PROBABILITY GAMES

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There is considerable, good evidence that games can be effective tools in teaching mathematics and that all games are not equally effective (Bright, Harvey, & Wheeler, 1985). One key to effectiveness may be the degree to which the mathematics content is involved in the play of a game, since there would seem to be a corresponding involvement of the game players with that content.

In order to embed the mathematics content in a game, it is necessary to have a clear understanding of both the game's instructional objectives and the highest taxonomic level (Bloom, 1956) required to play the game successfully This can be combined with an identification of the instructional level of the game. (See Bright et al., 1983, 1985, for the relevant definitions and a discussion of the appropriate techniques.)

Previous Research

Although there has been some in-depth analysis of young children's conceptions of probability concepts (e.g., Fischbein, 1975), there has been almost no research on the effectiveness of probability games with these children, in spite of the fact that there are many suggestions for games that ought to provide effective instruction for these children. Extrapolation of research with older students seems the only way currently to approach the task of trying to plan effective uses of probability games with young children.

Four different probability games have been used in experimental studies. The simplest is a microcomputer game, The Jar Game (Kraus, 1982), designed for elementary school students. The player is shown two "jars" on the monitor screen, each containing green and gold cubes, and asked to select the jar that gives the best chance of having a gold cube randomly selected. If a correct choice is made, the player receives bonus points. Cubes are then randomly selected from whichever jar was chosen by the player, and one point is awarded for each gold cube selected. Two studies have reported treatments using this game (Bright, 1985a, 1985b). In both cases, the subjects were preservice elementary school teachers, and there was no effect on probability learning attributable to the use of the game.

Another game used in research is a set of fair/unfair games (Romberg, Harvey, Moser, & Montgomery, 1976). Students are asked to play each of a pair of games several times, and then to decide if the games are fair or unfair, in the sense of whether one player has a better chance of winning. The games involve dice, chips, or spinners. Bright, Harvey, and Wheeler

(1980) demonstrated that grade 7 students learned to better identify fair and unfair situations through use of these games, even without any instruction from the teacher. In a replication and extension of this research, Bright et al. (1985) showed that for grades 6 and 8, the games were effective at teaching the notion of fair/unfair, but only when the test situations included hypothetical data like that generated in the play of the game.

A third game used in research is Number Golf (Bright, 1980). Students roll dice and add or subtract the sum generated to a cumulative total, with the objective of trying to attain specified "goal numbers." With only one of three groups of college-aged subjects did he find a weak relationship between students' knowledge of probability concepts and the use of a first-order strategy in playing the game. Bright et al. (1985) found learning effects only at a taxonomic level below that of the game with grade 7 and 9 students. However, there was no analysis of the strategies that students may have used in play of the game.

The fourth game is Capture (Schroeder, 1983). This game also involves strategy, and it appears to be the only game used with children aged 6 to 11; his subjects were from grades 4, 5, and 6. Schroeder concluded that some subjects had little difficulty applying probability concepts and explaining the strategies they used, while other students observed that positions on the gameboard did not change hands equally often but were unable to relate this observation to probability. (For more details, see the paper by Schroeder in this volume.)

In summary, there is clear evidence that probability can be taught through games, but the role of students' strategy use may be important for understanding the effects of these games. Although only limited attention has in the past been given to identification of students' strategies, techniques have now been developed which may allow relating strategy use to learning.

Measuring Game Playing Strategy

A game strategy is an algorithm that, when applied at any play of a game, determines the move to be made from among all of the possible moves at that play. This definition coincides with the one inherent in the definition both of an optimal strategy in mathematical game theory and of a winning strategy in artificial intelligence. The algorithms referred to in the definition are the ones used by artificial intelligence game-playing programs whenever it is possible to specify them. For example, in NIM the optimal strategy for the second player is "always pick up enough objects so that the number of objects remaining is equal to 1 (mod 4)."

Although this definition by default applies to expert strategies; that is, winning strategies; many games have less-than-expert strategies that students might use. These are called novice strategies. One of the goals of using probability games seems to be to help students develop better strategies; that is, to develop strategies that become more like expert/winning strategies.

A strategy instructional game, then, is an instructional game (Bright et al., 1985) that has at least one expert strategy which (a) involves the content of the primary instructional objective of that game and (b) requires use of the content at the taxonomic level of the game. Capture and Number Golf are examples of strategy instructional games, but The Jar Game is not, since there is no strategy for winning that game.

The analysis of the strategy used by a student can be made by considering the plays of the game as successive solutions to equivalent sets of problems. Prior to the analysis, the possible expert and novice strategies that might be applied in playing the game must be specified. The analysis uses techniques similar to those used to analyze the content of instructional sequences.

The analysis begins by the development of a move matrix. One column of the matrix is generated for each turn in a game. The number of rows of the matrix is one more than the number of identified strategies. Each row, save the last one, is labeled with one of the strategies; these rows are called strategy rows of the matrix. The entry in row *i* and column *j* is 1 if the *j*th move was consistent with strategy *i*, or 0 if the *j*th move was inconsistent with strategy *i*. The last row is used to record instances when none of the identified strategies is used; 1 is entered in column *j* if the *j*th move was inconsistent with all identified strategies, and 0 is entered if the *j*th move was consistent with at least one of the identified strategies.

The move matrix is then used to generate two numeric values. The first is a 0, 1, or 2, depending on whether (a) no discernable category of strategies was used, (b) novice strategies were most consistently used, or (c) expert strategies were most consistently used. The second value is a ratio of (a) moves consistent with the identified strategy category to (b) the total number of times that category could have been observed. (Since it is possible, for example, that expert strategies might not be applicable to each move, it is necessary to restrict the denominator of this ratio to the largest subset of moves in which the identified category of strategy use could have been observed.)

From these two generated values for each game, a player's use of strategies can be tracked across multiple plays of a game. This tracking is begun with the last few plays of a game, since use of strategies is more likely to be observed as students become familiar with the game. If no category of strategy use can be observed in this analysis, then a weighted average across all plays can be computed as an approximation of the use of strategies.

Conclusions

These research techniques can be used to determine (a) relationships between the development and use of game strategies and the acquisition of content knowledge, (b) whether replays of games affect the development and use of strategies or the acquisition of content knowledge, (c) whether the use of particular strategies by an opponent affects either the development and use of strategies or the acquisition of content knowledge, and (d) whether the development and use of strategies in one game transfers to the development and use of strategies in other games or affects the acquisition of content knowledge in other games. It is, of course, hypothesized that improvement in strategy over many plays of a game would be associated with increased knowledge, but this needs to be verified empirically.

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