statement, "that any subject can be taught effectively in some intellectually honest form to any child at any stage of development", truly fits the work that students do with data. While data analysis problems which engage young children deal with smaller sets of data, smaller numbers, and more personal topics than those which typically engage statisticians, many of the issues raised are of the same general nature. Deciding what to count or measure, how to count or measure, and how to classify and represent the results, are problems which lead the young student, just as they do the statistician, into complex and challenging mathematical thinking. By listening carefully to students we begin to see the complexity of their thinking and, as we learn to see through their eyes, we learn to ask questions which will help them think harder rather than parrot what they think we want to hear.

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