Probability and Statistics in the Victorian Certificate of Education

Gail FitzSimons and Robert Money - Melbourne, Australia

1. Introduction

The study of statistics in the post-compulsory years (11 and 12) in Victoria, Australia, has increased dramatically as a result of the introduction of the *Mathematics Study Design* (VCAB, 1990) within the Victorian Certificate of Education (VCE). The VCE is being phased in over a number of years and from 1991 will encompass all Year 11 and 12 mathematics in the State. Essential features of the innovation are its state-wide nature and the strong link established between three key work requirements and four common assessment tasks. This paper provides an overview of these developments, with particular emphasis on the statistics education component.

2. The *Mathematics Study Design*

The *Mathematics Study Design* for the VCE is based on the premise that mathematics should apply to real and meaningful situations, and should meet the needs of all students and a majority of people in their working lives. Its introduction is having a radical effect on senior classes, which have generally been fed on a steady diet of abstract content segments, followed by skills practice, ending up with applications related closely to the latest dose of theory. Students seem to have relied on rote learning, and a "recipe book" approach to solving problems. Higher education institutions and employers frequently complained that students do not know how to think. VCE Mathematics is attempting to change current practice.

2.1 *Structure of the study*

Breadth of study of mathematics is encouraged across up to four blocks, each of which would normally be studied at either Year 11 or Year 12, but not both. The four blocks can be taken as one or two semester units. They are:

(i) *Space and Number* (arithmetic, geometry, trigonometry, and algebra);
(ii) *Change and Approximation* (coordinate geometry, calculus, and algebra);
(iii) *Reasoning and Data* (probability, statistics, logic, and algebra);
(iv) *Extensions* - of any of the above three, by adding a new area of study.

It is through the Reasoning and Data block that statistics is having its greatest impact on the curriculum. While Space and Number is currently the preferred block for students doing only two units of mathematics, Reasoning and Data is being included in the great majority of four to eight unit mathematics programmes. This is a remarkable turnaround from the previous situation where many students had to wait until tertiary study (or never) for their first course in statistics.

2.2 *The work requirements*

Satisfactory completion of units is based on the completion of three work requirements. These are:

(i) *Projects:* extended, independent investigations involving the use of mathematics.
(ii) *Problem-solving and modelling:* the creative application of mathematical knowledge and skills to solve problems in unfamiliar situations, including real-life applications.
(iii) *Skills practice and standard applications:* the study of aspects of the existing body of mathematical knowledge through learning and practising mathematical algorithms, routines, and techniques, and using them to find solutions to standard problems.

Each of these three work requirements must take up at least 20% of the time devoted to each unit and completion of at least one of these work requirements must involve the use of computers or calculators.

These "new" work requirements are particularly suitable to Reasoning and Data, where teachers have been able to find a wide range of topics suitable for investigation and students have been able to build their understandings on a firm basis of real and simulated data. Teachers have been innovative in adapting computer use - particularly spreadsheets - to this purpose. Spreadsheets are more accessible than specialised statistics packages, often illustrate processes more clearly, and can be used in interactive "what if ...?" mode.

2.3 *The Reasoning and Data block*

If the block is studied at "semesters 1 and 2 level" then each unit must contain study of each of the four specified areas of study - probability, statistics, logic, and algebra. Where Common Assessment Tasks are involved, approximately half the year's course must contain specified material with the rest of the course covering at least four of eleven available clusters of additional material.

(A) *Specified Material*

Probability: introduction to probability, random variables, and probability distributions.

Statistics: descriptive univariate statistics and the confidence interval for a population proportion.

Logic: deductive reasoning applied to graphs and networks.

Algebra: relevant to the study of probability, statistics, and logic.

(B) *Clusters of additional material*

All courses must include at least four of the following eleven clusters.

