# DIFFICULTIES IN LEARNING PROBABILITY AND STATISTICS

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In the literature on education in probability and statistics, different issues of difficulty have been addressed rather independently by individuals from three different disciplines: college statistics faculty, specialists in pre-college mathematics education, and psychologists. A fairly complete bibliography of all three categories appears in Garfield & Ahlgren (in press).

The literature produced by teachers of statistics at the college level primarily complains that students in introductory "service" courses are not learning what they should and can not apply what they do learn to unfamiliar problems. There have been calls for new approaches to teaching statistics, such as problem solving or microcomputers, but there has been little empirical research to verify improved student learning achieved by the recommended approaches.

In the area of precollege mathematics education, the literature contains a mixture of the following (Garfield & Ahlgren, in press):

1. statements about the need for statistics instruction;

2. descriptions of the role statistics can play in school curricula;

3. suggestions for how to teach statistics;

4. descriptions of the difficulties students have in understanding concepts in probability and statistics;

5. descriptions of intuitive ideas that students already have.

(The intuitive ideas are often called "misconceptions" but sometimes, in recognition that some are a pervasive mode of thinking in humans, are efforts by mathematics educators to understand difficulties students have in areas related to probability and statistics, such as rational number skills, proportional reasoning, and problem solving ability.

Most of the actual research on difficulties in understanding probability appears in the work of psychologists, who initially seem to have seen their task as identifying common errors in probabilistic reasoning. (Some of these researchers also taught statistics courses at the college level to students in education and psychology and experienced firsthand the effects of students' faulty thinking.) More recently, their interest has been less in the error aspect and more in studying the nature of the prevalent intuitive preconceptions.

Research in science education may be further along in studying misconceptions, and offers some insights. Researchers in physics, chemistry, and biology education have found that misconceptions persist despite instrucanswer are. The artificial contexts typically considered in instruction (e.g., balls and urns) often may not support any questions that have any real meaning for the students, and so offer no opportunity for reorganizing their thinking. (For example, most students appear to have little interest in **series** of events, and focus instead on yes/no outcomes of **single** situations.)

## Implications for Teaching

The instructional implication of these distinctions is that teachers must determine where the difficulty lies before they can help the students. If the concept is abstract and intrinsically difficult, the students will need more experience with it, including exposure to different representations of it. If the students lack requisite mathematical skills, remedial work will be necessary first. If the students have interfering intuitions, they should be exercised rather than suppressed, and the practical superiority of the new conception should be demonstrated in contexts that the students care about. And if the questions themselves are misinterpreted, sufficient time should be given to students to talk about their thinking to allow detection and revision of their interpretations.

The multiplicity of possible underlying reasons for students' difficulties greatly complicates the teacher's task. Yet proceeding without diagnosis almost surely will be fruitless. What is needed first is that teachers themselves be well informed. They should correctly understand the concepts and be aware of the different sources of difficulty that students may have.

Teachers should also listen, a great deal more than they now do, to their students' explanations of their answers. Students will have to be encouraged to express their ideas in a non-threatening environment, with the teacher and with one another, so that their ways of thinking can be revealed. Eventually research may lead to diagnostic tests for concepts in probability and statistics, but for the present we know of no other means than interview and discussion.

## Implications for Research

The research, for its part, should expand in scope. Not only does much remain to be learned about how students actually think, but we know almost nothing about how they change their thinking. Longitudinal studies are needed that document the steps that occur in increasing sophistication of statistical reasoning – when it occurs. As part of this, the trial of new curriculum should include penetrating evaluation of how students' thinking is (or is not) changed.

The research should also become more cross-disciplinary and collaborative. Cross-disciplinary influence is already occurring: some mathematics educators have begun to see their task as something like the clinical psychologists'; some philosophical analysts have begun to analyze the logic of intuitive human thought as well as that of mathematics; some test makers are trying to craft instruments that not only determine correctness but also unveil viewpoints; and some psychologists have begun to devise their investigations in ways that illuminate issues of instruction. Progress might be more rapid, however, if there were more real collaboration – if psychologists, educators, and mathematicians (perhaps even social anthropologists) were to design and interpret research together.

## Selected References


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