USING SIMULATION TO MODEL REAL WORLD PROBLEMS

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Introduction

The primary source of the material used in this presentation is The Art and Techniques of Simulation, a book from the Quantitative Literacy Series. The series was written by members of the American Statistical Association-National Council of Teachers of Mathematics Joint Committee on the Curriculum in Statistics and Probability and funded in part by a grant from the National Science Foundation. These techniques are designed for use in middle school through senior high school. They feature statistical topics that are important to students, a wealth of hands-on activities, real data sets and active experiments which motivate student participation, and graphical methods instead of complicated formulas or abstract mathematical concepts. In particular, simulation is introduced as a technique for solving probability and statistics problems.

Objectives

Practical problems from the very simple to the most complex can be solved (or at least approximated) by using simple simulation. The simulation procedure involves conducting experiments which closely resemble an actual situation in order to provide answers to real life problems.

A hands-on approach in which the students actively participate in obtaining results is used. Beginning with simple models where emphasis is placed on estimating the probability of an event, a gradual progression eventually leads the students to the point where they are able to apply methods involving statistical inference to draw conclusions about their results. Students are taught a step-by-step procedure for applying the simulation technique which helps them answer questions about the behavior of real processes under varying conditions. Many of the applications discussed here can readily be adapted to the computer, but the approach is not computer dependent.

A Simulation Model

This method consists of an eight-step process which is outlined below:

Step 1:

State the problem clearly so that all necessary information is given and the objective is clear.

Example: Two evenly matched baseball teams play each other for a series of seven games. Estimate the probability that team I will win the series by winning at least four games from team 2.

Step 2:

Define the simple events which form the basis of the simulation.

Example: The seven games form a series of seven simple events, each of which can be simulated very easily.

Step 3:

State the underlying assumptions which simplify the problem so that a solution can be found.

Example: We make the assumption that the teams are evenly matched so that for any single game (simple event) the probability that team 1 will win is 1/2. We also assume that the games are independent so that the outcome of any one game is not affected by outcomes of the previous games.

Step 4:

Select a model for a simple event by choosing a device to generate chance outcomes with the probabilities as dictated by the real event.

Example: Since the probability that team 1 wins a game is 1/2, we can model a single game by tossing a coin and letting a head represent a win for team 1 and a tail represent a win for team 2.

Step 5:

Define and conduct a trial which consists of a series of simple event simulations that stop when the event of interest has been simulated once.

Example: Since teams 1 and 2 are to play a seven game series, tossing a fair coin seven times would model a single playing of the series.

Step 6:

Record the observation of interest by tabulating the information necessary to reach the desired objective. Most often, this simply requires a notation of favorable or nonfavorable for each trial. Occasionally, a numerical outcome will be noted.

Example: After tossing the coin several times, we observe the number of heads. If the number of heads is at least four, the trial is classified as favorable to the event that team 1 wins the series. It might be useful to keep a record of the number of games won by team 1 on each trial.

Step 7:

Repeat steps 5 and 6 at least 50 times. An accurate estimate of a probability from empirical results requires a large number of trials. If the simulation is done with the aid of a computer, then 1000 or more trials can be run without any inconvenience.

Example: Toss the coin seven more times and record the number of heads. Repeat this procedure until at least 50 trials of seven coin tosses are obtained.

Step 8:

Summarize the information and draw conclusions. We can estimate the probability of an event of interest, A, by evaluating:

$$\frac{\text{the number of trials favorable to A}}{\text{the total number of trials in the experiment}}$$

Example: We can estimate the probability that team 1 wins at least four games by evaluating:

$$\frac{\text{the number of trials with at least four heads}}{\text{the total number of trials in the experiment}}$$

A coin provided a simple way to generate outcomes in the experiment above because it was necessary to use a device that would generate two outcomes with equal frequency. Many other devices could be used equally as well as long as there are two outcomes with an identical chance of occurring. For example, we could use a die toss and classify the outcomes as either even or odd.

Simulation When Probabilities Differ From One-Half

In the preceding example we generated the outcomes of the experiment by tossing a coin because each outcome had an equal chance of occurring. Now let us consider a simulation problem in which we have a simple event where the probability is not equal to 1/2. Obviously, the coin is not an appropriate device in this case. However, there are still a number of easy ways to simulate this type of experiment. One possibility, illustrated below, is to use a random number table.

Step 1: State the problem clearly.

Joe runs a bus with eight seats. On the average 10% of the people who buy tickets in advance do not show up. So Joe sells ten tickets for each trip. Estimate the probability that more than eight people show up with tickets.

Step 2: Define the simple events.

The simple events are whether a person holding a ticket shows up for the trip.

Step 3: State the underlying assumptions.

The probability that any one person with a ticket fails to show up for the trip is 10%. Whether any given ticketholder shows up or not is independent of what happens to the other ticketholders.

Step 4: Select a model for a simple event.

Draw a number from a random number table. The number 0 will represent a ticketholder who didn't show up for the trip.

Step 5: Define and conduct a trial.

Since ten tickets are sold for each trip, one trial will consist of drawing ten random numbers representing the ticketholders for one trip. If the first trial resulted in 0 6 4 9 3 1 8 6 6 9 , then nine of the ten ticketholders showed up for the trip.

Step 6: Record the observation of interest.

If nine ticketholders showed up, one person did not get a seat. We then keep a tally of the number of ticketholders who did not get seats for each trip (trial).

Step 7: Repeat step 5 & 6 until 100 trials are completed.

The results of 100 such trials are summarized below:

Number not getting seats	Number of trials
0	26
1	31
2	43
	100

Step 8: Summarize the information and draw conclusions.

The data from step 7 show that more than eight people showed up 74 times out of 100. Therefore the probability that more than eight ticketholders show up for any one trip is estimated to be 74/100 = .74.

Conclusion

Simulation techniques following the same eight step approach can also be devised for situations where the number of simple events in a trial is not predetermined, i.e. the length of a trial changes from one performance to the next, as well as more complex problems where the event of interest may have more than one characteristic. There are numerous examples of practical problems which can be simulated using this approach and which are interesting and motivating for 12 to 18 year old students. A partial listing could include predicting the outcome of sporting events such as basketball games, the results of an election, the outcomes of games of chance, the waiting times in customer lines, and the event of passing or failing on multiple choice tests. Clearly, simulation is an ideal mechanism for providing the teacher with the opportunity to develop a systematic progression from estimating probabilities to drawing conclusions and making inferences.

References

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