1. Combinatorics
2. Sampling processes
3. Probability models - Bernoulli trials and the Poisson process
4. Time series and economic statistics
5. Correlation and regression
6. Random sampling
7. Estimation and confidence intervals
8. Non-parametric statistics - small sample
9. Logic and proof
10. Boolean algebra
11. Directed graphs

2.4 *Assessment*

Considering planned curriculum change, Burkhardt (1988) believes that assessment has a strong influence on implemented curriculum, as distinct from the actual ideal curriculum. His solution to the problem of public examinations emphasising narrow content skills, which dominate the curriculum, is for assessment to encompass high quality tasks, with the balance matching that of the curriculum.

In VCE, judgement of satisfactory completion of work requirements and of a unit as a whole (i.e. "getting the unit for your certificate") is a matter which is determined by the school. If a block is taken at "semesters 1 and 2 level" (normally Year 11) then assessment of level of performance is determined by the school. If a block is taken at "semesters 3 and 4 level" (normally Year 12) then assessment of level of performance is determined through four Common Assessment Tasks (CATs), each of which is graded on an A to E scale. These are designed to be:

(i) accessible yet challenging for students with a wide range of different abilities;
(ii) appropriate to a range of different courses which can be developed within each block;
(iii) closely linked to the key work requirements of the study.

The first two of these tasks involve an "investigative project" and a "challenging problem", respectively, and are assessed through grade descriptions and an Assessment Checklist. The second two tasks are assessed through multi-choice and open-book examinations set by the Victorian Curriculum Assessment Board (VCAB). The four tasks are described in greater detail in Section 4.

3. The VCE Mathematics classroom

The VCE *Mathematics Study Design* has been written so that teachers have the time to implement good pedagogical practices - there is not the same pressure to cover such a vast amount of content. The *National Statement of Principles for Mathematics in Australian Schools* (Australian Education Council, 1990) suggests that learning is likely to be enhanced by:

(i) activities which build on students' experience;
(ii) feedback to students;
(iii) discussion and collaboration;
(iv) activities which the learner regards as purposeful and interesting;
(v) talking and writing about one's ideas;
(vi) challenge within a supportive framework.

In addition to these, richness in mathematical activities is enhanced by the use of concrete materials, videotapes, calculators, and computers as integral parts of the lesson.

For example, to practise the skills of data collection, students at Box Hill College of Technical and Further Education conducted a survey of the smoking habits of other college students; formulating their own questions, displaying and analysing the data. To contrast experimentation with surveys, students designed and conducted an experiment to determine the optimum slit length for a gyrocopter, learning about experimental techniques as they needed them. Using data from their own families, students learned a variety of methods for presenting and analysing data, including stem-and-leaf diagrams and boxplots, and were able to display them on the computer. Students measured first each other, and then a second group, collecting data on height, arm span, and head circumference, in order to gain practice in displaying bivariate data, drawing lines of best fit, and calculating correlations.

To model the finding of probabilities, simulation techniques were investigated, using material from the Quantitative Literacy Series, prepared in association with the National Council of Teachers of Mathematics (USA). On the Australian scene, the NCTP materials have proved extremely successful in involving students in memorable lessons, leading them to discover for themselves probabilistic insights about Tattslotto, bubblegum card collections, and the final five of football teams. Although each of these activities can be used with lower grades, they lend themselves to more sophisticated analysis. Later in the year, students simulated the spread of an epidemic, with assistance from the Key Centre for Statistics at Melbourne University.

The work on graphs and networks was a welcome relief for students after so much probability and statistics. They enjoyed the practical nature of the material, and had an intuitive feel for it.

4. The Common Assessment Tasks

4.1 *CAT 1 : Investigative project*

This is a project based on a theme set annually by the VCAB and undertaken as part of the project work requirement for the first unit taken (Unit 3). The purpose of the task is (*Mathematics Study Design*, p.54):

"to enable students to demonstrate their ability to
* carry out an extended piece of independent work
* define important variables
* simplify complex situations
* formulate useful questions
* formulate and interpret problems mathematically
* seek out and use available resources
* synthesise and analyse information
* organise, structure and communicate mathematical ideas and results."

In 1989, the theme for the investigative project was "Prediction of Uncertain Events". Students chose such diverse topics as the prediction of: primary school enrolments in the Northcote-Thornbury area; world record and Olympic times for the men's and women's 100m and 200m sprints; when women would make up 50% of the Australian workforce; the likelihood of a person attaining a black belt within three years of beginning karate classes; whether women's sporting times will ever be as good as men's; how much money banks and building societies will lend in 1990; Australia's population in the year 2000; and life expectancy. Some of these projects would have been ideal as joint projects with other subject areas. Students worked at their projects with an enthusiasm we have never seen previously in conventional mathematics classes; they set about visiting libraries (college, municipal, and state), contacting public and private information sources, cooperating with one another to share their skills. Most students presented an attractive piece of work, showing pride in their achievements. This project presented an ideal opportunity for the students to see a concrete use for the skills they were learning, and there was a real need to learn more about data collection, analysis, presentation, and interpretation.

4.2 *CAT 2 : Challenging problem*

This task comprises a problem selected from a number of problems set annually by the VCAB and undertaken over a specified two weeks in Term 3 as part of the Problem Solving and Modelling work requirement. The task is intended to assess students' ability to understand mathematical problems, to use a number of strategies in solving problems and/or constructing models and to interpret results.

The problems were set so that everyone could make a start, and encouraged students to look for patterns or relationships. Those who found solutions were expected to justify them, providing a meaninful use for algebra, and in some cases, probability theory.

One of the problems available in the 1989 Trial CAT for Reasoning and Data was:

"Runs of heads and tails

When you toss a fair coin a number of times you get a sequence of heads (H) and tails (T). If you toss the coin a large number of times you might expect the number of heads and the number of tails to be about equal. However, when you repeatedly toss a coin you will also notice that you get runs of heads and runs of tails of varying length.

One possible outcome of tossing a coin five times is HHHTH. This sequence contains a run of three heads, a run of one tail, and a run of one head, a total of three runs.

* Investigate the distribution of runs of different length for the 5 coin situation.
* How does the number of coin tosses affect the distribution of runs of heads and tails?
* What if there was a different number of equally likely outcomes (for example if you were rolling a die instead of tossing a coin) or if the outcomes were not equally likely (for example if you were using an unfair coin)? Investigate the distribution of runs in these situations."

4.3 *CAT 3 : Facts and Skills task*

This consists of a set of about 50 multiple-choice questions undertaken under test conditions (90 minutes plus reading time) near the end of Term 3. Approximately half of these questions are on content which must be included in all courses; the other questions are grouped in clusters from which students choose those which are appropriate to the additional material covered in the courses they have undertaken.

The task is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms, and their ability to apply concepts and skills in standard ways. The following example, from the 1989 Reasoning and Data paper, illustrates the "cut and dried" nature of this paper, in contrast with that of CAT 4.

"For a large set of sales figures (y), it is found that the sample mean is 22.30, and the sample standard deviation is 2.50. Which one of the following intervals contains about 95% of the sales figures?

A. $9.80 < y < 34.80$ B. $14.80 < y < 29.80$ C. $16.05 < y < 28.55$
D. $17.30 < Y < 27.30$ E. $19.80 < Y < 24.80$"

4.4 *CAT 4 : Analysis task*

The final task comprises a set of between four and six short answer questions involving multi-stage solutions of increasing complexity. The task is undertaken under test conditions (90 minutes plus reading time) near the end of Term 4. The questions are based on the content which must be included in all courses.

The task is intended to assess students' abilities in interpretation and analysis of this mathematics and their ability to communicate their understandings. It should enable students to demonstrate their ability to:

(i) understand, interpret, and communicate mathematical ideas appropriate to the focus of the block;
(ii) transfer conceptual understandings to new situations;

(iii) analyse complex situations, select suitable solution strategies, and apply logical argument and other appropriate techniques for solution of problems related to the areas of study and the broad focus of the block;

(iv) make appropriate checks and estimates.

Students were allowed to bring in four pages of written material, so that they could concentrate on thinking mathematically rather than trying to remember slabs of information. This had the effect of making their revision more organised so that they could locate the information easily, and also decreased the anxieties typically associated with this form of assessment.

A typical statistics question (time allocation - about 20 minutes) from the 1989 CAT in Extensions (Space and Number) was:

"The table shows how the sum of squares of differences is used to measure Lou-Lou's performance in predicting an end of season football ladder.

Final position	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Lou-Lou's prediction	1	5	2	3	4	9	7	11	6	8	10	14	12	13
Difference, d	0	3	1	1	1	3	0	3	3	2	1	2	1	1
Squares of differences, d^2	0	9	1	1	1	9	0	9	9	4	1	4	1	1

$$\text{Lou-Lou's score, } s = \Sigma d^2 = 50$$

(a) Using this method, what would be the score for the best possible prediction?

(b) Calculate the score for the following prediction.

Final Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mr Football's prediction	5	9	1	6	10	3	7	11	8	2	12	13	14	4

(c) Calculate the score for the worst possible prediction.

(d) These scores are to be adjusted or plotted on a linear scale or graph which gives a measure of success, p, of +1 for the best possible prediction and -1 for the worst possible prediction. Find the measure of success for Lou-Lou's score.

(e) Obtain a formula for the measure of success, p, for any score s.

(f) Extend the formula to allow for a competition involving an even number, n, teams.
[Note that $1^2 + 3^2 + 5^2 + 7^2 + ... + (n-1)^2 = n(n^2-1)/6$.]

(g) If we ignore final positions and instead use differences between Lou-Lou's and Mr Football's predictions, what interpretation could then be given to the value of p found?"

5. Conclusion

The radical changes to senior mathematics in Victoria are intended to have the effect of making this most feared subject accessible and meaningful to a great many more people. Students will gain maximum benefit if the course material can be related to other subjects or their personal lives, and if it is presented in the most practical way possible with abstractions developing naturally, instead of the other way round. The students involved in the 1989 trial year grew in confidence, developed a spirit of cooperation, and actually enjoyed mathematics lessons.

Acknowledgements

We wish to acknowledge the work done by staff of the Victorian Curriculum and Assessment Board, in particular the members of the Mathematics Project Team and the four Mathematics CAT Panels. We have drawn heavily on their work but take full responsibility for the opinions expressed in this paper.

References

Australian Education Council (1990) *A (National) Statement of Principles for Mathematics in Australian Schools.* (Draft)

Burkhardt, H (1988) National testing - liability or asset? *Mathematics Teaching* **122**, 33-35.

Gnanadesikan, M, Scheaffer, R L and Swift, J (1987) *The Art and Techniques of Simulation* (Teacher's edition). Dale Seymour Publications, Palo Alto, California.

Lipson, K, Sharpe, K and Watson, R (1990) *Projects in Probability and Statistics.* Statistical Education Unit, The Mathematical Association of Victoria, Parkville.

Lovitt, C and Clarke, D M (1988) *Mathematics Curriculum and Teaching Program (MCTP) Activity Bank* (Vols 1 and 2). Curriculum Development Centre, Canberra.

Money, R and Sentry, K (1990) *VCE Mathematics - The 1989 Trials of Common Assessment Tasks.* Paper presented at the 13th Annual Conference of the Mathematics Research Group of Australasia.

Money, R and Sentry, K (1990) *Common Assessment Tasks in VCE Mathematics.* Paper presented at the 13th Biennial Conference of the Australian Association of Mathematics Teachers.

Victorian Curriculum and Assessment Board (1990) *Mathematics Study Design.* VCAB, Melbourne.

Victorian Curriculum and Assessment Board (1990) *Course Development Support Material.* VCAB, Melbourne.